Synthesis of digital correction filters

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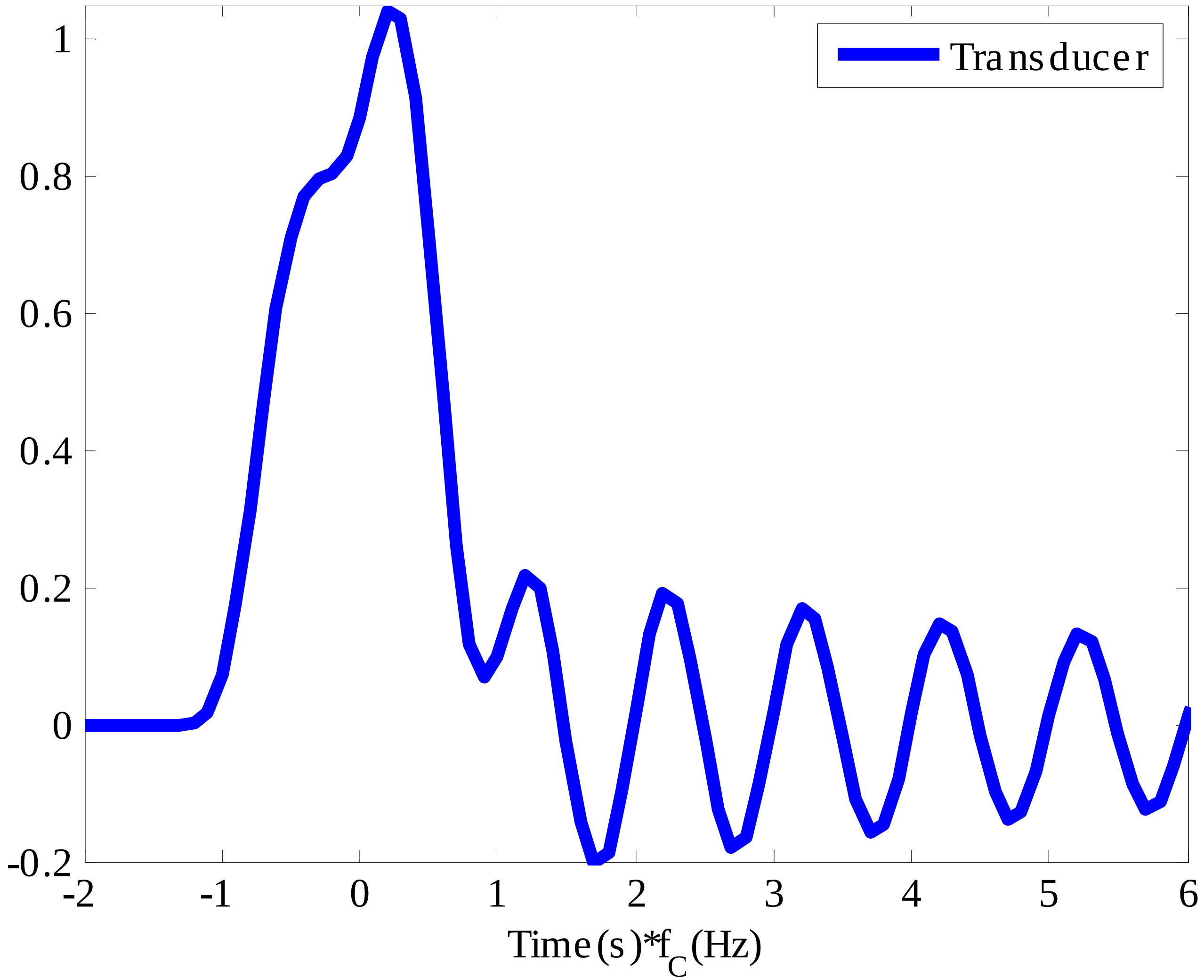
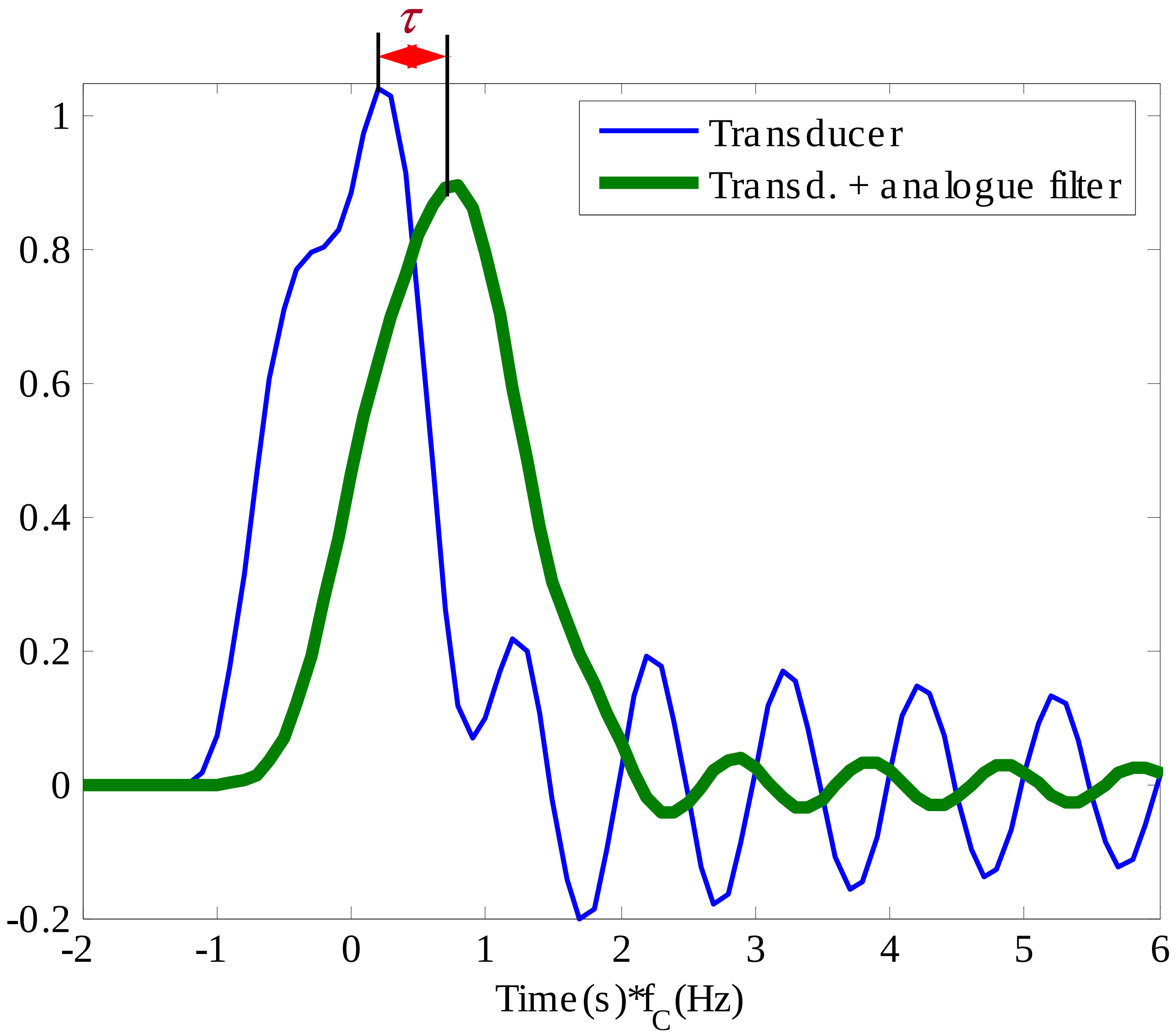
Disposition

- Introduction – result
- Prelude
  - Dynamic error and time delay of signal
  - Characterization: ”Dynamic calibration”
  - Digital filter signal correction (…of measured signals…)
- ***Digital correction filter synthesis (9 steps)***
- Conclusions

