Uncertainty in the measurement of LED luminous intensity: an example of the treatment of integral quantities

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ORM Club AGM, 2006
• Traditional method: Calibrate with a photometer – a broad band detector with a $\lambda$ filter

$$I_v = \frac{S_{phot}}{R_{phot}} \times CCF$$

Does not work for narrow band sources like LEDs
Luminant intensity of a LED source from spectral intensity data:

\[ I_v = \int I(\lambda)V(\lambda) d\lambda \]

• Approximate to:

\[ I_v = \sum I(\lambda_i)V(\lambda_i) \Delta\lambda \]

• Calibration of the LED against a tungsten reference lamp of known spectral intensity

NPL
Denote each term in the summation by $x_i$
Then: $C = \sum x_i$

“First Approximation” uncertainty calculation (chain rule):

$$(U_c)^2 = \sum_{i=1}^{N} u(x_i)^2$$

i.e.

$$(U_c)^2 = \sum_{i=1}^{N} (V_i)^2 (u(I_i))^2$$

(since $u(bx) = b.u(x)$)

- a model with $N$ inputs: $C = f(x_1, x_2, \ldots, x_N)$
Problems with this approximation

- Uncertainty due to series approximation:
  - assess by taking larger wavelength interval?

- Correlation!
Example of uncorrelated uncertainty

- Errors in individual data points uncorrelated
BUT: Problem of correlation: $x_i$ and $x_j$ not independent

Correlation due to instrument offset uncertainty: data points affected in "concerted fashion"
For $C = f(x_1, x_2, \ldots, x_N)$: (N input terms)

$$(U_c)^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} c_i u(x_i) c_j u(x_j) r(x_i, x_j)$$

$r(x, x_j)$ is the correlation coefficient. $0 \leq r(x_i, x_j) \leq 1$

$r(x_i, x_i) = 1$, always. (All uncertainty terms correlated with themselves!)

If $r(x_i, x_j) = 0$ for $i \neq j$ (no correlation), gives the familiar:

$$(U_c)^2 = \sum_{i=1}^{N} (c_i)^2 u(x_i)^2$$
• This gives the overall uncertainty for the LED luminous intensity:

\[(U_c)^2 = \sum_i \sum_j u(I_i)u(I_j)V_i V_j r(I_i, I_j)\]
Dealing with partial correlation

• In optical radiation measurements the \( u(A_i) \) typically include correlated (e.g. offset) and uncorrelated (e.g. noise) contributions. Correlation coefficients cannot readily be calculated.

• Therefore break down each \( u(x_i) \) into “sub-components”, \( u(x_i^k) \) where \( r(x_i^k, x_j^l) \) is either one or zero:

\[
(U_c)^2 = \sum_{i=1}^{N} \sum_{k=1}^{n} \sum_{j=1}^{N} \sum_{l=1}^{n} u(x_i^k)u(x_j^l)C_i^k C_j^l r(x_i^k, x_j^l)
\]

• The output is now a function of \( N \times n \) inputs
GUM definition of correlation

- “If inputs $x_i$ and $x_j$ are correlated, and a change $\delta_i$ in $x_i$ is associated with a change $\delta_j$ in $x_j$ “:
- $r(x_i, x_j) \approx u(x_i) \delta_j / u(x_j) \delta_i$
- Thus:
  - for noise in signal, $r = 0$ for all $i,j$
  - for constant wavelength offset $r = 1$ for all $i,j$

- $U(x_i)$ constant for all $i$
- $\delta_i$ constant for all $i$
• Most commonly occurring uncertainty components in optical radiation measurement meet this assumption

• Exception: quantities determined by interpolation. These can be treated separately.
\[
(U_c)^2 = \sum_{i=1}^{N} \sum_{k=1}^{n} \sum_{j=1}^{N} \sum_{l=1}^{n} u(I^k_i)u(I^l_j)C^k_i C^l_j r(I^k_i, I^l_j)
\]

Using \( r(I^k_i, I^l_j) = 0 \) for \( k \neq l \):

\[
= \sum_{k \ (corr)} \left( \sum_{i=1}^{N} u(I^k_i)^2 C^k_i \right)^2 + \sum_{k \ (uncorr)} \left( \sum_{i=1}^{N} u(I^k_i)C^k_i \right)^2
\]

(+ partially correlated terms)
Refined measurement model:

\[ C \equiv \sum_{i=1}^{N} V_i \{I_i\}_{(meas)} = \sum_{i=1}^{N} V_i \{I_i\}_{(true)} + f_1(x_1^i) + f_2(x_2^i) + \ldots + f_n(x_n^i) \]

- \( x_k^i \) denote the input quantities, e.g. wavelength offset, noise, stray light at \( \lambda_i \). The model now has \( N \times n \) inputs.

