

# Signal processing case study: Analysis of Watt balance vibration data

## Wavelet Analysis

Jane Jiang University of Huddersfield

Ian M Smith, National Physical Laboratory

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# Outline

- Introduction to Wavelet
  - What is wavelet, why wavelet, wavelet History
  - Typical applications
- Analysis of Watt balance vibration data
  - Detection of Potential datasets with transient events;
  - Time/frequency Localization of Transient signal using wavelet packet analysis;
  - Transient signal extraction using wavelet Denoising;
- Summary

# What is wavelet analysis?

A wavelet is a waveform of effectively limited duration that has an average value of zero.

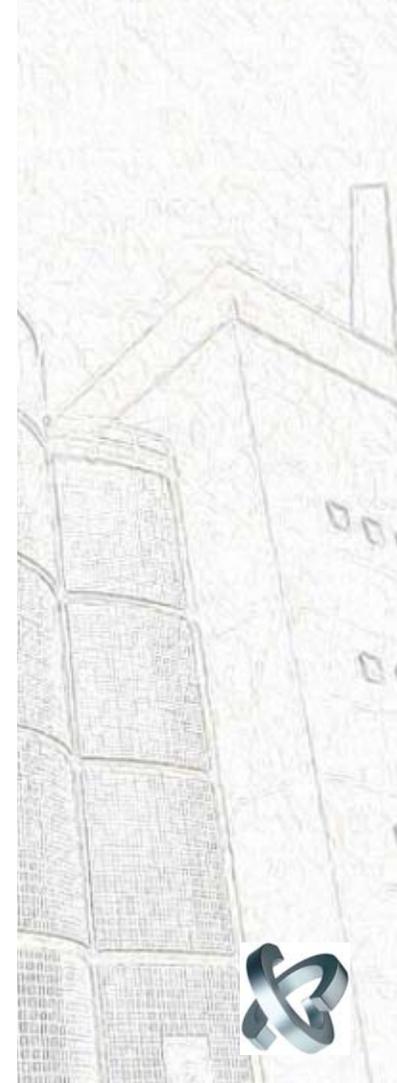
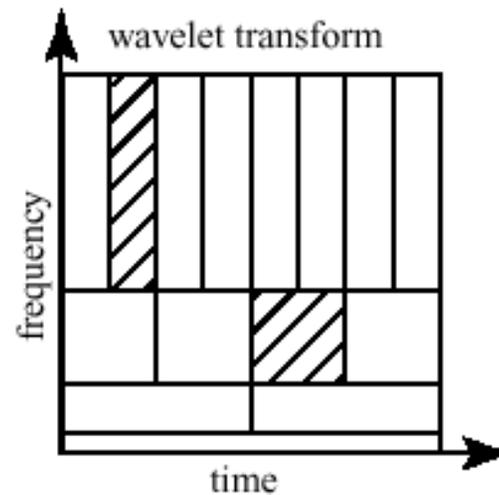
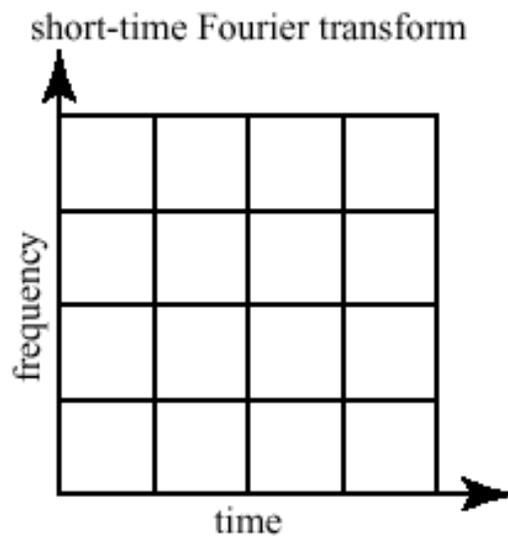
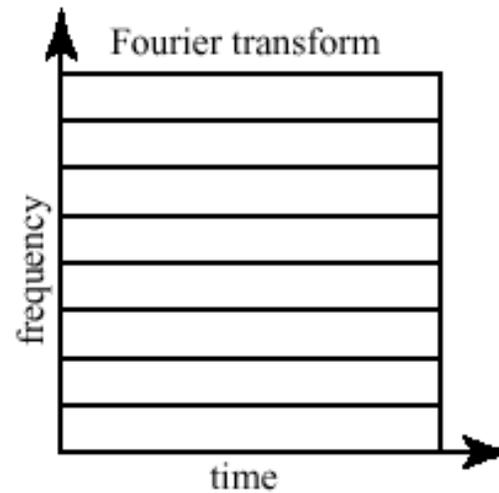
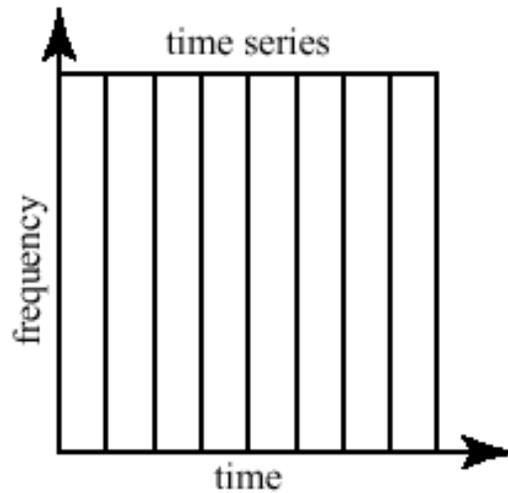


# Why Wavelet?

- **Locality:** each wavelet coefficient represents signal local content local in space/time and frequency;
- **Multiresolution:** the wavelet transform represents the signal at a nested set of scales;
- **Energy compaction:** the wavelet transform tend to be sparse, (Compression)
- **Edge detection,** wavelet act as local edge detectors

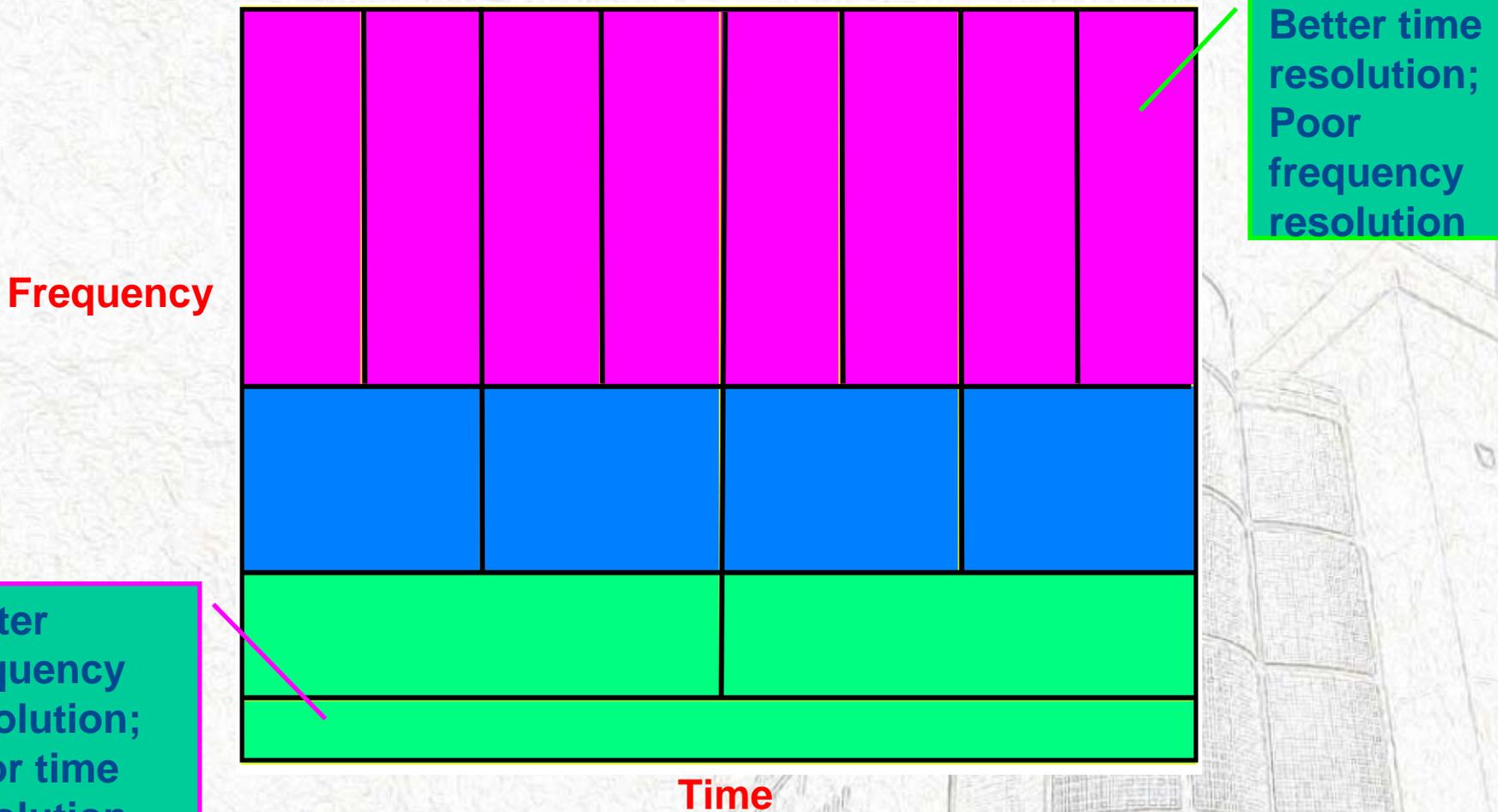
# What is wavelet analysis?