Introduction: Signal correction



Introduction: Signal correction



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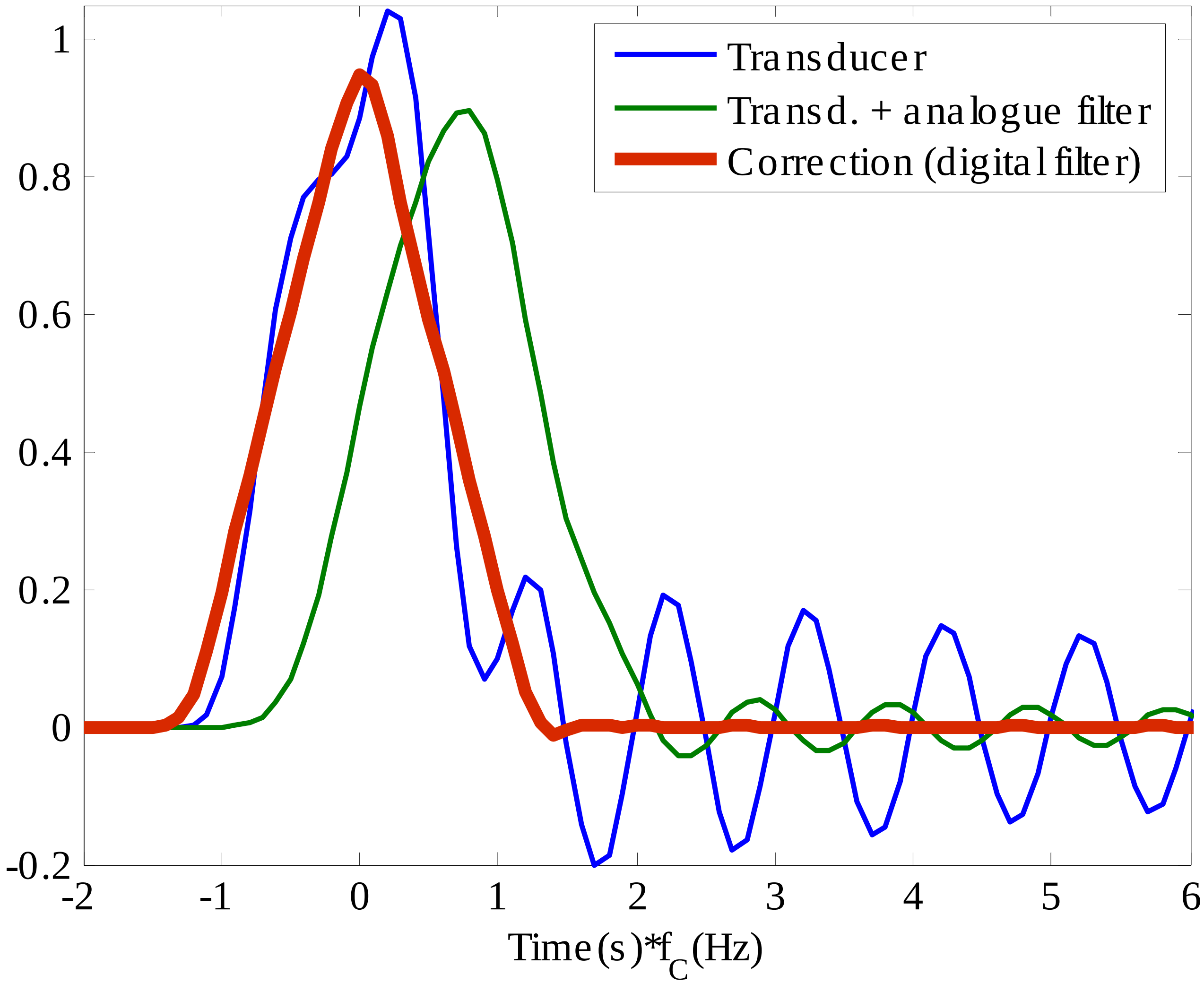
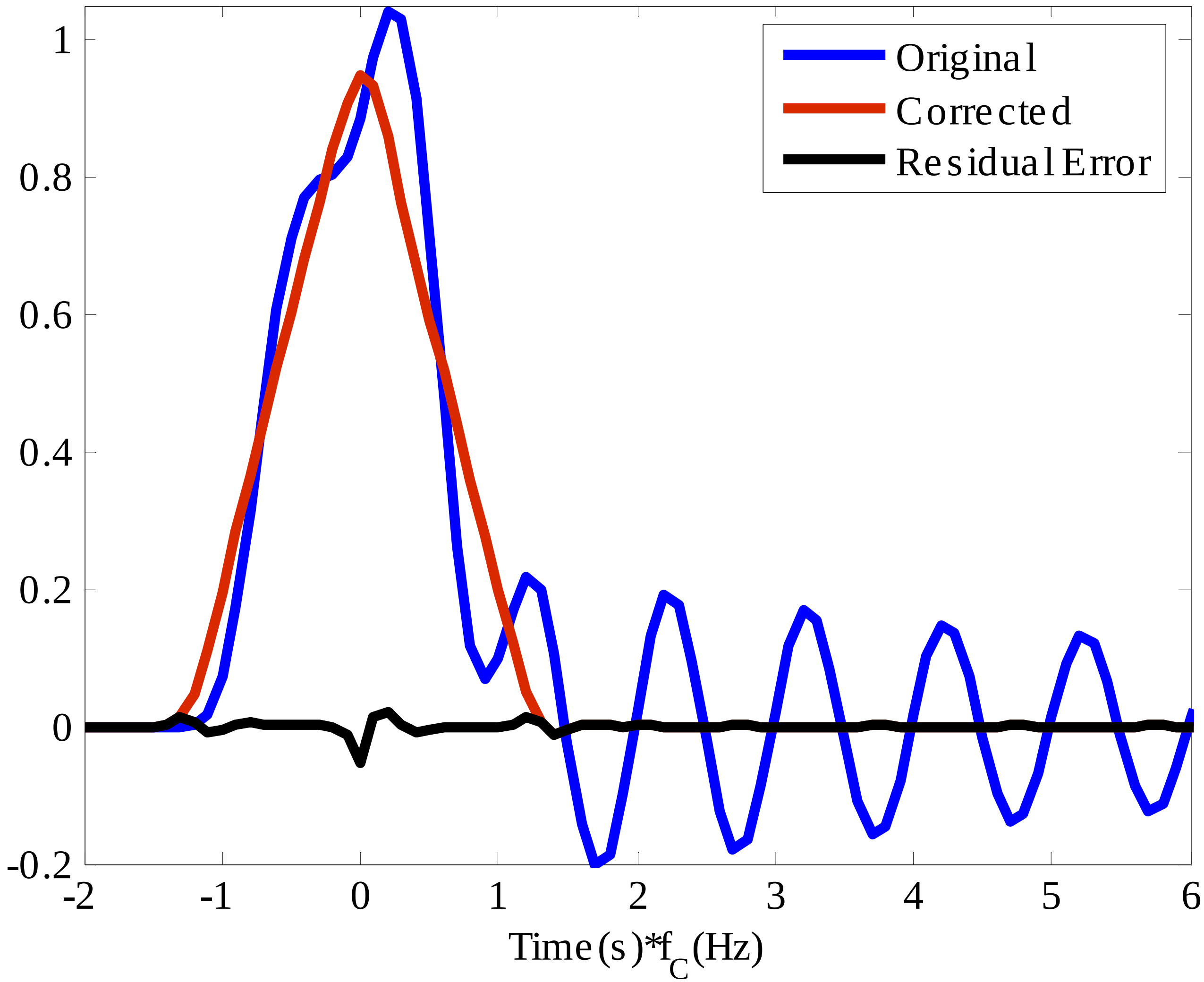
Introduction: Signal correction



Introduction: Signal correction



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Introduction:
Potential for improving dynamic response?

Signal processing often not optimal:

- General filters – not designed for actual sensors
- Analogue filters – limited flexibility
- Real time applications – requires causality
- 'Correction' limited – not the entire system considered

Here:

- Custom filter synthesis – set by actual measurement system
- Digital filters – for flexibility
- Post-processing, allows for:
  - Non-causal filtering
  - Time-reversed filtering
  - Translation and interpolation in time
- Dynamic characterization – determines filter parameters
- Holistic perspective – include various types of subsystems
- Tools of control theory – system described by differential eq's

Dynamic error / time delay of signal

Propose measure: ***’Response error’:***

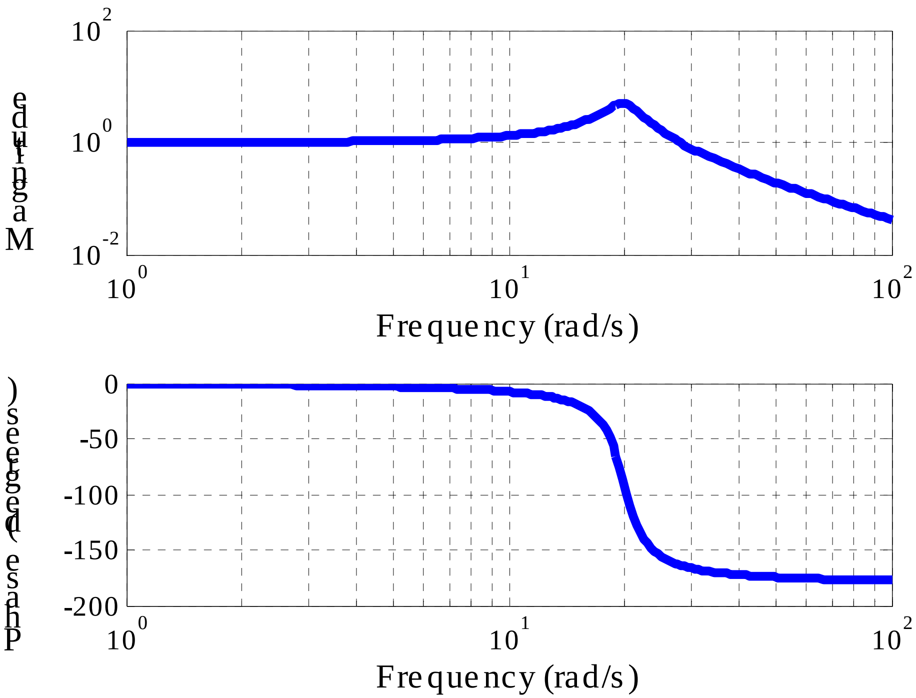
- Article submitted to *Meas. Science and Techn.* (IOP)
- Manuscript available

Simple force transducer – (FT)

