MODELLING CREEP IN TOUGHENED ADHESIVES FOR FINITE ELEMENT ANALYSIS

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1 INTRODUCTION

Finite element analyses can be used to predict stress and strain distributions in an adhesive joint under an applied load. The results of an analysis can be used to explore variations in the design of the joint in order to reduce stress or strain levels in regions of stress concentration. In conjunction with a valid failure criterion, it should be possible to predict the ultimate load that the joint can sustain. Because the deformation behaviour of tough adhesives is highly non-linear, elastic-plastic models are generally used to describe the behaviour of structural adhesives. The properties and parameters required by these models are usually obtained from short-term tests on bulk or joint test specimens.

Since adhesives are viscoelastic materials, their properties will vary with strain rate or time under load. This means that under long-term loading, properties will change with time, and stress and strain levels in the adhesive will be very different from predictions based on a short-term analysis under monotonic loading. In order for a stress analysis to assist with the design of bonded joints under long-term loading, models are needed that describe deformation of the adhesive under creep, intermittent or fatigue loads. The work reported here is concerned with modelling non-linear creep in a toughened adhesive. A procedure is outlined for implementing this model in a finite element system. The redistribution of stress and strain in a specimen with time under load is illustrated using the results of finite element analyses of a lap joint.

2 A MODEL FOR CREEP IN TOUGHENED ADHESIVES

2.1 Tensile behaviour

In a tensile creep test, a time-dependent tensile strain $\varepsilon(t)$ is produced by a constant applied stress $\sigma_o$. A tensile creep compliance function $D(t)$ is then defined by the expression

$$D(t) = \frac{\varepsilon(t)}{\sigma_o} \quad (1)$$

The creep behaviour of glassy polymers is commonly modelled using the function

$$D(t) = D_0 \exp \left( \frac{t}{t_o} \right)^m \quad (2)$$

This function will only model the short-time tail of the relaxation process, but this is usually a valid approximation, even for long times under load, as long as the test temperature is not close to the glass transition temperature. In equation (2), the exponent $m$ characterises a broad spectrum of relaxation times whose mean value is $t_o$.

Figure 2 shows creep compliance curves for a rubber-toughened adhesive measured under different levels of stress $\sigma_o$. At short creep times, there is a small dependence of the compliance on stress which is consistent with slight curvature in tensile stress/strain curves obtained from constant displacement rate tests at speeds of around a few mm/min. This non-linear behaviour is observed to increase significantly with time under load, the limiting stress for linear behaviour being near or below 10 MPa. The effect of stresses above this level is to give a significant reduction in the mean relaxation time $t_o$ of the creep process and to, thereby, shift curves to shorter times. This can be demonstrated by using equation (2) to obtain best fitting curves to the data in figure 2. The continuous curves in the figure have been obtained with constant values for $D_0 = 0.44$ GPa$^{-1}$ and $m = 0.33$ and...
selecting values for \( t_0 \) that decrease with increasing stress. The variation of \( t_0 \) with \( \sigma_0 \) can be described with satisfactory accuracy by the empirical relationship

\[
t_0 = A \exp \left( -\alpha \sigma_0^2 \right)
\]

(3)

Values for the parameters in equations (1) and (2) obtained from the results in figure 2 are listed in table 1.

<table>
<thead>
<tr>
<th>( D_0 ) (GPa(^{-1}))</th>
<th>0.44</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>0.33</td>
</tr>
<tr>
<td>( A ) (s)</td>
<td>( 4.10^7 )</td>
</tr>
<tr>
<td>( \alpha ) (MPa(^{-2}))</td>
<td>0.0061</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1.7</td>
</tr>
</tbody>
</table>

2.2 Extension of the model to creep under multiaxial stresses

Equations (1) and (2) describe creep behaviour under a uniaxial tensile stress. Under uniaxial compression, compliance curves are the same as tensile curves at low stresses where creep behaviour is linear. Under higher stresses, where behaviour is non-linear, the reduction in relaxation time \( t_0 \) with stress is less under compression than under tension. The results of tensile and compressive creep tests indicate that the stress in equation (2) should be replaced by an effective stress \( \sigma \) that is a function of both the shear and hydrostatic components of the creep stress. The simplest function to consider is

\[
\sigma = \frac{(\lambda + 1)}{2\lambda} \sigma_e + \frac{3(\lambda - 1)}{2\lambda} \sigma_m
\]