- Sensitivity coefficients for our refined model are \( \partial C/\partial x_k^i \)

- In view of the number of inputs it’s advisable to write down the model!
Uncertainty budget for each spectral point

<table>
<thead>
<tr>
<th>symbol</th>
<th>Source</th>
<th>Standard Value</th>
<th>Sensitivity coefficient</th>
<th>Correlated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Calibration of reference</td>
<td>0.0018(rel.)</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Test lamp repeatability</td>
<td>0.0006(rel.)</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_3$</td>
<td>Reference lamp current setting</td>
<td>.0001(rel.)</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_4$</td>
<td>radius of notional sphere</td>
<td>.0006(rel.)</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Cap correction</td>
<td>0.0006(rel.)</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Reference lamp current stability</td>
<td>.00001(rel.)</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Ageing of reference</td>
<td>0.0035(rel.)</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>$x_8$</td>
<td>ratio of reference lamps</td>
<td>0.0035(rel.)</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>$x_9$</td>
<td>Internal stray light</td>
<td>k = 0.0001</td>
<td>?</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Wavelength</td>
<td>0.3 nm</td>
<td>?</td>
<td>No</td>
</tr>
</tbody>
</table>

\[
I_{\text{true}}(\lambda_i) = I_{\text{meas}}(\lambda_i) + I(\lambda_i)x_1 + I(\lambda_i)x_2 + 8I(\lambda_i)x_3 + 2I(\lambda_i)x_4 + I(\lambda_i)x_5 + 8I(\lambda_i)x_6 + I(\lambda_i)x_7 + I(\lambda_i)x_8 + \ldots?
\]
Stray light evaluation

\[ k = \frac{S_{\text{out}}}{S_{\text{in}}} \]
# Uncertainty budget with sensitivity coefficients: Differentiate the measurement equation

| symbol | Source                   | Standard Value, \( u_i(x_i) \) | Sensitivity coefficient, \( C_i \) | Correlated?
|--------|-------------------------|---------------------------------|----------------------------------|-----------
| \( x_1 \) | Calibration of reference | 0.0018 (rel.)                  | \( I(\lambda) \)                | Yes       
| \( x_2 \) | Test lamp repeatability  | 0.0006 (rel.)                  | \( I(\lambda) \)                | Yes       
| \( x_3 \) | Reference lamp current setting | .0001 (rel.)                  | \( 8I(\lambda) \)                | Yes       
| \( x_4 \) | radius of notional sphere | 0.006 (rel.)                   | \( I(\lambda) \)                 | Yes       
| \( x_5 \) | Cap correction          | 0.0006 (rel.)                  | \( I(\lambda) \)                | Yes       
| \( x_6 \) | Reference lamp current stability | .00001 (rel.)                 | \( 8I(\lambda) \)                | No        
| \( x_7 \) | Ageing of reference     | 0.0035 (rel.)                  | \( I(\lambda) \)                | No        
| \( x_8 \) | ratio of reference lamps | 0.0035 (rel.)                  | \( I(\lambda) \)                | No        
| \( x_9 \) | Internal stray light    | \( k = 0.0001 \)              | *                                | Yes       
| \( x_{10} \) | Wavelength             | 0.3 nm                         | *                                | No        

Problem comes with \( x_9 \) and \( x_{10} \). A refined model is required.
Wavelength offset and stray light

\[ F(x_1, x_2, \ldots, x_{10}) = \frac{S_{LED} + \delta S_{LED, wave} + \delta S_{LED, stray}}{S_R + \delta S_{R, wave} + \delta S_{R, stray}} (I_R + I_R \sum_{i=1}^{8} c_i x_i) \]

(where \( c_k = C_k/I_{LED} \). The \( c_k \) are independent of wavelength!)