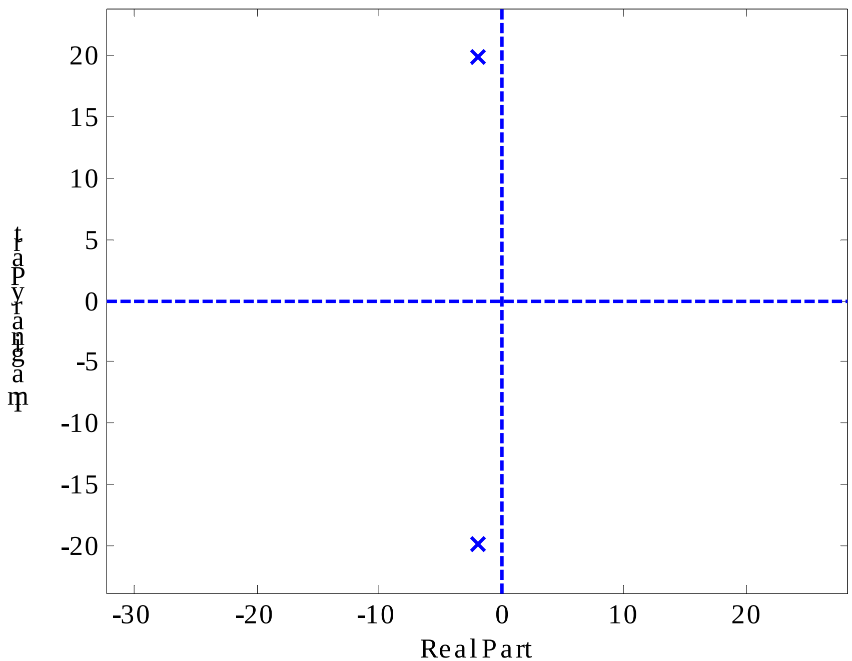
- Mechanical: Simple resonance (masses + damped spring)
- Electrical: Strain gauge (resistive, static model - large bw)

Bode diagram

(ω_C=20 rad/s, ζ=0,1)



Zeros and Poles

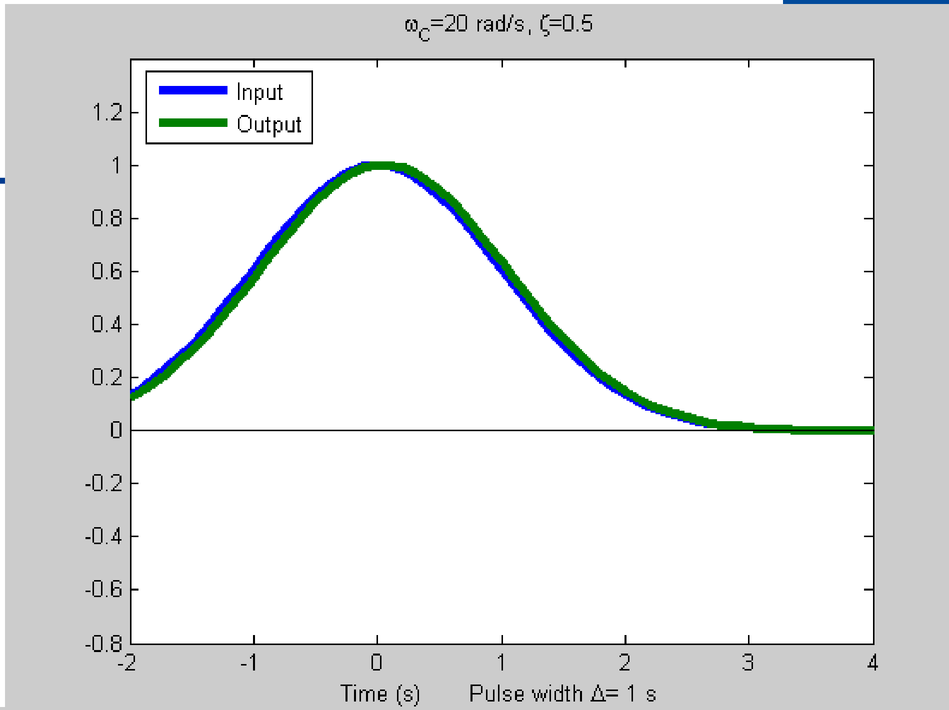


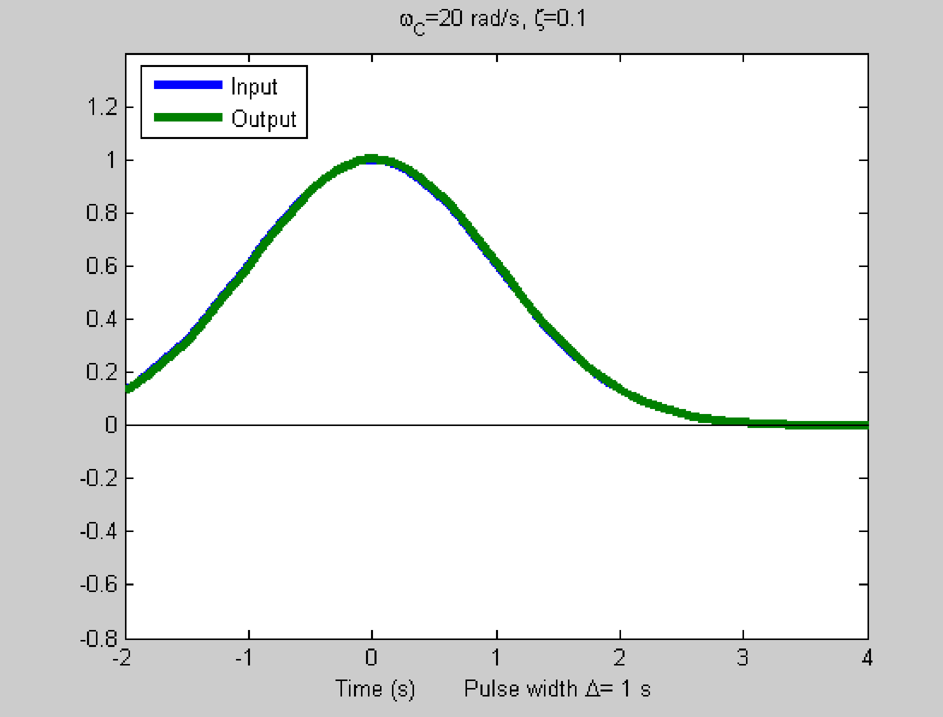
Pulse [Gaussian] deformation 'FT'

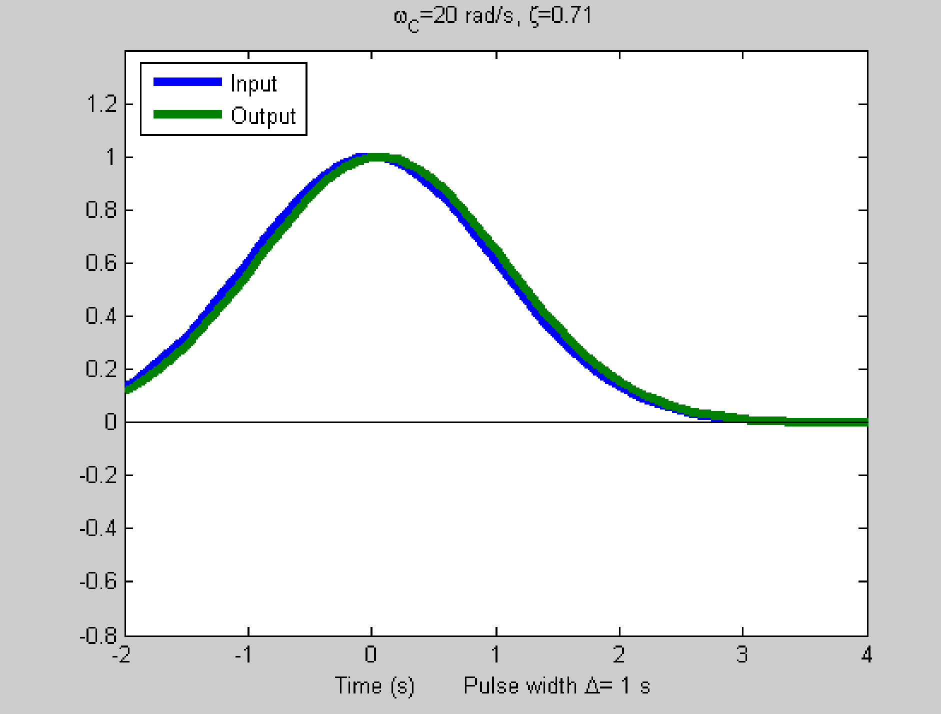
$$\omega_B = 1 \rightarrow 10\ rad/s$$

$$\omega_C / \omega_B = 20 \rightarrow 2$$

$$\zeta = 0.1, 0.5, 1/\sqrt{2} \approx 0{,}707$$







Characterization
- ’Dynamic Calibration’

Characterization (mechanical) of accelerometer

$$p_{1,2} = 2\pi f_0 \left(-\delta \pm i\sqrt{1-\delta^2} \right)$$

Identifikation von Beschleunigungsaufnehmern mit hochintensiven Stößen

Alfred Link, Wolfgang Wabinski, Hans-Jürgen von Martens, PTB Braunschweig und Berlin

Manuskripteingang: 08. November 2004; zur Veröffentlichung angenommen: 13. November 2004