## Time-frequency Tiles



# What is wavelet analysis?

## Time-frequency Tiles

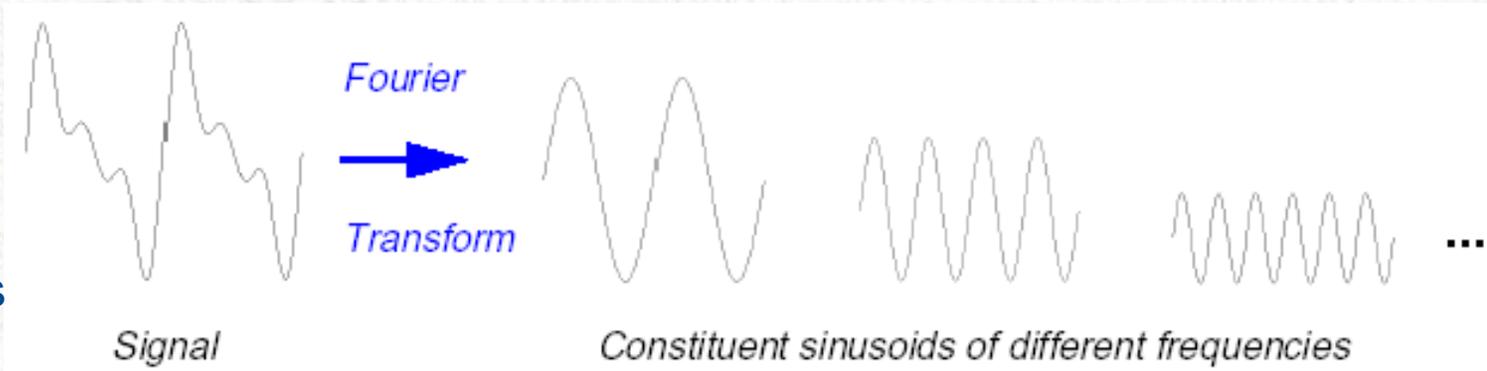


Better frequency resolution; Poor time resolution

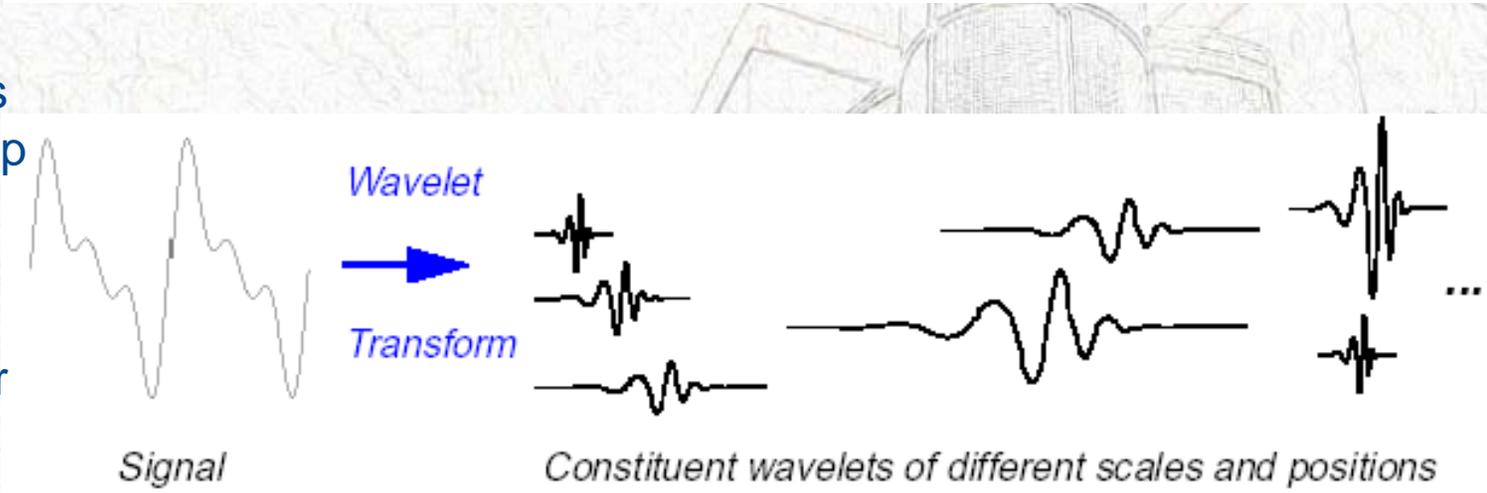
- Each box represents a equal portion
- Resolution in STFT is selected once for entire analysis

# What is wavelet analysis

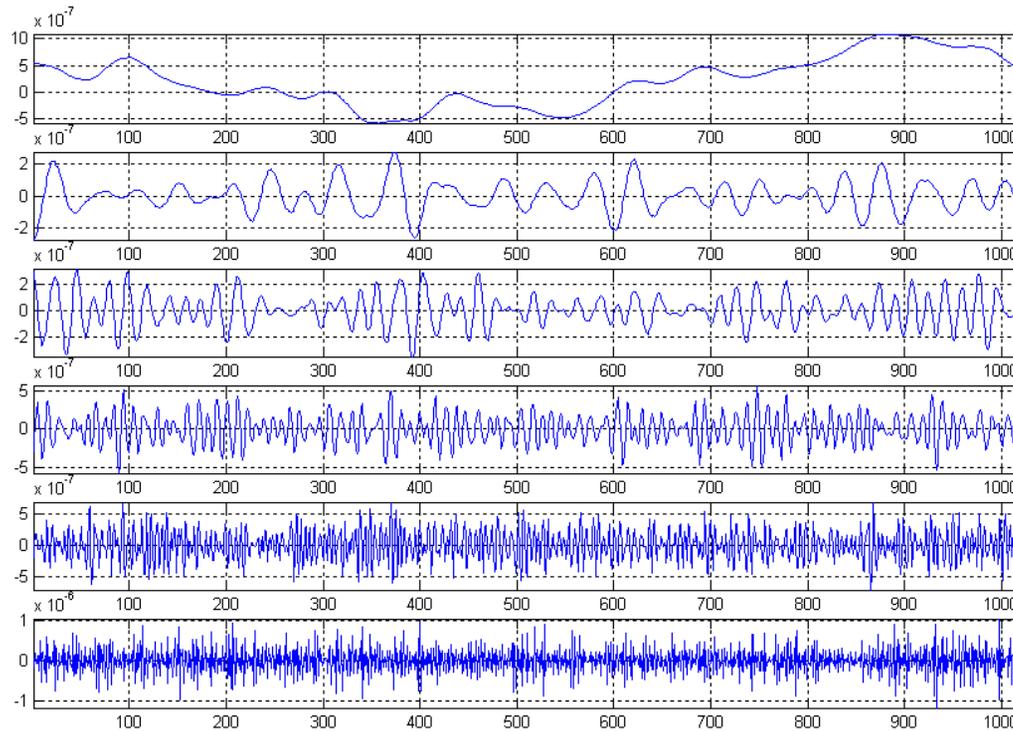
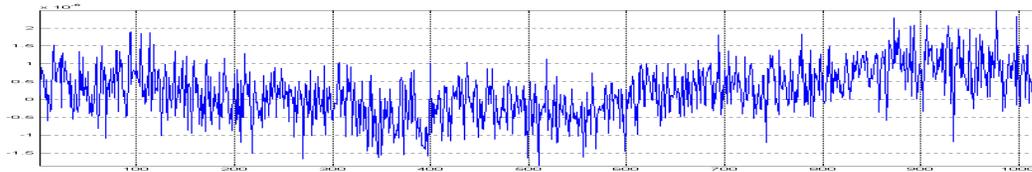
Fourier analysis consists of breaking up a signal into sine waves of various frequencies.



Wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or *mother*) wavelet.



# Multi-scalar Analysis



# Typical Applications

- New JPEG standard; (JPEG2000)
- FBI fingerprint compression;
- Image Denoising;
- Texture analysis;
- Feature extraction;
- Speech processing;
- Seismic;
- Metrology; (GPS TC213 ISO16610)



# Analysis of Watt balance vibration data

The transient signal is featured by:

- Abrupt changes in amplitudes, phases, or frequencies;
- Rapid decays in amplitudes;
- Fast transitions in frequencies and amplitudes;

The challenge of this case is:

- Transient signals will typically appear as broadband energy on the frequency display. Fourier based methods are suitable only for stationary signal detection, and are not suitable for the detection of transient signal.
- A large amount of datasets, which dataset contain transient?

# Our strategy

Find potential dataset with  
transients

(Time domain analysis;  
Multi scale analysis )

Time/frequency localization

Wavelet Packet analysis

Extraction of Transient events

Wavelet denoising

# Potential dataset detection

The abrupt change of the signal in time or frequency domain leads to the local maximum in the wavelet coefficient at the position where the changes appear

Multi scale root mean square deviation

$$Pq_{ij} = \sqrt{\frac{1}{m} \sum_{k=1}^m w_{i,j,k}^2}$$

Multi scale absolute height

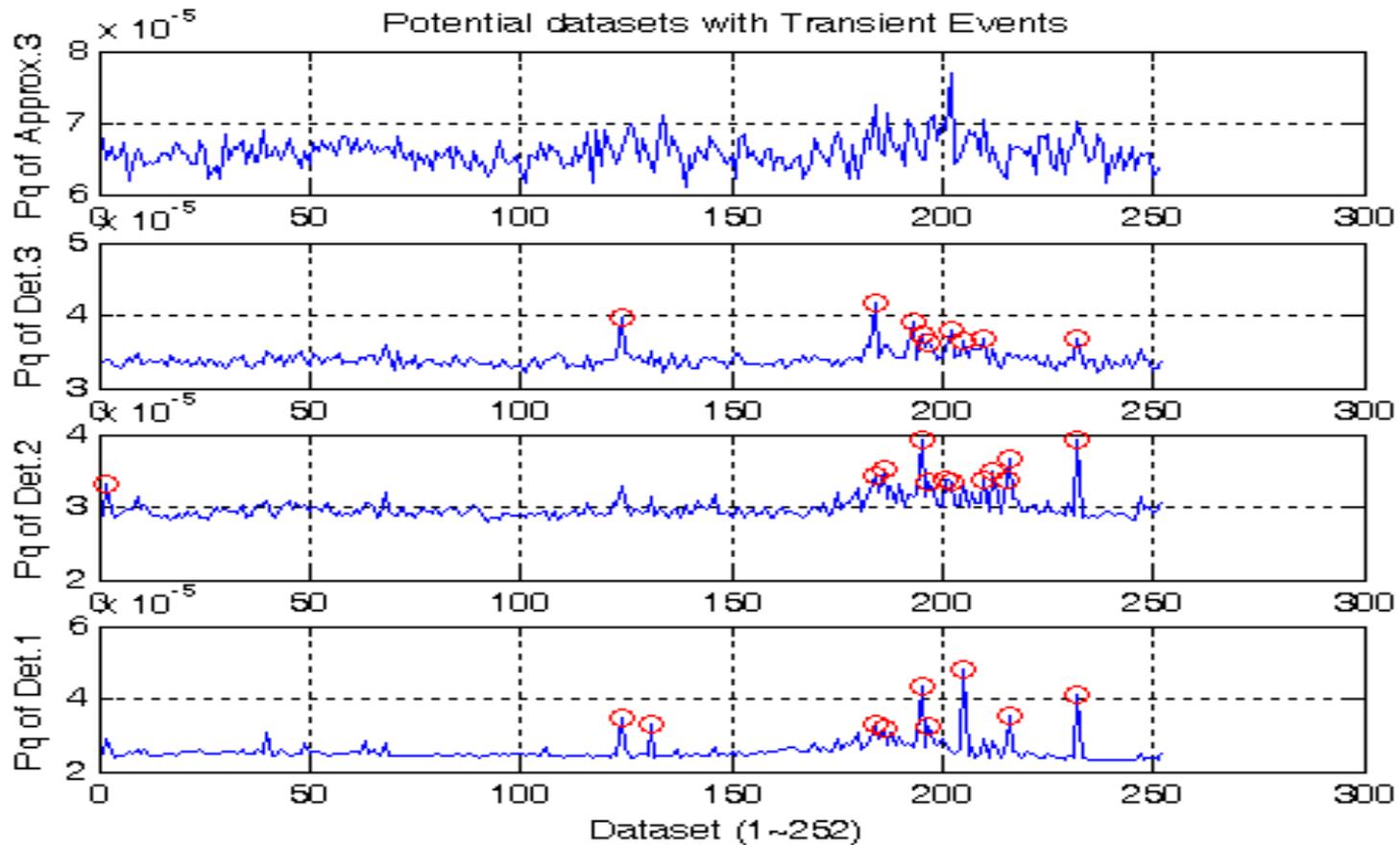
$$Pt_{ij} = \max(w_{i,j}) - \min(w_{i,j})$$

$W_{ij}$  is the wavelet coefficients of the dataset  $i$  on wavelet decomposition level  $j$