Using first order approximations \( 1/(1-x) = 1+x \) and \( 1+x_1 + x_2 = (1+x_1)(1+x_1) \) gives:

\[
= \frac{I_R S_{LED}}{S_R} \left( 1 + \delta \frac{S_{LED, stray}}{S_{LED}} - \delta \frac{S_{R, stray}}{S_R} \right) \times \\
\left( 1 + \delta \frac{S_{LED, wave}}{S_{LED}} - \delta \frac{S_{R, wave}}{S_R} \right) \times \sum_{i=1}^{8} (1 + c_i x_i)
\]

This has the form:

\[ F = f_9(x_9) f_{10}(x_{10}) \sum_{i=1}^{8} (1 + c_i x_i) \]
Wavelength offset and stray light

- Now F has the form:

\[
F = f_9(x_9) f_{10}(x_{10}) \sum_{i=1}^{8} (1 + c_i x_i)
\]

Making differentiation straightforward, e.g.

\[
\frac{\partial F}{\partial x_9} = \frac{F}{f_9(x_9)} \frac{\partial f_9(x_9)}{\partial x_9}
\]

- Similarly for \(x_{10}\)
What is \( f_{10}(x_{10}) \)?

Now:
\[
\delta S = \frac{\partial S(\lambda)}{\partial \lambda} \cdot x_{10}
\]

Thus:
\[
f_{10}(x_{10}) = \frac{\partial S_{\text{LED}}(\lambda)}{\partial \lambda} \cdot \frac{x_{10}}{S_{\text{LED}}} - \frac{\partial S_R(\lambda)}{\partial \lambda} \cdot \frac{x_{10}}{S_R}
\]

Differentiate:
\[
\frac{\partial f_{10}}{\partial x_{10}} = I_{\text{LED}} \left\{ \frac{1}{S_{\text{LED}}} \frac{\partial S_{\text{LED}}}{\partial \lambda} - \frac{1}{S_R} \frac{\partial S_R}{\partial \lambda} \right\}
\]
\[
= I_{\text{LED}} \frac{S_R}{S_{\text{LED}}} \frac{\partial}{\partial \lambda} \left\{ \frac{S_{\text{LED}}}{S_R} \right\}
\]

Difference Approx’n:
\[
\approx I_{\text{LED}} \frac{S_R}{S_{\text{LED}}} \frac{1}{\lambda_i - \lambda_{i-1}} \left\{ \left[ \frac{S_{\text{LED}}}{S_R} \right]_i - \left[ \frac{S_{\text{LED}}}{S_R} \right]_{i-1} \right\}
\]
Similarly for stray light, \( f_9(x_9) \):

\[
\frac{\partial F}{\partial x_9} = I_{LED}(\lambda_0) \left\{ \frac{\int_{\text{total range of spectrograph}} S_{LED}(\lambda)d\lambda}{S_{LED}(\lambda_0).\Delta\lambda} - \frac{\int_{\text{total range of spectrograph}} S_{R}(\lambda)d\lambda}{S_{R}(\lambda_0).\Delta\lambda} \right\}
\]

(See, e.g. Kostkowski, "Reliable Spectroradiometry")

To calculate this, we can approximate integrals to summations: then use a spreadsheet to calculate!

These examples show that it is necessary to write down the relevant measurement equation to determine the sensitivity coefficients
The expression for uncertainty in $I_v$

\[
(U_c)^2 = \sum_{k=1}^{5} (u(x_k)c_k)^2 \left( \sum_{i=1}^{N} (I_{LED})_i V_i \right)^2 + \left( \sum_{i=1}^{N} (u(x_9^i))(C_i^9 V_i) \right)^2 \\
+ \sum_{k=6}^{8} (u(x_k)c_k)^2 \left( \sum_{i=1}^{N} (I_{LED})_i^2 (V_i) \right)^2 + \sum_{i=1}^{N} (u(x_{10}^i))^2 (C_i^{10} V_i)^2
\]