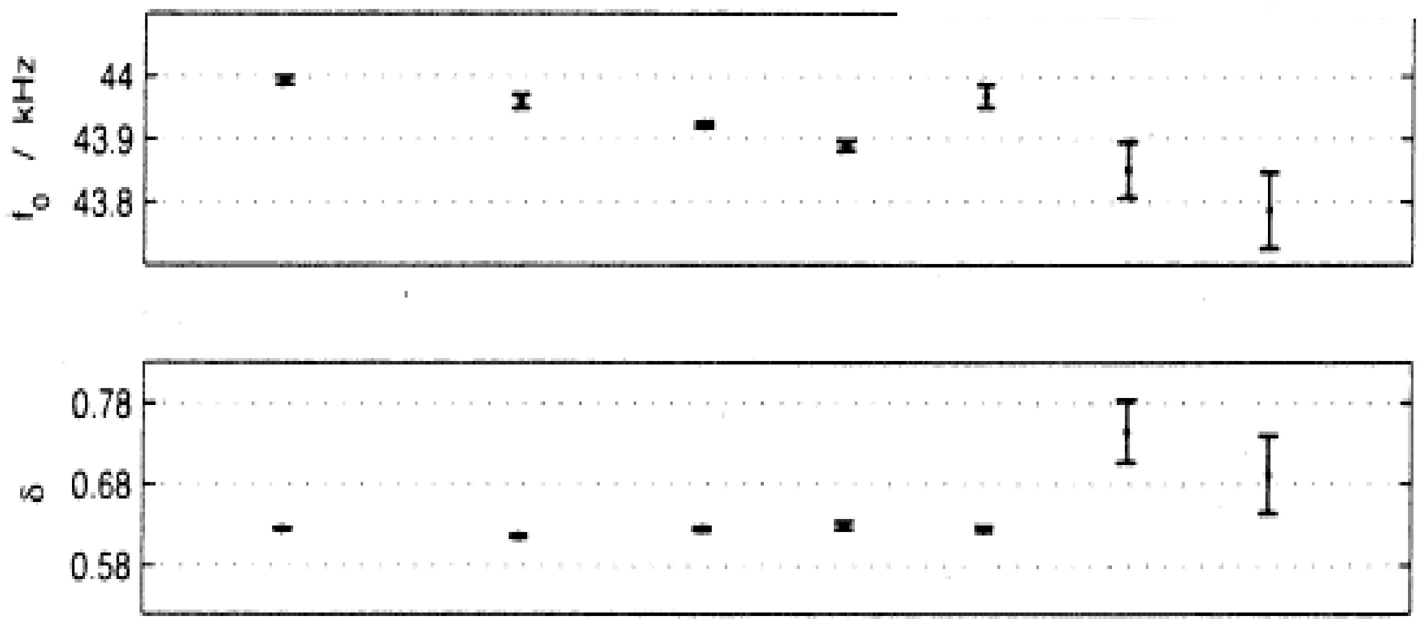
Das Ein-/Ausgangsverhalten von Beschleunigungsaufnehmern wurde in einem breiten Frequenz- und Amplitudenbereich durch ein System 2. Ordnung modelliert. Die Identifikation der Systemparameter erfolgte mit impulsförmigen Beschleunigungen hoher Intensität. Diese parametrische Systemidentifikation erweitert und verbessert die Bestimmung des Übertragungsverhaltens von Beschleunigungsaufnehmern gegenüber den bekannten nichtparametrischen Verfahren.

Schlagwörter: Beschleunigungsaufnehmer, Identifikation, Übertragungsfunktion, Kalibrierung

Accelerometer Identification by High Shock Intensities

The input/output behaviour of accelerometers has been modeled within a broad frequency and amplitude range by a second order system. The system parameters have been identified using impulse-like accelerations of high intensity. This parametric system identification extends and improves the determination of the system behaviour of accelerometers in relation to the known nonparametric identification methods.

Keywords: Accelerometer, identification, transfer function, calibration



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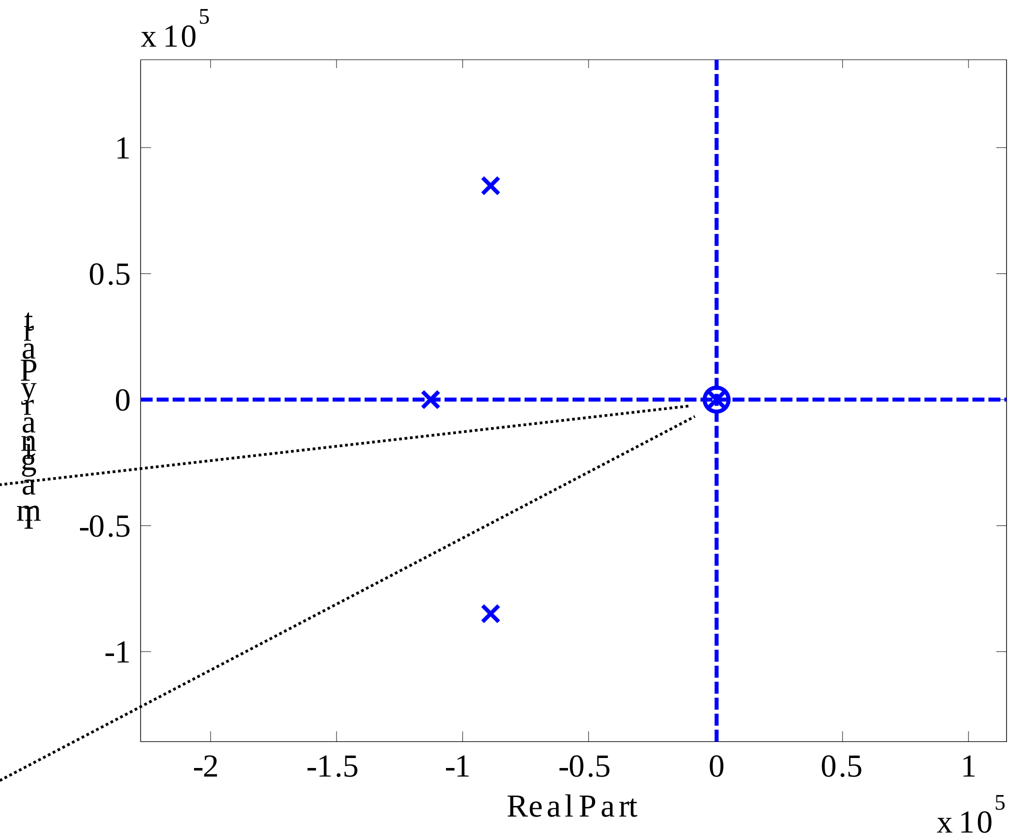
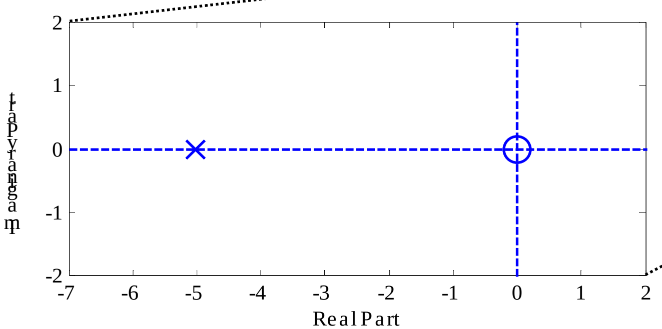
Characterization (electrical) signal conditioning

- National Instrument DAQ SC2345 signal conditioning
  - Amplifier (High bandwidth)
  - Filter
    - Bessel
    - Low-pass
      - n=3
      - f_C=19 kHz
    - High-pass
      - n=1
      - f_C=0.8 Hz