(4)

where \( \sigma_e \) is the effective shear stress given, in terms of principal components of the applied creep stress, by

\[
\sigma_e = \left[ \frac{1}{2} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right) \right]^{1/2}
\]

(5)

and \( \sigma_m \) is the hydrostatic component of the creep stress given by

\[
\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)
\]

(6)

The parameter \( \lambda \) is a measure of the sensitivity of the mean creep relaxation time to the hydrostatic component of stress, and the value for the toughened adhesive studied here is recorded in table 1.

3 APPLICATION OF THE CREEP MODEL IN A FINITE ELEMENT ANALYSIS

A generalised creep function in ABAQUS arising from flow by rate-dependent plasticity takes the form

\[
\dot{\varepsilon}_{ij}(t) = \dot{\varepsilon}_s \left( \frac{\partial \sigma_e}{\partial \sigma_{ij}} \right) + \dot{\varepsilon}_{sw} \delta_{ij}
\]

(7)
The terms $\dot{\varepsilon}_s$ and $\dot{\varepsilon}_{sw}$ are associated with contributions to the creep strain rate arising from shear and dilatational (swelling) flow processes.

Equation (7) can be expressed in the form

$$
\dot{\varepsilon}_i^j (t) = \dot{\varepsilon}_s \frac{3\sigma_{ij}}{2\sigma_e} + \left( \dot{\varepsilon}_{sw} - \frac{3\sigma_m \dot{\varepsilon}_s}{2\sigma_e} \right) \delta_{ij}
$$

Equation (8) can be expressed in the form

$$
\dot{\varepsilon}_i^j (t) = \frac{2\sigma_{ij}}{3} \left( \frac{t}{t_o} \right)^{m-1} \exp \left( \frac{t}{t_o} \right)^m
$$

The creep function for adhesives, equation (1), can be generalised to describe multiaxial creep and takes a form similar to equation (8), thus

$$
\dot{\varepsilon}_i^j (t) = \left( (1+\nu)D_o \sigma_{ij} - 3\nu D_o \sigma_m \delta_{ij} \right) \frac{mt^{m-1}}{t_o^m} \exp \left( \frac{t}{t_o} \right)^m
$$

where $\nu$ is Poisson’s ratio assumed to be independent of time, and $t_o$ is given by equation (3) with $\sigma_e$ replaced by equation (4). This can be identified with equation (8) if

$$
\dot{\varepsilon}_s = \frac{2\sigma_{ij}}{3} \left( \frac{t}{t_o} \right)^{m-1} \exp \left( \frac{t}{t_o} \right)^m
$$

and

$$
\dot{\varepsilon}_{sw} = \frac{\sigma_{ij}}{t_o^{m-1}} \left( \frac{t}{t_o} \right)^m
$$

Through equations (10) and (11), a model for non-linear viscoelasticity has been associated with a model for time-dependent plasticity. Coding for a user subroutine has been written so that equations (10) and (11) can be implemented in equation (7) in ABAQUS for the solution of creep analyses by finite element methods.

4 CREEP OF A LAP-JOINT SPECIMEN

A creep analysis has been carried out of a lap joint having the dimensions and geometry shown in figure 1. A load of 2300 N was applied to the specimen for a duration of $10^6$ s. The material parameters for the adhesive required by the creep model are given in table 1. The results show a redistribution of stress and strain in the adhesive resulting from non-linear creep behaviour. The levels of stress in regions of stress concentration are reduced whilst strains increase significantly. Figure 3 shows distributions of maximum principal strain at the beginning and end of the loading period, and it can be seen that the maximum strain is predicted to increase by more than a factor of 2 in this time.
Figure 2. Creep compliance curves for a toughened adhesive at different stress levels.

Figure 3. Contours of maximum principal strain in the ends of a lap joint after 10 s and 1,000,000 s under load predicted using the implementation of the creep model in ABAQUS.

ACKNOWLEDGEMENT

This work was funded by the UK Department of Trade and Industry under the Measurements for Materials Performance research programme.