

Multiscale Properties IAG Meeting

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Project SM06

Knowledge Based Design with Plastics

Louise Crocker

Greg Dean

Neil McCartney

Richard Mera

Simon Roberts

Project Outline

- **Objective** - to develop a capability to design with polymers under long-term load
- Model long-term deformation behaviour
- Develop non-linear viscoelasticity theory to produce solutions for arbitrary stress states and stress histories
- Prepare code, based on this theory, for ABAQUS to obtain solutions using finite element methods
- Evaluate the code

Design for linear elastic behaviour

- Analysis of measurements under tension

$$\sigma_T = E\varepsilon_T$$

$$\varepsilon_T = D\sigma_T$$

- Theory – linear elastic
 - Isotropic materials

$$\varepsilon_{ij} = \frac{J}{2}(\sigma_{ij} - \sigma_m \delta_{ij}) + \frac{B}{3}\sigma_m \delta_{ij}$$

- Material properties
 - J and B
 - or E and ν from tensile tests

Design for non-linear, elastic behaviour

- Analysis of measurements under simple stress states

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad \sigma_T = f(\varepsilon_T^p) = \sigma_{yT}(\varepsilon_T^p)$$

$$\sqrt{3}\sigma_{ys} = \frac{(3+\lambda)}{3}\sigma_{yT} = \frac{(3-\lambda)}{3}\sigma_{yC}$$

- Theory – elastic-plastic

$$- \quad \sigma_0(\varepsilon^p) = f(q, \sigma_m) \quad \text{Yield criterion}$$

$$- \quad \varepsilon_{ij}^p = c \frac{\partial F}{\partial \sigma_{ij}} \quad \text{Plastic flow law}$$

- Material properties

- $\sigma_T(\varepsilon_T^p)$ and $\sigma_S(\varepsilon_S^p)$ hardening curves in tension and shear