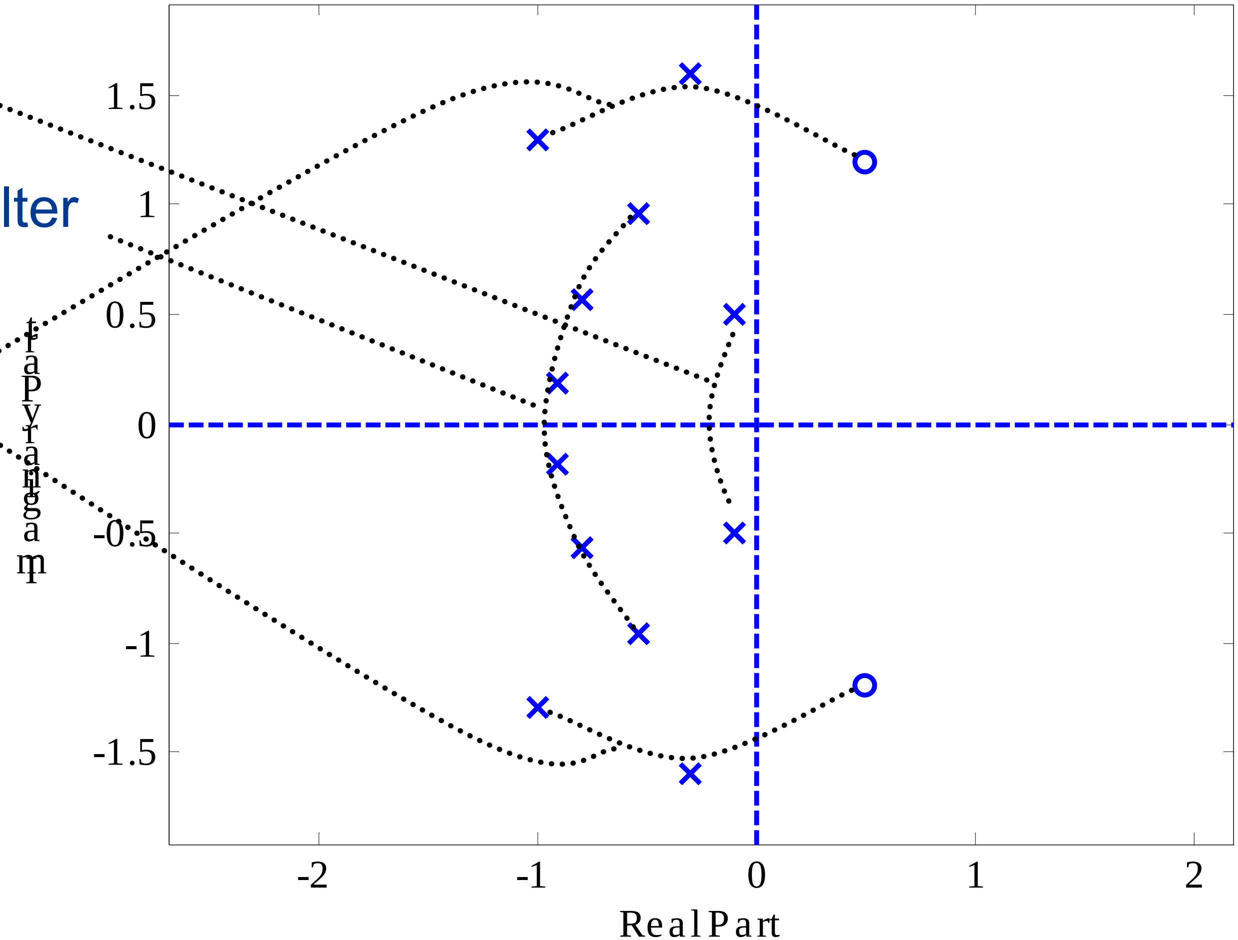


Characterization total measurement system

All linear subsystems

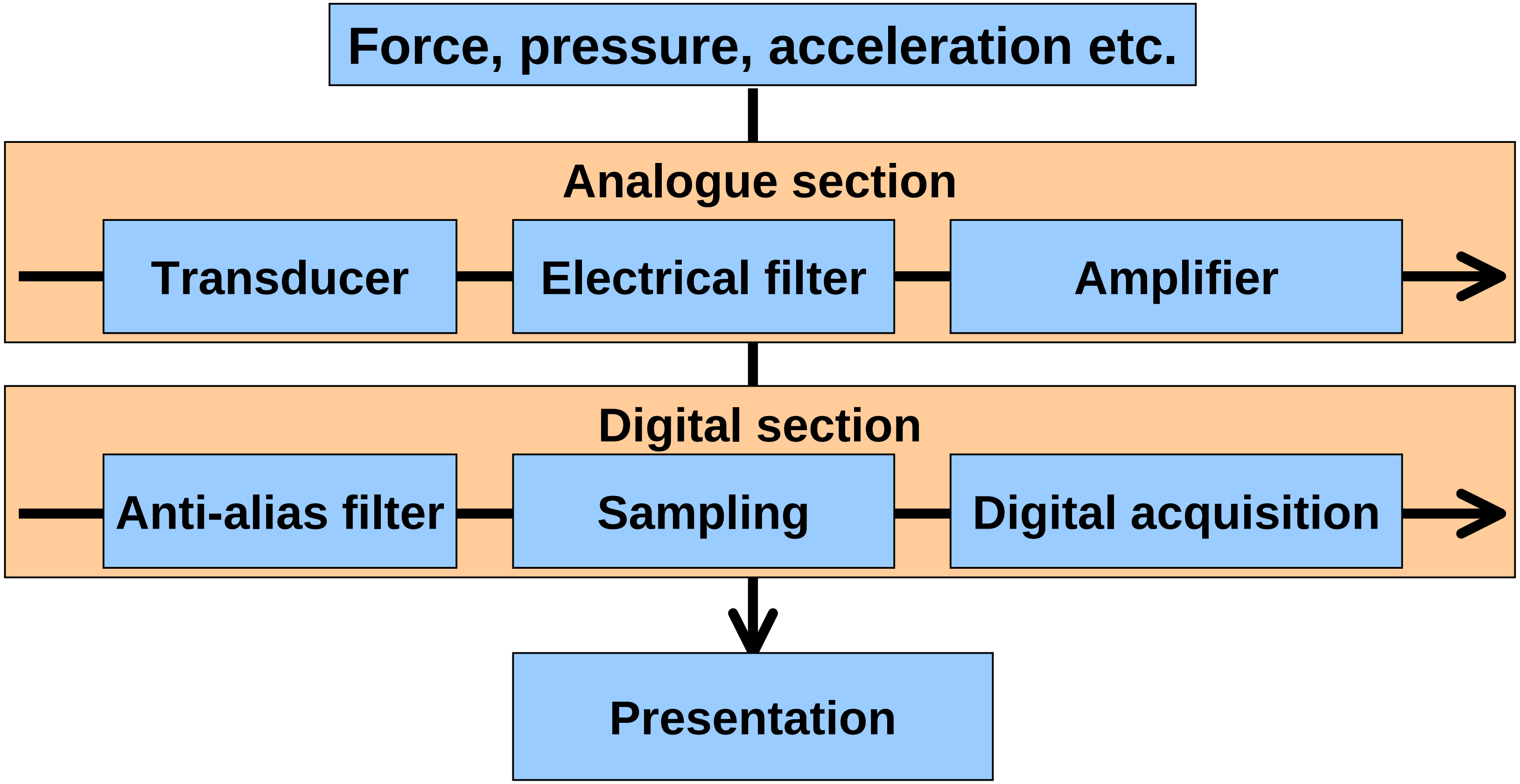
- Similar dynamic description!

1. Sensor
   – Mech.
2. Application filter
   – Electr.
3. Amplifier
   – Electr.



Digital filter signal correction

Typical measurement system



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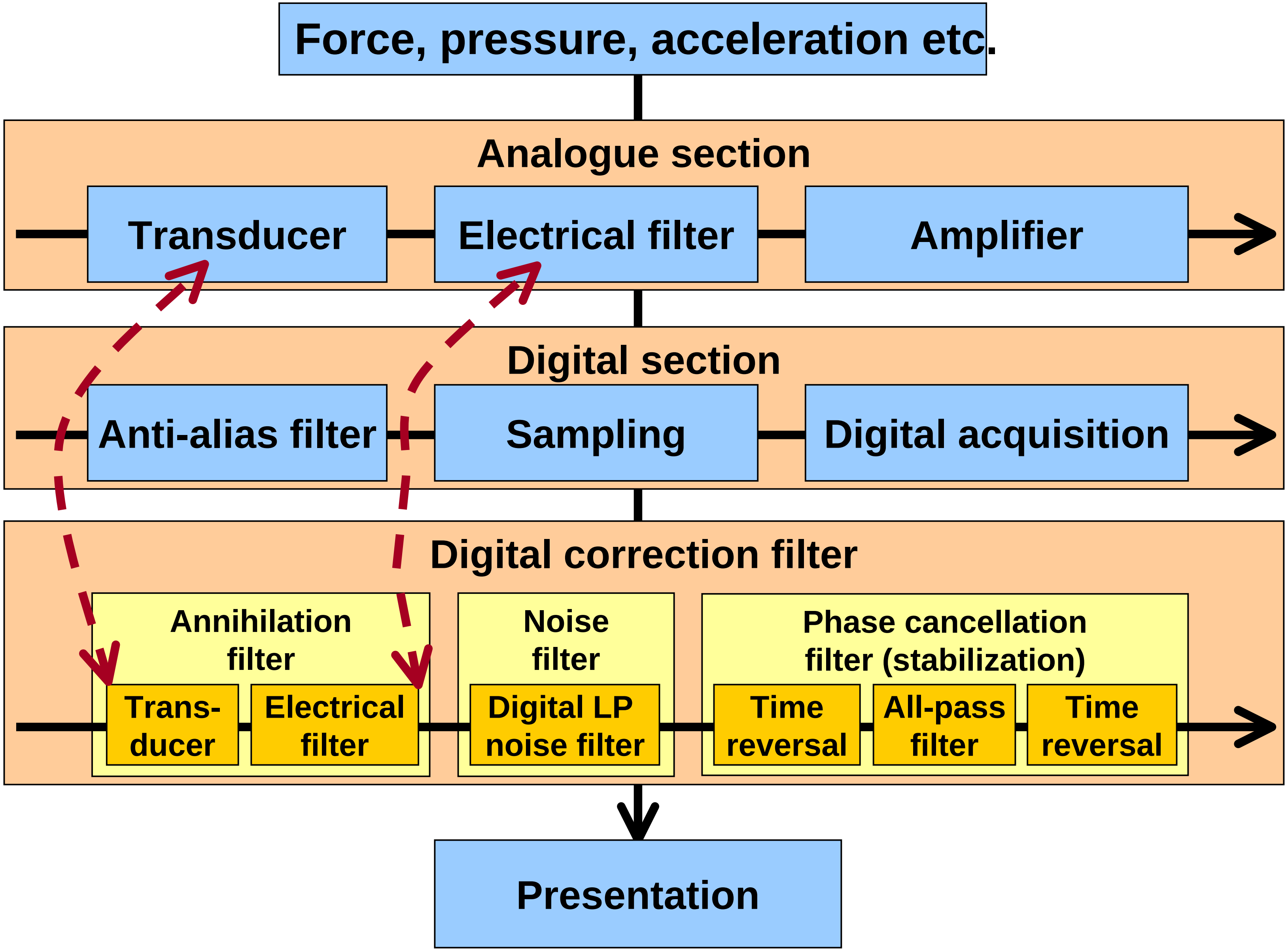
Correction of measurement system