Bigger  $Pq_{ij}$ ,  $Pt_{ij}$

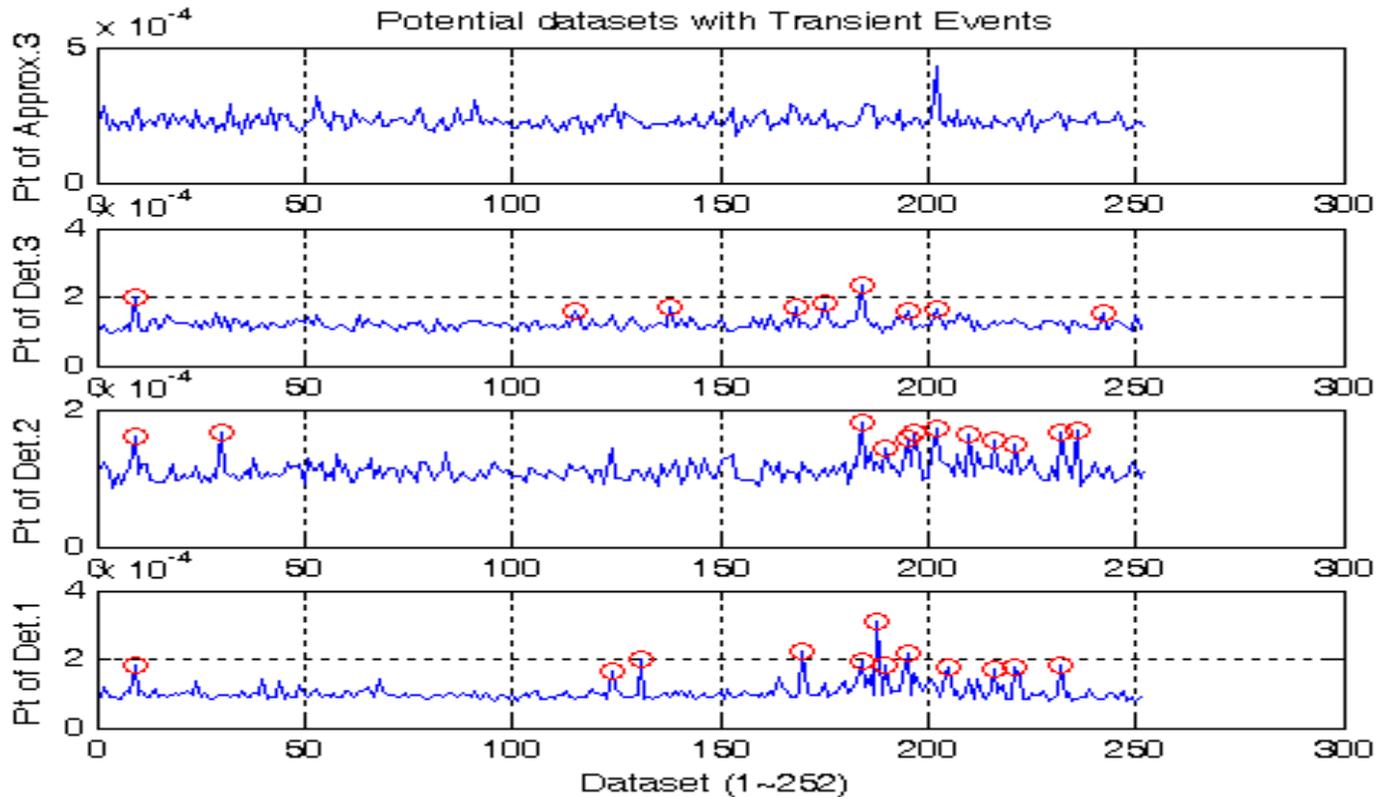
Greater potential to have transient events

# Potential dataset detection



X axis is the dataset no, y axis is Pqij

# Potential dataset detection



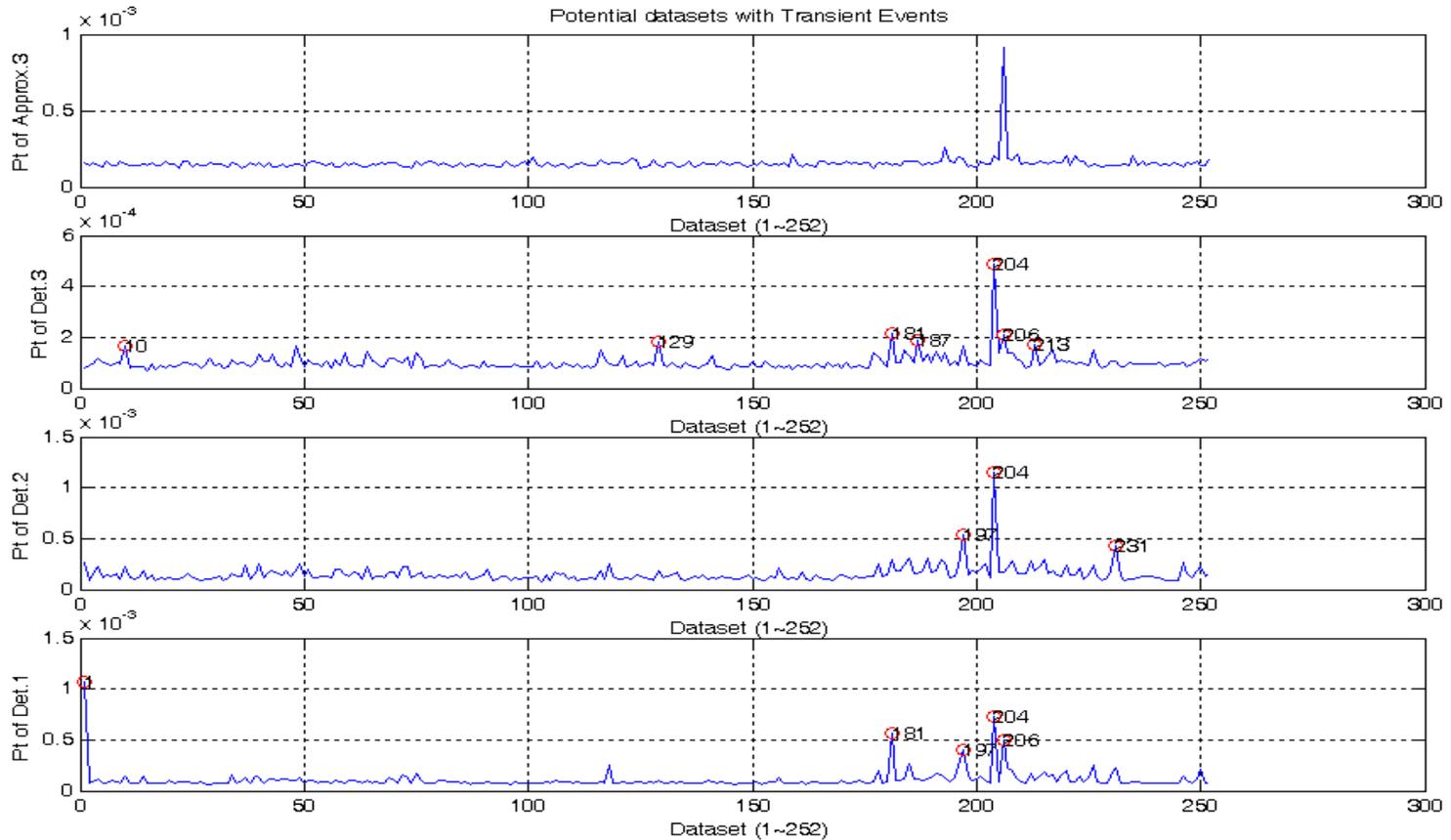
Pt analysis; X axis is the dataset no, y axis is Pt

# Potential dataset detection

		Possible datasets containing transient signals	
sensor		Pt	Pq
Acce   0	Det. 1	9, <u>124</u> , <u>131</u> ,170,184, <u>188</u> , 190,195,205, <u>216</u> ,221	<u>124</u> , <u>131</u> ,184,186,195, <u>197</u> ,205, <u>216</u> , <u>232</u>
	Det. 2	9,30,184,190,195, <u>197</u> , <u>202</u> , 210,216,221, <u>232</u> ,236	2,184,186,195,197,201 , <u>202</u> ,210,212,215,216, <u>232</u>
	Det. 3	9,115,138,168,175,184, 195,202,242	124,184,18 <u>32</u> 195,197, 202,205,210,232,

The bold number are the detected datasets by using time domain analysis;  
Wavelet analysis detected more datasets containing transient events