- Use a spreadsheet to perform summations.
### Uncertainty Budget with Sensitivity Coefficients: Differentiate the Measurement Equation

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<td>Yes</td>
</tr>
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<td>Test lamp repeatability</td>
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<td>1</td>
<td>Yes</td>
</tr>
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<td>$x_3$</td>
<td>Reference lamp current setting</td>
<td>0.0001 (rel.)</td>
<td>8</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_4$</td>
<td>radius of notional sphere</td>
<td>0.0006 (rel.)</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Cap correction</td>
<td>0.0006 (rel.)</td>
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<td>Wavelength</td>
<td>0.3 nm</td>
<td>*</td>
<td>No</td>
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</tbody>
</table>
Implementation of spreadsheet calculation

Assume green LED, reference tungsten lamp at 2800K, responsivity of spectrometer maximum at 500 nm, falling off slowly to longer and shorter wavelengths.
Table 2.2(b): Illustration of calculation of sensitivity coefficients for wavelength uncertainty term in the determination of LED illuminance

<table>
<thead>
<tr>
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<th>SB</th>
<th>SC</th>
<th>SD</th>
<th>SE</th>
<th>SF</th>
<th>SG</th>
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</thead>
<tbody>
<tr>
<td>λ</td>
<td>I(LED)</td>
<td>S(LED)</td>
<td>S(LED) * δλ</td>
<td>S(standard)</td>
<td>S(standard) * δλ</td>
<td>Sensitivity Coefficient</td>
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<tr>
<td>390</td>
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<td>11.13215</td>
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</tbody>
</table>

Intervening wavelengths omitted for conciseness

| λ  | I(LED) | S(LED) | S(LED) * δλ | S(standard) | S(standard) * δλ | Sensitivity Coefficient |
| 520 | 0.535261 | 0.534619 | 0.534619115 | 1.0050413 | 1.005041298 | -569.273 |
| 521 | 0.67032 | 0.669433 | 0.669433213 | 1.0143697 | 1.014369688 | -709.053 |
| 522 | 0.798516 | 0.797357 | 0.797356773 | 1.023723 | 1.023722969 | -838.959 |
| 523 | 0.904837 | 0.903401 | 0.903401441 | 1.0331006 | 1.033100595 | -943.411 |
| 524 | 0.97531 | 0.973625 | 0.973624577 | 1.042502 | 1.042502014 | -1008.48 |

Intervening wavelengths omitted for conciseness

| λ  | I(LED) | S(LED) | S(LED) * δλ | S(standard) | S(standard) * δλ | Sensitivity Coefficient |
| 549 | 5.57E-07 | 5.53E-07 | 5.5375E-07 | 1.2836148 | 1.283614841 | 11.26899 |
| 550 | 1.64E-07 | 1.63E-07 | 1.6251E-07 | 1.2934347 | 1.293434705 | 11.2727 |
| 560 | 1.3919851 | 13.91985126 |
| 570 | 1.4907562 | 14.90756204 |
| 580 | 1.5891501 | 15.89150073 |

Intervening wavelengths omitted for conciseness

| λ  | I(LED) | S(LED) | S(LED) * δλ | S(standard) | S(standard) * δλ | Sensitivity Coefficient |
| 980 | 1.3582387 | 13.58238745 |
| 990 | 1.2333534 | 12.33353381 |
| 1000 | 1.1045429 | 11.04542919 |

sum 11.20998 11.18829099 1089.936768
Table 2.2(a): Edited version of the main spreadsheet for the calculation of uncertainty in luminous intensity

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ</td>
<td>I(LED)</td>
<td>I(standard)</td>
<td>R(spectrometer)</td>
<td>S(LED)</td>
<td>S(standard)</td>
<td>v(λ)</td>
<td>v(λ) x (LED)</td>
<td>stray light sensitivity coefficient</td>
<td>stray light sensitivity coefficient</td>
<td>x k x v(λ)</td>
<td>previous col. uncert.</td>
<td>previous stray light sensitivity coefficient</td>
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</tr>
<tr>
<td>500</td>
<td>1.6374E-07</td>
<td>1.000000</td>
<td>1.6374E-07</td>
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<td>0.8245162</td>
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<td>0.3546858</td>
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<td>5.128E-19</td>
<td>0.84199234</td>
<td>1.8049E-06</td>
<td></td>
</tr>
</tbody>
</table>