Step 1:
Annihilation filter
– Annihilation

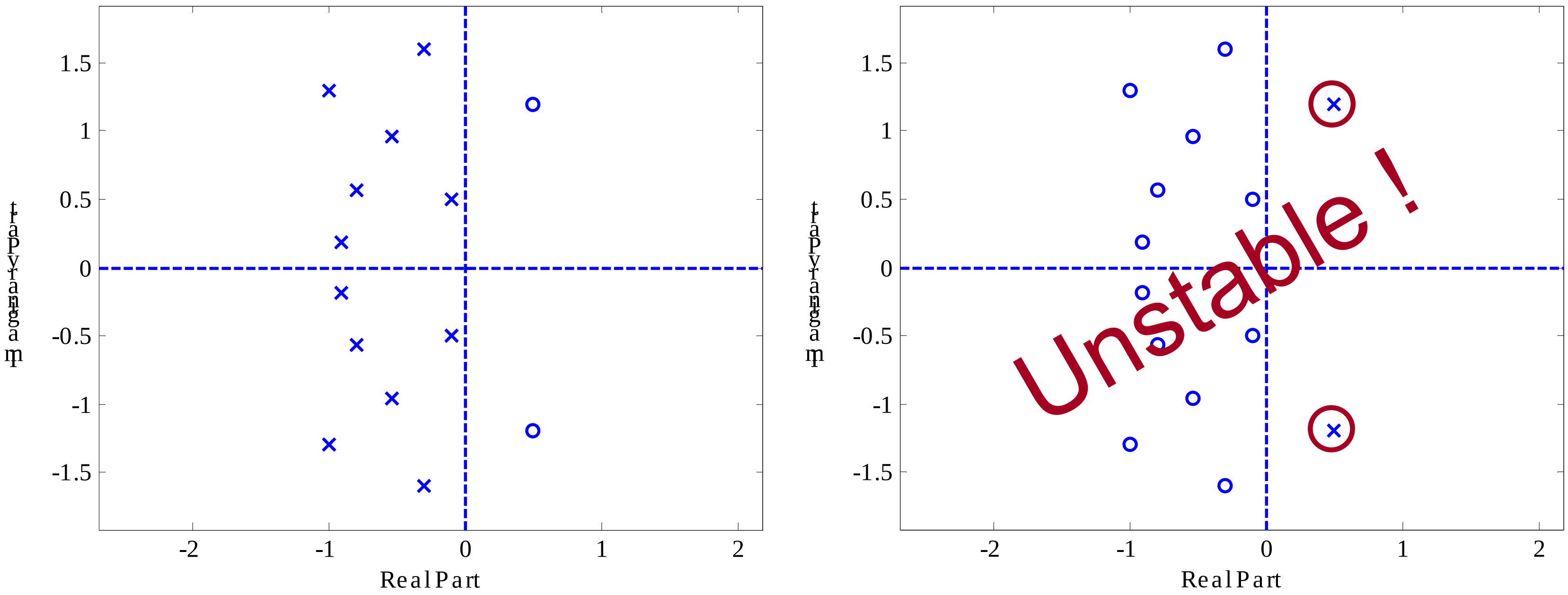
Annihilation $\Rightarrow H^{-1}$

(Here: annihilate continuous time subsystems only)

Recipe:

1. Create correction prototype:
   - Include all *relevant* s-plane poles and zeros
2. Exchange poles with zeros and vice versa

Measurement system | Correction prototype



Step 2:
Annihilation filter
– Stabilization

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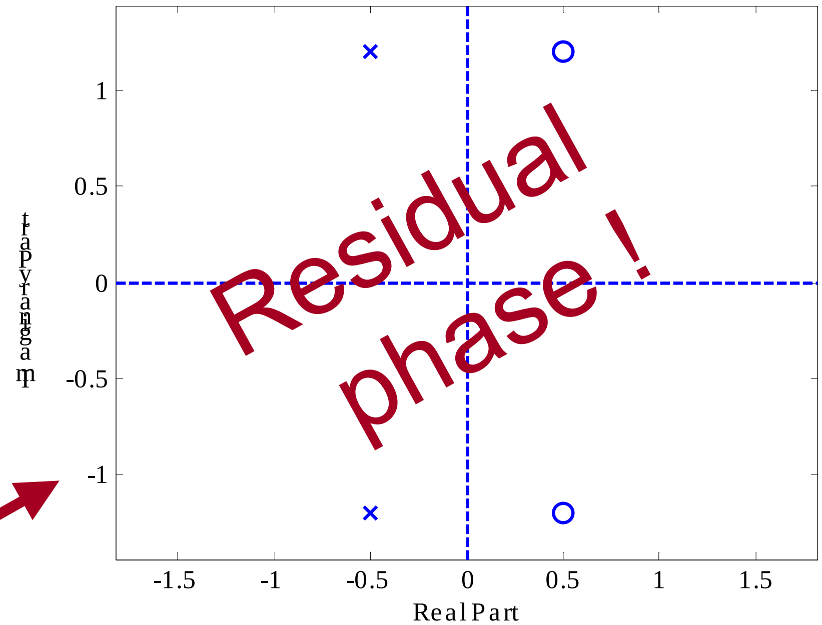
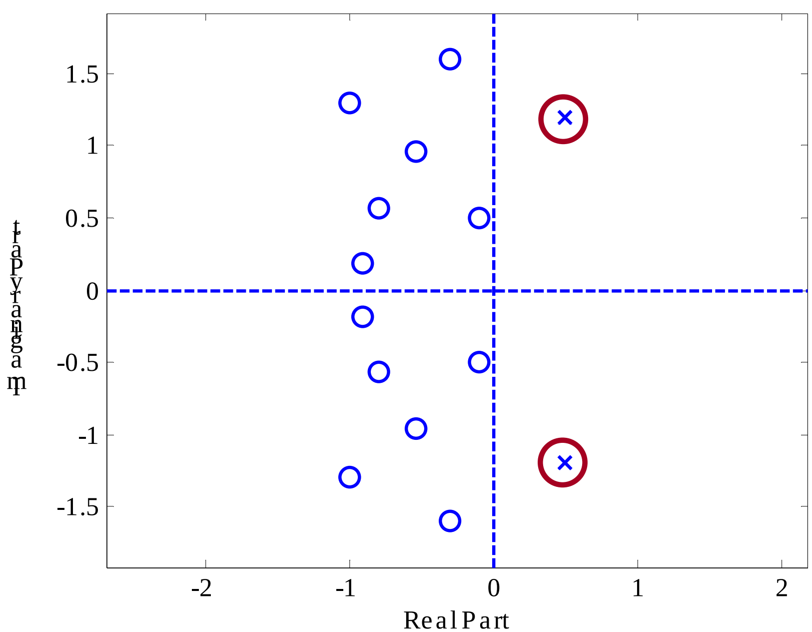
Stabilization

Recipe:

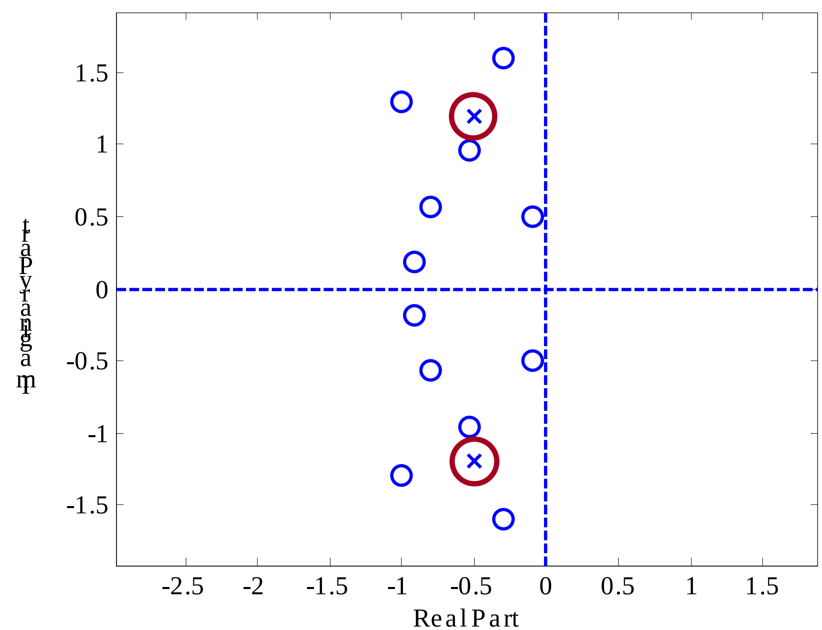
1. Reflect unstable poles through the imaginary axis
   - Preserves amplification
   - Introduces undesired additional all-pass filter