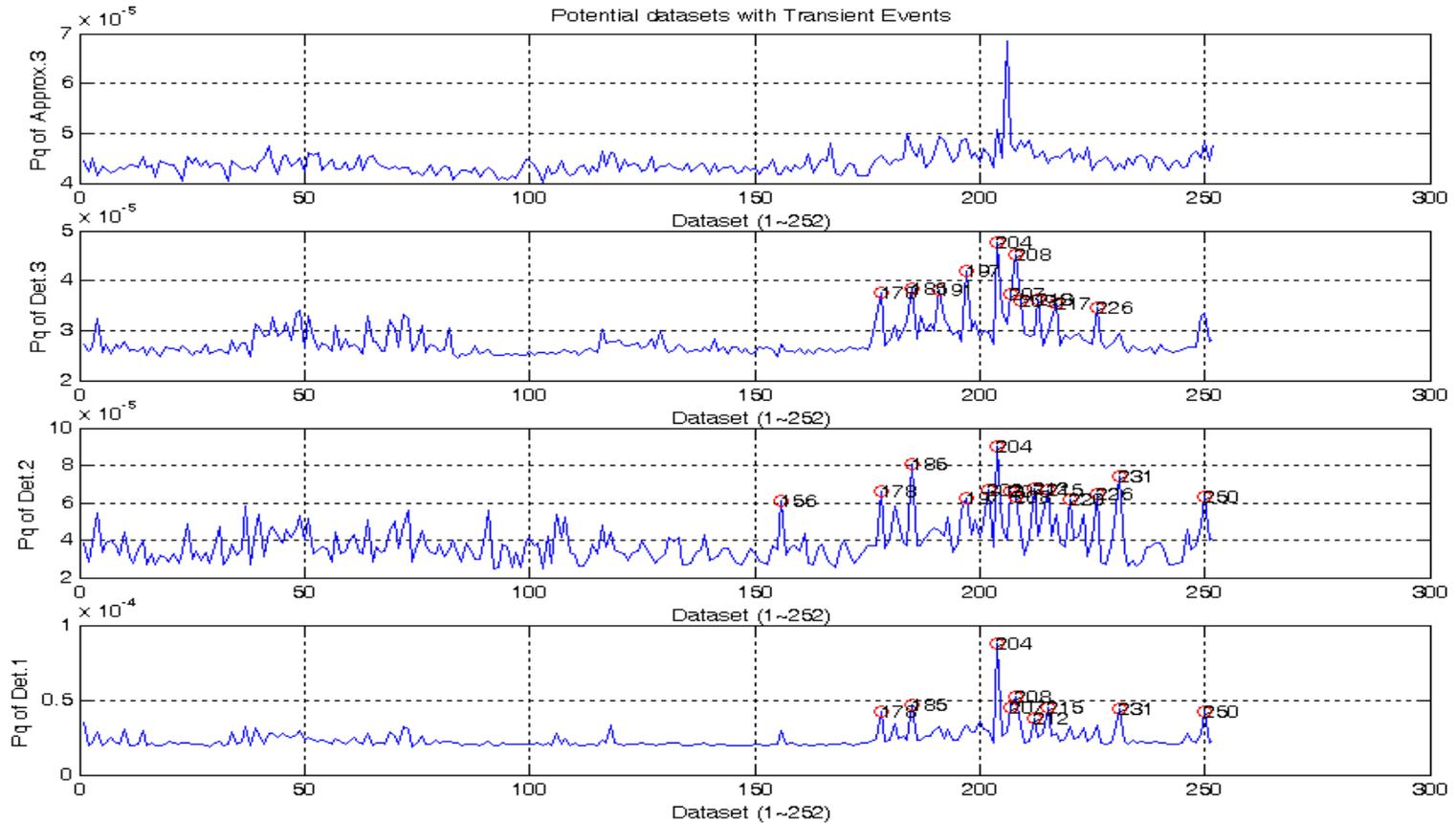
# Potential dataset detection



Sensor Accel 3 Pq analysis; X axis is the dataset no, Y axis is  $Pq_{ij}$



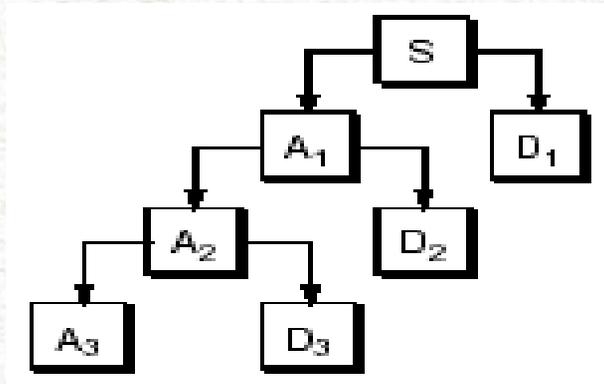
# Potential dataset detection



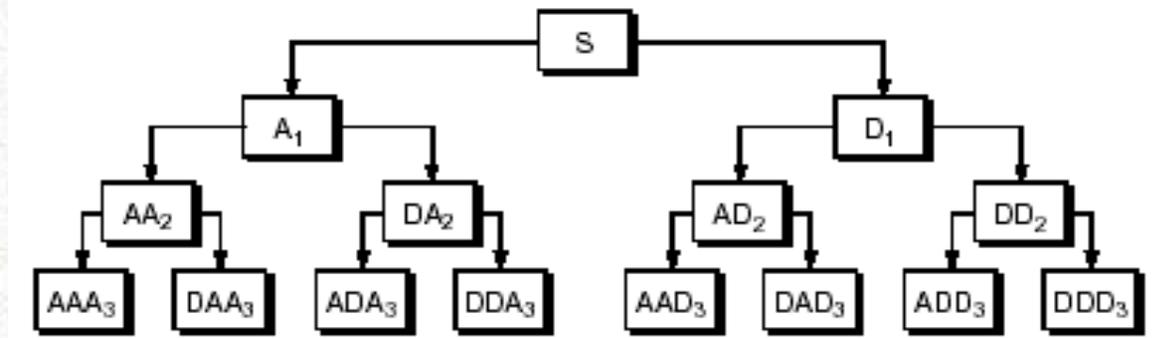
Sensor Accel 3 Pt analysis; X axis is the dataset no, Y axis is  $Pt_{ij}$

# Time/frequency Localization

- the wavelet packet analysis to reconstruct the transient events.
- Find the scale in which the coefficients are matched the most to the transient signals.