- $\nu^p(\varepsilon_T^p)$ Poisson's ratio with plastic strain

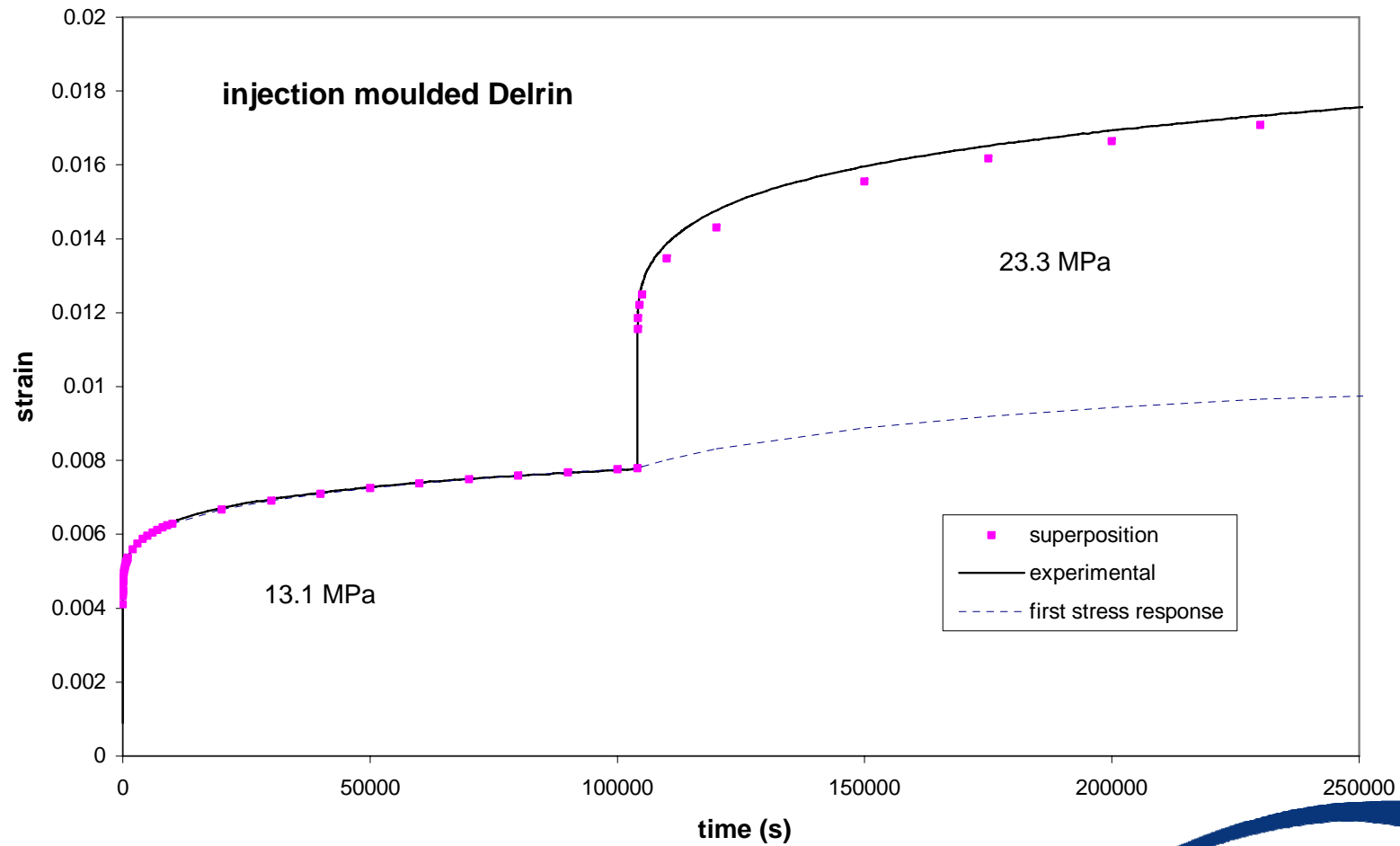
Design for non-linear, viscoelasticity

- Analysis of measurements on polymers under simple stress state and stress history (creep)

$$D(t) = D_0 + \Delta D \left(1 - \exp - \left(\frac{t}{t_0} \right)^n \right)$$
$$t_0 = A \exp - \alpha \bar{\sigma}$$

- Theory – non-linear viscoelastic
 - Not available in FE packages
 - Closest models are for time-dependent plasticity
 - These do not describe
 - Creep results
 - Response to step loading

2-step loading – measurement and prediction



Non-linear viscoelasticity in SM06

- Stress/strain equations for multiaxial and time-dependent stresses

$$\varepsilon_{ij}(t) = \varepsilon_{ij}^d(t) + \frac{1}{3}\varepsilon_m(t)\delta_{ij}$$

$$\varepsilon_{ij}^d(t) = \frac{1}{2} \int_0^t \dot{\sigma}_{ij}^d(\tau) J(t-\tau) d\tau$$

$$J(t-\tau) = J_0 + \Delta J \left[1 - \exp \left[- \left(\int_{\tau}^t \frac{du}{t_0(\overline{\sigma}(u))} \right)^n \right] \right]$$

$$\varepsilon_m(t) = \int_0^t \dot{\sigma}_m(\tau) B(t-\tau) d\tau$$

$$B(t-\tau) = B_0 + \Delta B \left[1 - \exp \left[- \left(\int_{\tau}^t \frac{du}{t_0(\overline{\sigma}(u))} \right)^n \right] \right]$$

- Material properties

- $J(t) = 2(1+\nu)D(t)$

- $B(t) = 3(1-2\nu)D(t)$ assuming constant Poisson's ratio

Next steps

- Evaluation of FE code
 - Compare FE predictions with measurements of creep under step loading and unloading
 - Compare predictions and measurements of creep under flexural loading
 - Compare predictions and measurements under a constant strain (stress relaxation)

New project proposal

- SM06 is a small project
- New project proposal for 2008+
 - Accelerated methods for determining long-term properties of plastics
 - Excluded from submission to the MAC for 2nd year running