Before stabilization



Stabilized prototype



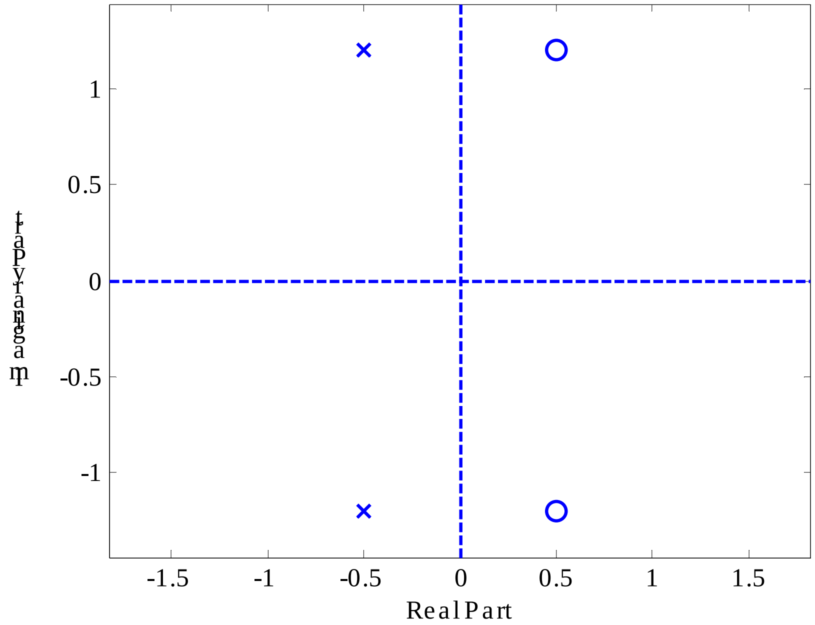
Step 3:
Phase cancellation filter
– Phase caused by stabilization

Cancellation of phase caused by stabilization

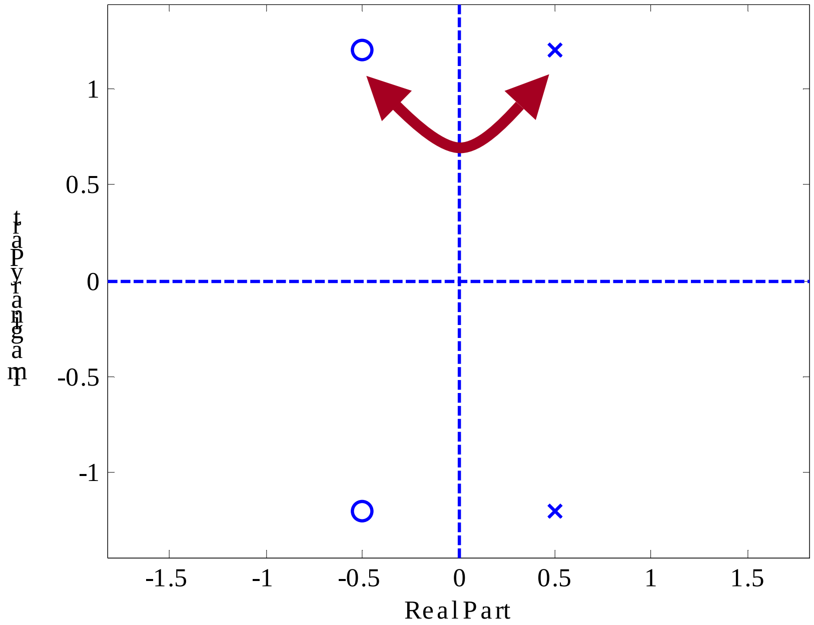
Recipe:

1. Reverse signal in time $t \rightarrow -t$
2. Filter with residual all-pass filter $|H| = 1$
3. Reverse back in time $t \rightarrow -t$

Residual all-pass
due to stabilization



Time-reversed all-pass filtering
= negative residual phase



Step 4:
Continuous to discrete time filter
– Exponential mapping

’s’ → ’z’

Continuous time

Governed by

- ***Differential*** equations

Modeled by

- ***s***-transform
- Poles/zeros

Fundamental component

$$\lim_{h\to 0}\left(\frac{x(t+h)-x(t)}{h}\right)$$

Frequency response function

- Complex-valued!
- Imaginary s-axis

Discrete time

Governed by

- ***Difference*** equations

Modeled by

- ***z***-transform
- Poles/zeros

Fundamental component

$$\frac{x(t+T_S)-x(t)}{T_S}$$

Frequency response function

- Complex-valued!
- Unit circle z-plane

$$s = i\omega \quad \longleftarrow \quad T_S \to 0 \quad z = \exp(i\omega T_S) = \exp(sT_S) \to 1 + i\omega T_S$$

Continuous-time (CT) to discrete-time (DT) mapping

Recipe:

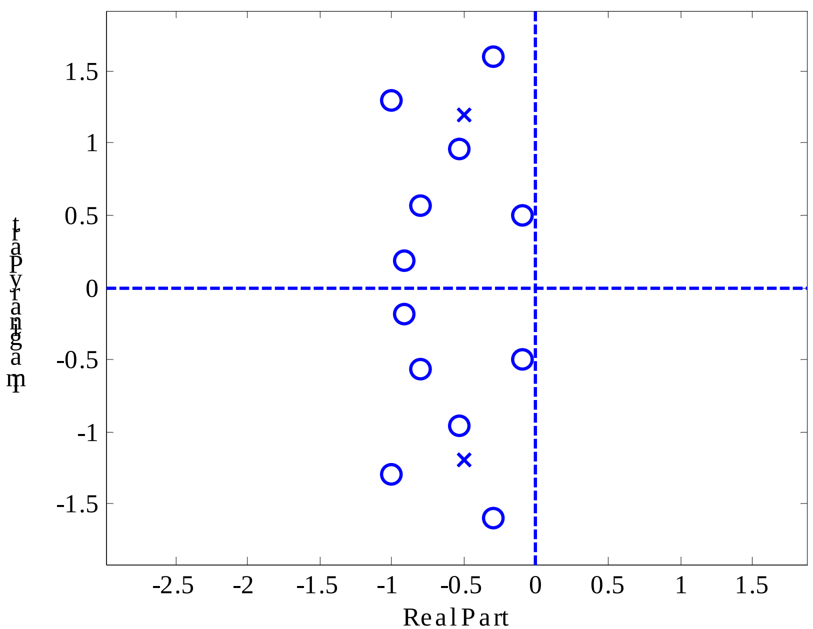
1. CT zeros and poles ➡ DT zeros and poles: $r_k = \exp\left(\tilde{r}_k T_S\right)$

   $\tilde{r}_k = \tilde{z}_k, \tilde{p}_k$ $\qquad$ $r_k = z_k, p_k$

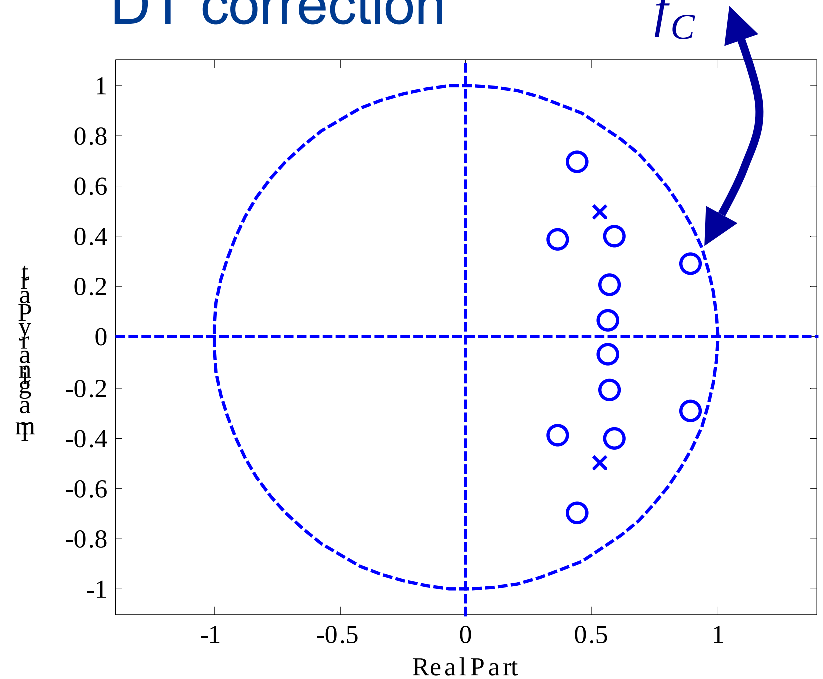
   - Approximation
   - Asymptotically identical FRF!!! $\left(T_S \to 0\right)$