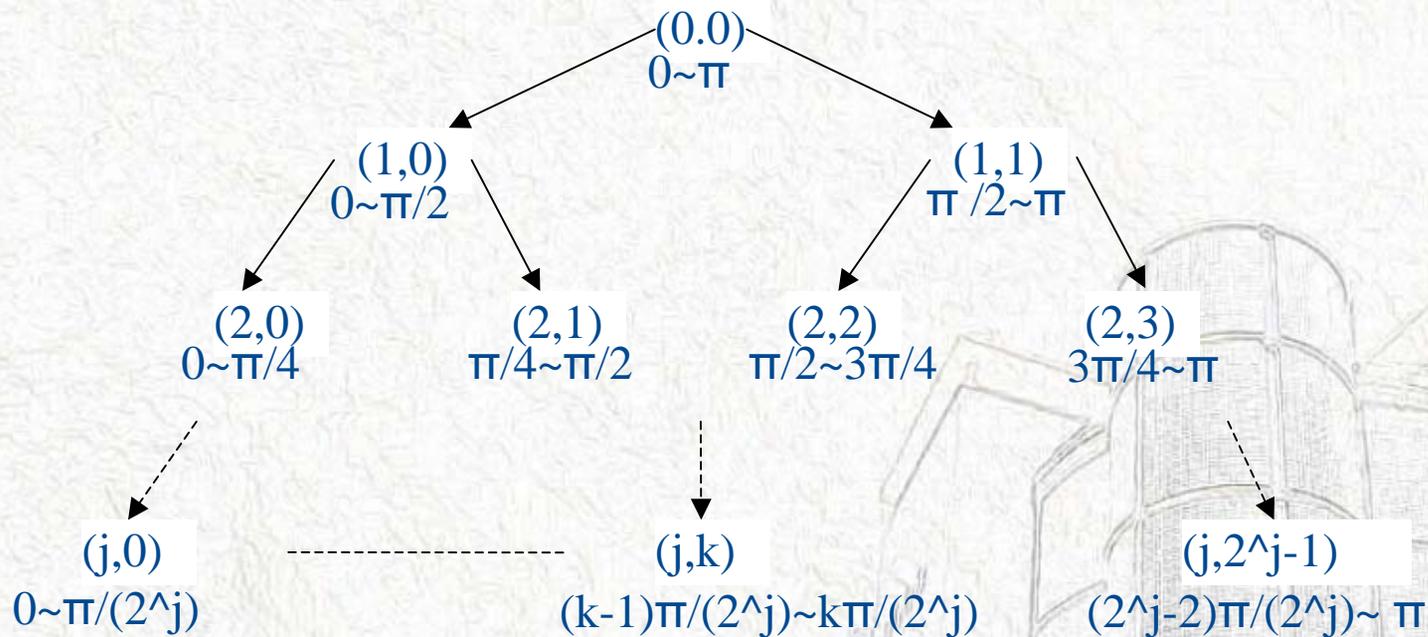
(a) tree structure of wavelet analysis



(b) tree structure of wavelet packet analysis

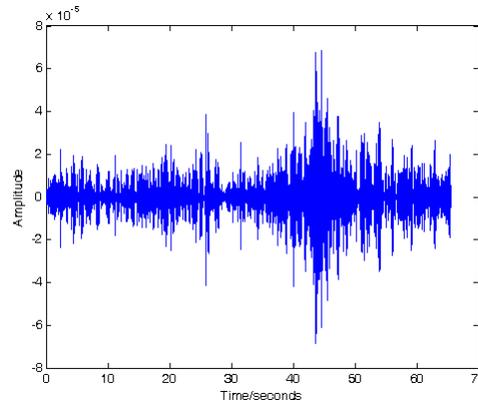
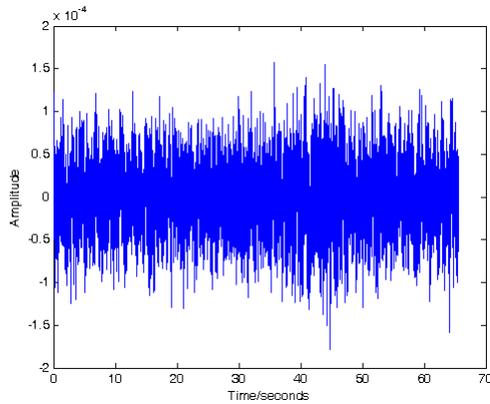
# Time/frequency Localization

## Wavelet packet analysis

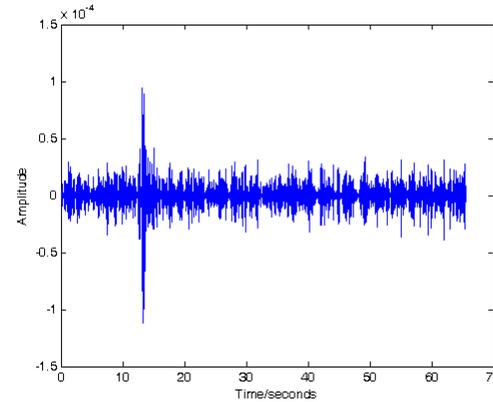
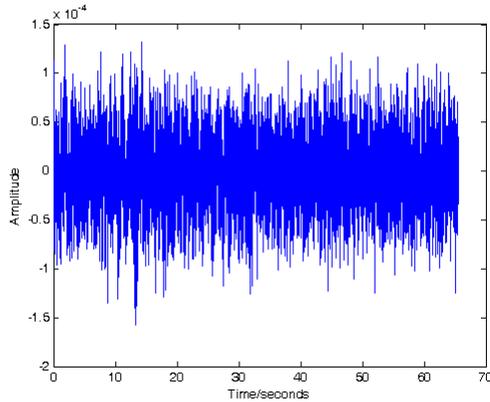


# Time/frequency Localization

## Wavelet packet analysis



(a) Original (b) Location and amplitude of transient (Rec from node 5.28)  
data 40 of Accel0, Transient signal analysis



(a) Original (b) Location and amplitude of transient (Rec from node 5.2)  
data 71 of Accel0, Transient signal analysis

From these results, one can see that wavelet transform can increase the Transient to Residual Ratio significantly. This has verified the ability of wavelets to detect transient signals, while keep their time and frequency local property.

# Time/frequency Localization

## Wavelet packet analysis

There still exist some problems:

- One can see that, if one only uses one or two nodes from the wavelet decomposition tree and then reconstruct from these coefficients one can not separate the transient signals precisely, which will still include much noise or stationary signal.
- The transient will propagate through many scales in the wavelet decomposition tree.
- For different signals, the most significant Transient to Residual Ratio are not at the same level.

# Wavelet Denoising

Simple denoising algorithms that use the wavelet transform consist of three steps:

- 1) Calculate the wavelet transform of the noisy signal;
- 2) Modify the noisy wavelet coefficients according to some rules;
  - Soft-thresholding
  - Hard-thresholding
- 3) Compute the inverse wavelet transform using the modified coefficients to reconstruct the denoised signal;

$$\delta_T^{soft}(\hat{w}_s) = \begin{cases} 0 & \text{if } |w_z| \leq T \\ w_z - T & \text{if } w_z > T \\ w_z + T & \text{if } w_z < -T \end{cases}$$

Soft-thresholding

$$\delta_T^{hard}(\hat{w}_s) = \begin{cases} 0 & \text{if } |w_z| \leq T \\ w_z & \text{if } |w_z| > T \end{cases}$$

Hard-thresholding

!!!! The key issue is how to select the threshold value



# Wavelet Denoising

## Marginal model

$$\begin{aligned}\hat{w}_s(w_z) &= \operatorname{argmax}_{w_s} p_{w_s|w_z}(w_s|w_z) \\ \hat{w}_s(w_z) &= \operatorname{argmax}_{w_s} [p_{w_z|w_s}(w_z|w_s) \cdot p_{w_s}(w_s)] \\ &= \operatorname{argmax}_{w_s} [p_{w_n}(w_z - w_s) \cdot p_{w_s}(w_s)] \\ &= \operatorname{argmax}_{w_s} [p_{w_n}(w_n) \cdot p_{w_s}(w_s)]\end{aligned}$$

$W_s$  is wavelet coefficients of real signal or transient;

$W_z$  is wavelet coefficients of measured Signal;

$W_n$  is the wavelet coefficients of the noise Or the stationary signal;

Assuming:

$$p_{w_n}(w_n) = \frac{1}{\sigma_n \sqrt{2\pi}} \cdot \exp\left(-\frac{w_n^2}{2\sigma_n^2}\right)$$

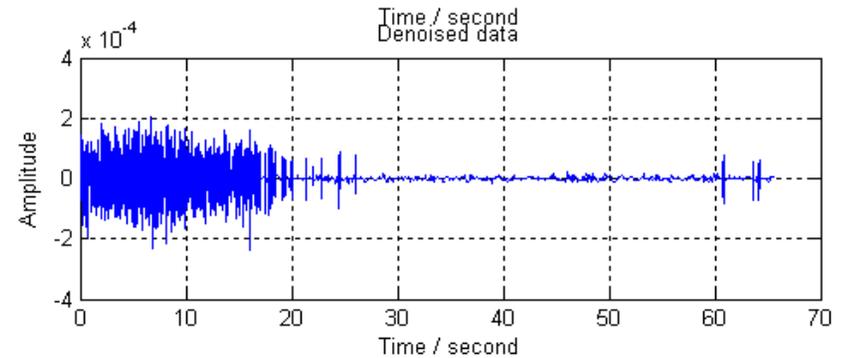
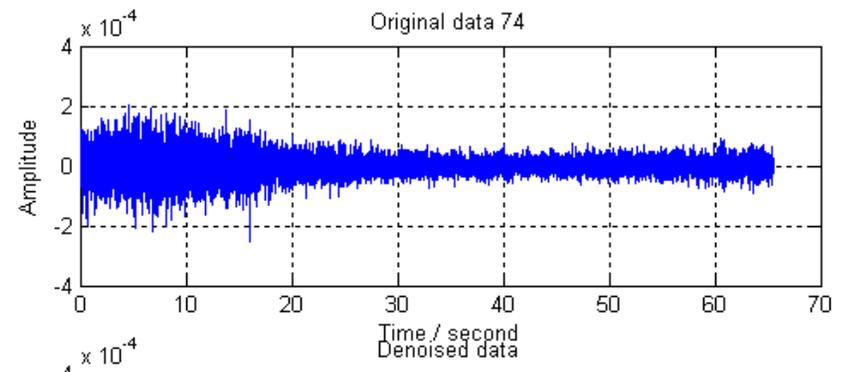
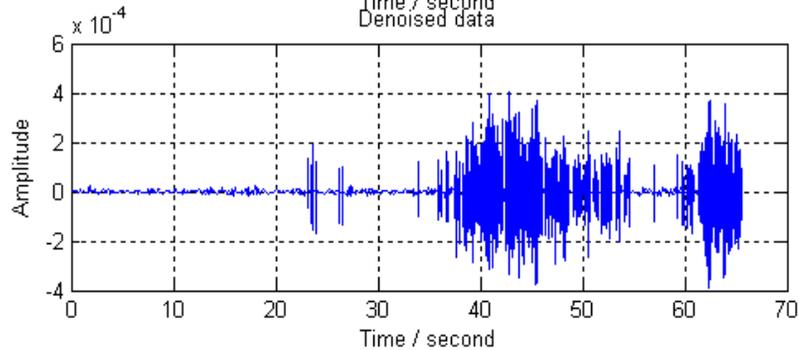
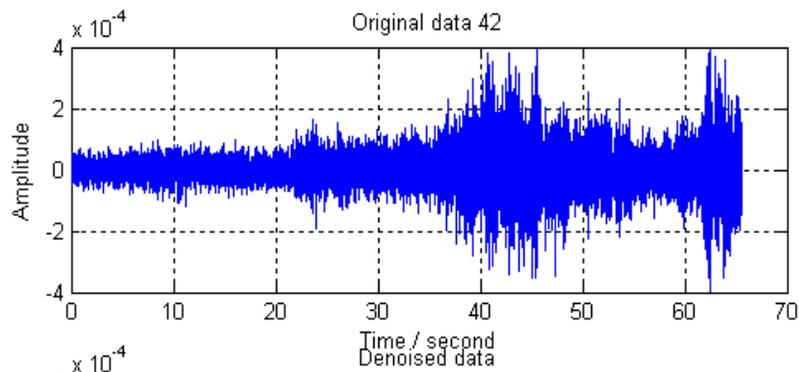
$$p_{w_s}(w_s) = \frac{1}{\sigma_s \sqrt{2}} \cdot \exp\left(-\frac{|w_s|}{2\sigma_s}\right)$$