Intervening wavelengths omitted for conciseness

|   |     |           |         |         |           |           |           |           |           |           |           |           |           |
|   |     |           |         |         |           |           |           |           |           |           |           |           |           |
| 523 | 0.90483742 | 1.03474273 | 0.99841 | 0.90340144 | 1.03310059 | 0.7619694 | 0.68945842 | 0.47535292 | -943.41 | -0.0650443 | 0.0042307 | 0.022607 | 0.000511 |
| 524 | 0.97530991 | 1.04430658 | 0.99827 | 0.97362459 | 1.04250201 | 0.7778368 | 0.75863194 | 0.57552242 | -1008.4 | -0.0769065 | 0.0058532 | 0.014493 | 0.000210 |
| 525 | 1.000000 | 1.05390274 | 0.99813 | 0.998125 | 1.05192667 | 0.7932 | 0.7932 | 0.62916624 | -1024.9 | -0.081297 | 0.0066092 | 0.003742 | 1.4007E-05 |
| 526 | 0.97530991 | 1.06353066 | 0.99797 | 0.97333198 | 1.06134702 | 0.8081104 | 0.78815808 | 0.62119316 | -990.34 | -0.0780549 | 0.0060925 | -0.008200 | 6.724E-05 |
| 527 | 0.90483742 | 1.07319056 | 0.99781 | 0.90283654 | 1.0708435 | 0.8224962 | 0.74422534 | 0.55387136 | -909.75 | -0.0677065 | 0.0045841 | -0.019574 | 0.000383 |

Intervening wavelengths omitted for conciseness

|   |     |           |         |         |           |           |           |           |           |           |           |           |           |
|   |     |           |         |         |           |           |           |           |           |           |           |           |           |
| 548 | 1.8049E-06 | 1.28266957 | 0.99309 | 1.7924E-06 | 1.27380376 | 0.9903128 | 1.7874E-06 | 3.1949E-12 | 11.264 | 2.0135E-09 | 4.054E-18 | -1.129E-06 | 1.2740E-12 |
| 550 | 1.6374E-07 | 1.30320877 | 0.9925 | 1.6251E-07 | 1.2934347 | 0.9949501 | 1.6291E-07 | 2.654E-14 | 11.272 | 1.8364E-10 | 3.372E-20 | -1.188E-07 | 1.411E-14 |

sum 8.82492848 | 4.97004622 | -0.6385635 | 0.0367782 | -0.0748374 | 0.011501 |

sum of squares 4.97004622 | 0.0367782 | 0.011501 |

square of sum 77.8793627 | 0.4077633 | 0.005600 |

"correlated spectrally invariant" terms: 0.00039

"uncorrelated spectrally invariant" terms: 0.00012

For integrated quantity:

Total absolute uncertainty squared: 0.4197

Total absolute uncertainty: 0.6479

Relative uncertainty: 0.07342 %
The expression for uncertainty in $I_v$

\[
(U_c)^2 = \sum_{k=1}^{5} (u(x_k)c_k)^2 \left( \sum_{i=1}^{N} (I_{LED})_i V_i \right)^2 + \left( \sum_{i=1}^{N} (u(x_9))(C_i^9 V_i) \right)^2 \\
+ \sum_{k=6}^{8} (u(x_k)c_k)^2 \left( \sum_{i=1}^{N} (I_{LED})_i^2 (V_i) \right)^2 + \sum_{i=1}^{N} (u(x_{10}))^2 (C_i^{10} V_i)^2
\]

- Use a spreadsheet to perform summations.
Prediction of uncertainty as a function of stray light parameter

Variation of overall relative uncertainty with value of stray light parameter, $k$

![Graph showing the variation of overall relative uncertainty with value of stray light parameter, $k$. The x-axis represents $k$ values ranging from $1.0 \times 10^{-8}$ to $1.0 \times 10^{-4}$, and the y-axis represents uncertainty percentage ranging from 0% to 8%. The graph shows an increasing trend in uncertainty as $k$ increases.](image-url)