$$\hat{w}_s = \begin{cases} 0 & \text{if } |w_z| \leq \frac{\sqrt{2}\sigma_n^2}{\sigma_s} \\ w_z - \frac{\sqrt{2}\sigma_n^2}{\sigma_s} & \text{if } w_z > \frac{\sqrt{2}\sigma_n^2}{\sigma_s} \\ w_z + \frac{\sqrt{2}\sigma_n^2}{\sigma_s} & \text{if } w_z < -\frac{\sqrt{2}\sigma_n^2}{\sigma_s} \end{cases}$$

$\sigma_n$  From the first level detail coefficients;  $\hat{\sigma}_s = \sqrt{\sigma_z^2 - \sigma_n^2}$

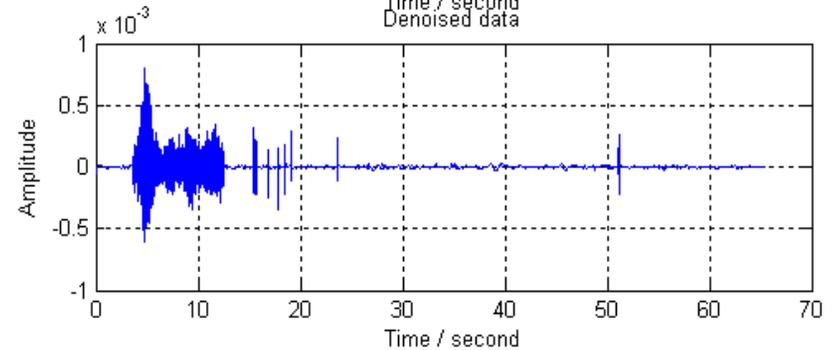
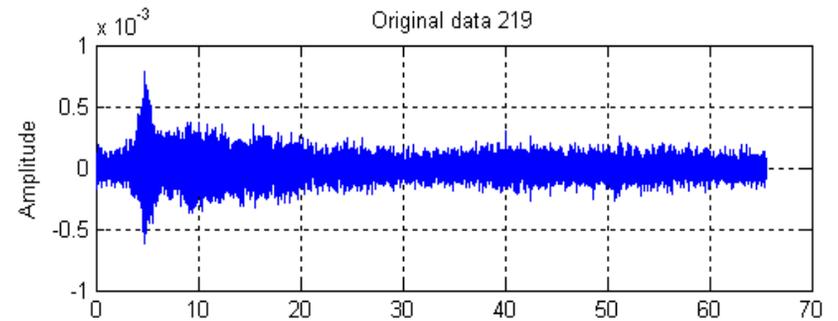
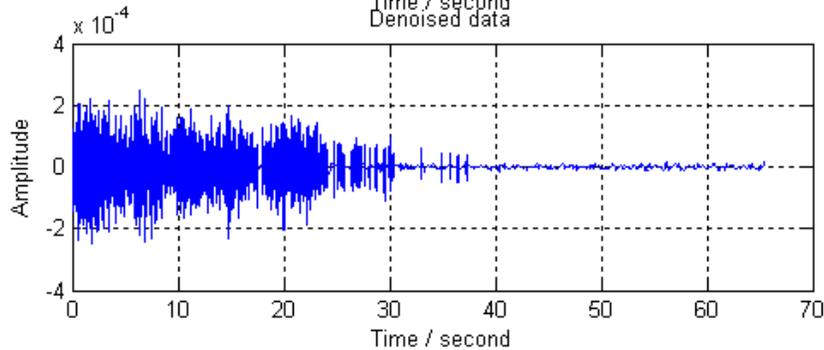
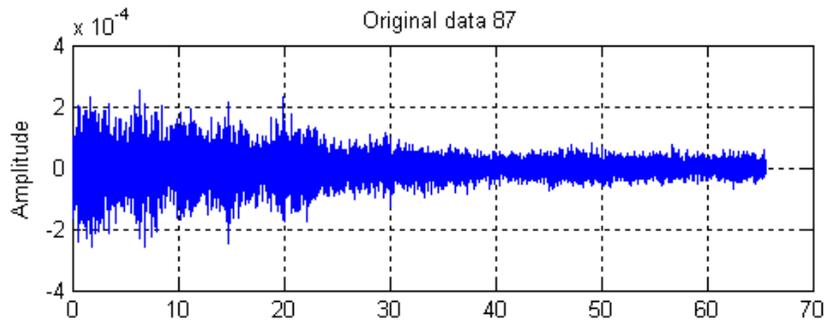
# Wavelet Denoising

## Marginal model



# Wavelet Denoising

## Marginal model



# Wavelet Denoising

## Bivariate shrinkage

- Marginal models do not model the statistical dependencies between wavelet coefficients.
- However, there are strong dependencies between neighboring coefficients such as between a coefficient, its parent (adjacent coarser scale locations), and their siblings (adjacent spatial locations).
- The bivariate shrinkage

# Wavelet Denoising

## Bivariate shrinkage

$$\begin{aligned}\hat{w}_s(w_z) &= \operatorname{argmax}_{w_s} p_{w_s|w_z}(w_s|w_z) \\ \hat{w}_s(w_z) &= \operatorname{argmax}_{w_s} [p_{w_z|w_s}(w_z|w_s) \cdot p_{w_s}(w_s)] \\ &= \operatorname{argmax}_{w_s} [p_{w_n}(w_z - w_s) \cdot p_{w_s}(w_s)] \\ &= \operatorname{argmax}_{w_s} [p_{w_n}(w_n) \cdot p_{w_s}(w_s)]\end{aligned}$$

Assuming:

$$P_{w_n}(w_n) = \frac{1}{2\pi\sigma_n^2} \exp\left(-\frac{n_c^2 + n_p^2}{2\sigma_n^2}\right)$$

$$P_{w_s}(w_s) = \frac{3}{2\pi\sigma_s^2} \exp\left(-\frac{\sqrt{3}}{\sigma_s} \sqrt{w_{s_p}^2 + w_{s_c}^2}\right)$$

The suffix c refers to the child coefficients  
The suffix p refers to the parent coefficients

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$W_z$  is wavelet coefficients of measured Signal;

$W_n$  is the wavelet coefficients of the noise Or the stationary signal;

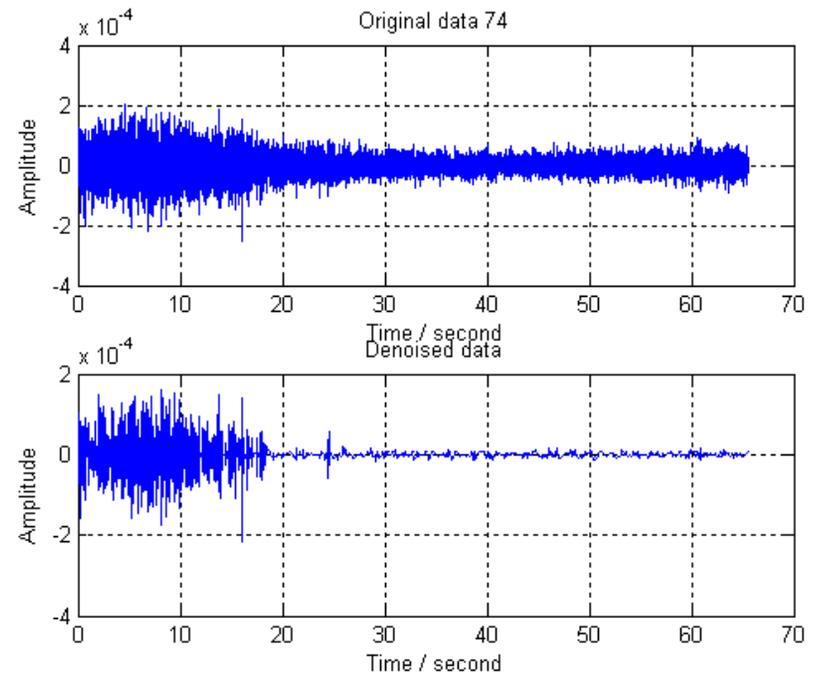
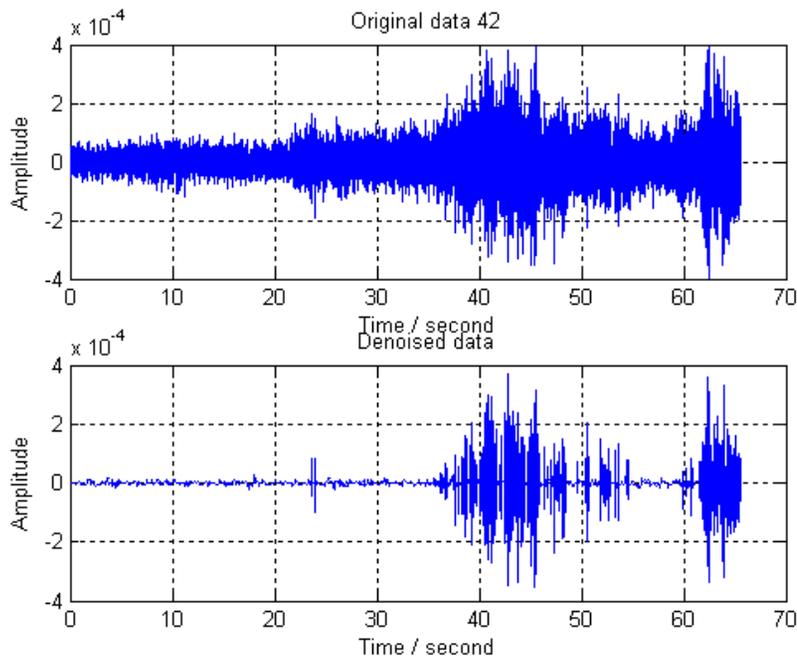
$$\hat{W}_{s_c} = \frac{\left( \sqrt{w_{z_c}^2 + w_{z_p}^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma_s} \right)_+}{\sqrt{w_{z_c}^2 + w_{z_p}^2}} \cdot w_{z_c}$$

$$(g)_+ = \begin{cases} 0, & \text{if } g < 0 \\ g, & \text{otherwise} \end{cases}$$



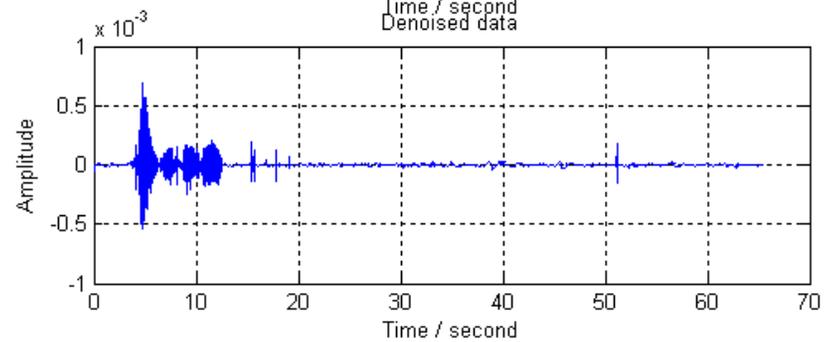
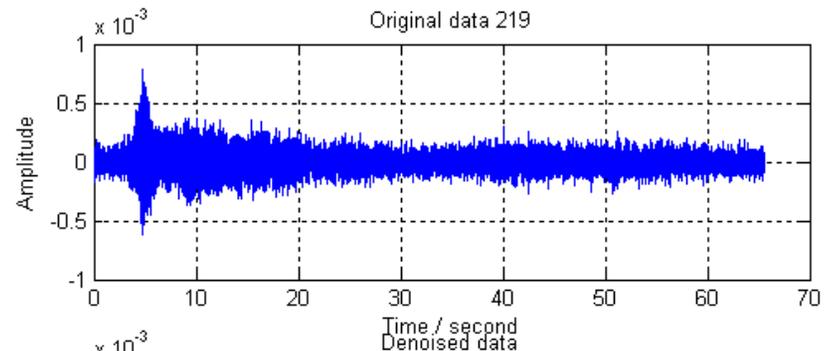
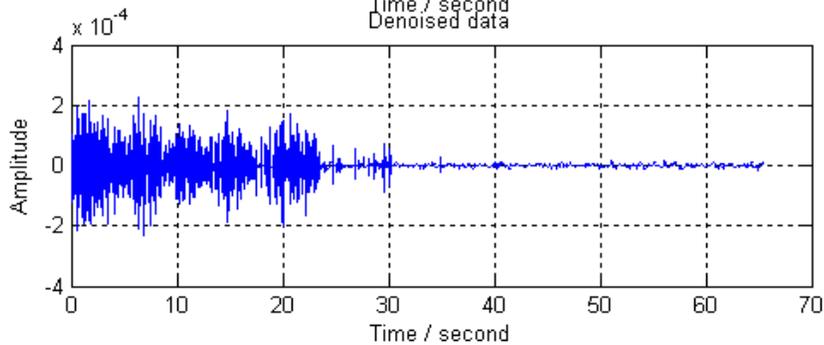
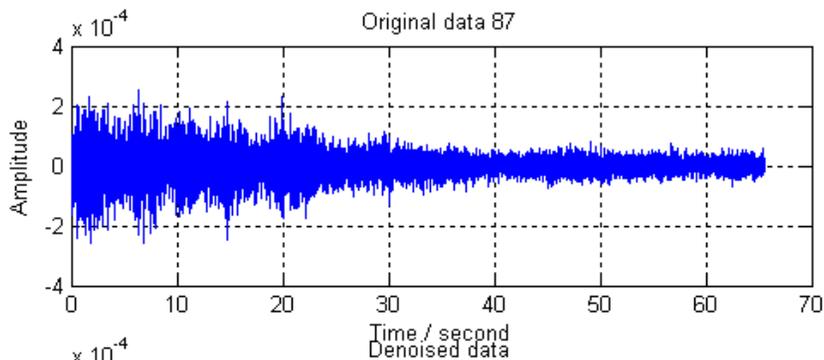
# Wavelet Denoising

## Bivariate shrinkage



# Wavelet Denoising

## Bivariate shrinkage



# Summary

- The Multi-scale root-mean-square analysis have been successfully used to find the potential data that contain transient events, which significantly reduce the work load to analysis all the data sets.
- Wavelet packet analysis have been used the get the time and frequency structure of the transient events;
- Wavelet domain denoising techniques are used for the extraction of transient events from measured dataset. Three typical wavelet domain denoising methods: Marginal model; Bivariate model and HMT, have been used to identify the transient events.

# Thanks for your attention !

Further discussion please email to: [x.jiang@hud.ac.uk](mailto:x.jiang@hud.ac.uk)