$$\frac{f_S}{f_C} = 20$$

CT correction

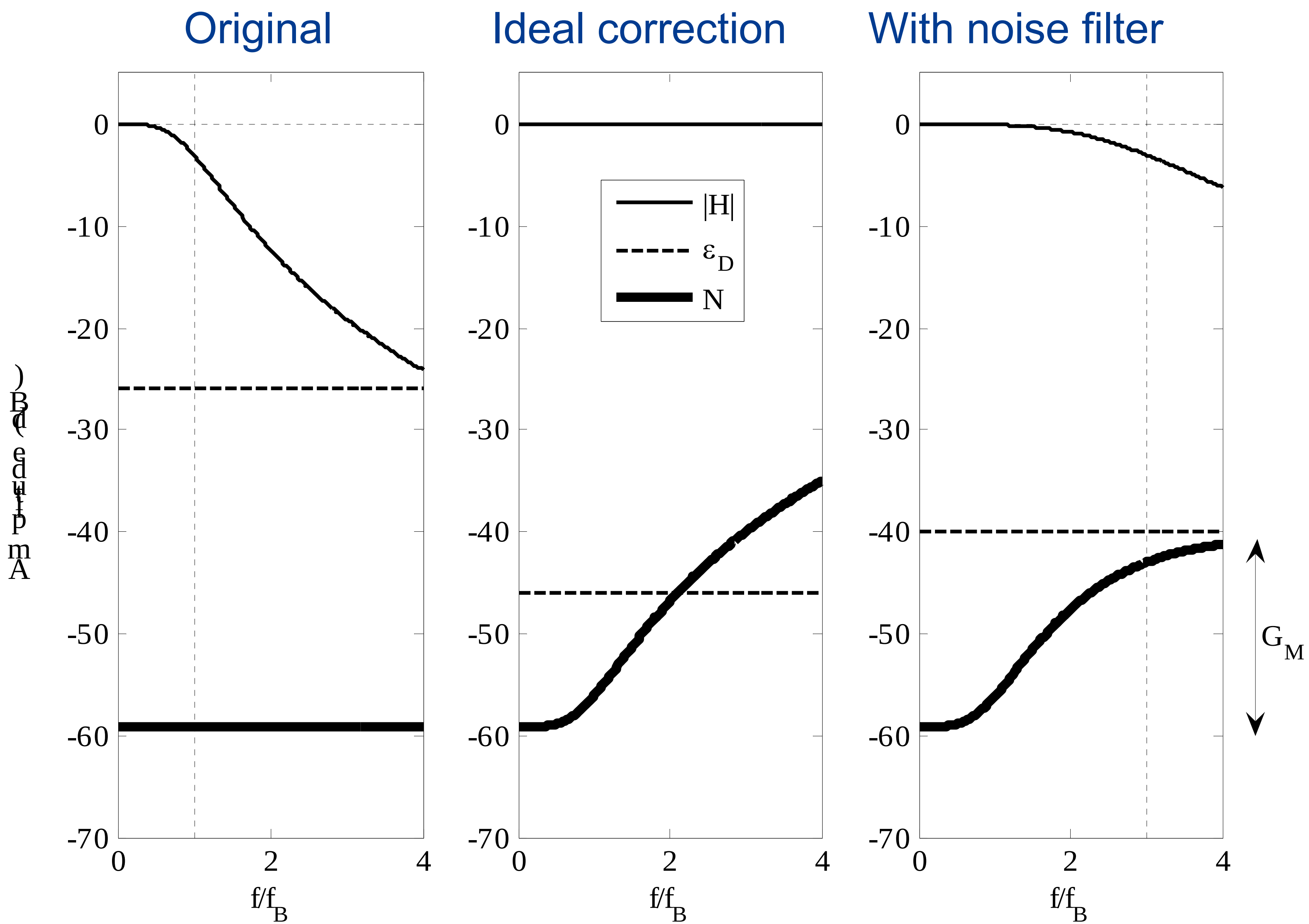


DT correction



Step 5:
Noise filter
– Bandwidth optimization

Noise level N, system amp. |H|, residual dynamic error ε_D → Noise filter LP cut-off



Step 6:
Calculation of filter coefficients

Zeros, poles => Filtercoefficients

- DT zeros z_K ⇒ Input coefficient: b_K
  DT poles, p_K ⇒ Output coefficients: a_K
  (Coefficient a_K and b_K weights sample *q-k* to filter sample *q*)

Original transfer function

$$H_M(s) = H_0 \frac{\prod_{k=1}^{\tilde{n}_z} \left(1 - s/\tilde{z}_k\right)}{\prod_{k=1}^{\tilde{n}_p} \left(1 - s/\tilde{p}_k\right)}$$

Correction filter:

$$G_C(z) \rightarrow \frac{\sum_{k=n_p-n_z}^{n_p} b_k z^{-k}}{\sum_{k=0}^{n_p} a_k z^{-k}}$$

Recipe:

1. Multiplication:

Step 7-8:
Further limitations…

Model and implementation inaccuracy…

- Sensitivity analysis
  - Uncertainty of dynamic parameters amplified by correction filter

$$\left\langle \sqrt{\sum_{r=\{\tilde{z}_k,\tilde{p}_k\}} \left| H_{HP}(i\omega, r) \right|^2} \right\rangle$$

  - => Requirement precision of characterization

- Numerical robustness
  - High sampling frequency
  - => Filter coefficients
    - Large magnitude
    - Varying sign
  - => Numerical cancellation

$$\frac{\sigma(y_m)}{\langle y_m \rangle} = \sigma(\varepsilon)\cdot 10^{\eta/20},$$

  - *'Numerical noise gain':*

$$\eta(\mathrm{dB}) \equiv 10\log\left(\sum_{k=n_P-n_Z}^{n_P} \left|b_k\right|^2 + \sum_{k=1}^{n_P} \left|a_k\right|^2 \right)$$

  - => Requirement numerical accuracy of filter implementation (# of digits)

Step 9: Verification Transducer system

Verification: Tranducer system

Subsystems to correct:

- Force transducer
  - Time scale f_C^{-1}
  - Relative damping $\zeta = 0.02$
- Application filter
  - Analogue Bessel
  - Order $n = 4$
  - Cross-over freq. $f_A = 0.75 f_C$
- Sampling frequency:
  - $f_S = 10 f_C$

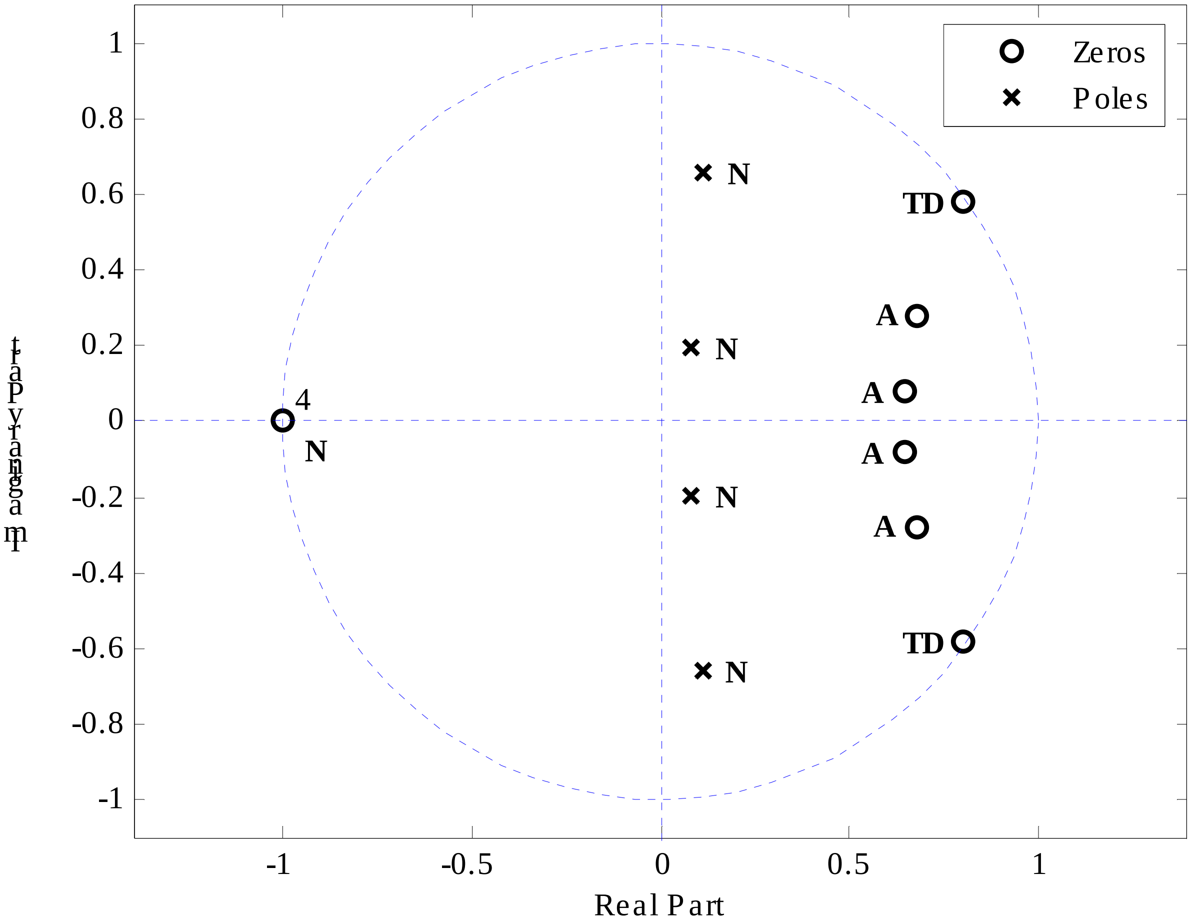
    $\Rightarrow f_{NYQ} = 5 f_C$

Correction filter:

- No zeros to compensate
  - no stabilization/phase cancellation
- Original S/N ratio: 85 dB
- Residual dynamic error: 1% = 40 dB
- Noise filter:
  - Max filter gain = 85-40=45 dB
  - Digital Butterworth
  - Order $n = 4$
  - Cross-over freq. $f_N = 2.2 f_C$
- Numerical noise gain
  - 36 dB
  - S/N ratio (num) ≥ 36+40=76 dB
  - 5 decimal digits
- Amplification of parameter uncertainty
  - $1.5 @ f_B = 0.4 f_C$
- Simulations real time signals (Matlab/Simulink)

Verification: Transducer system

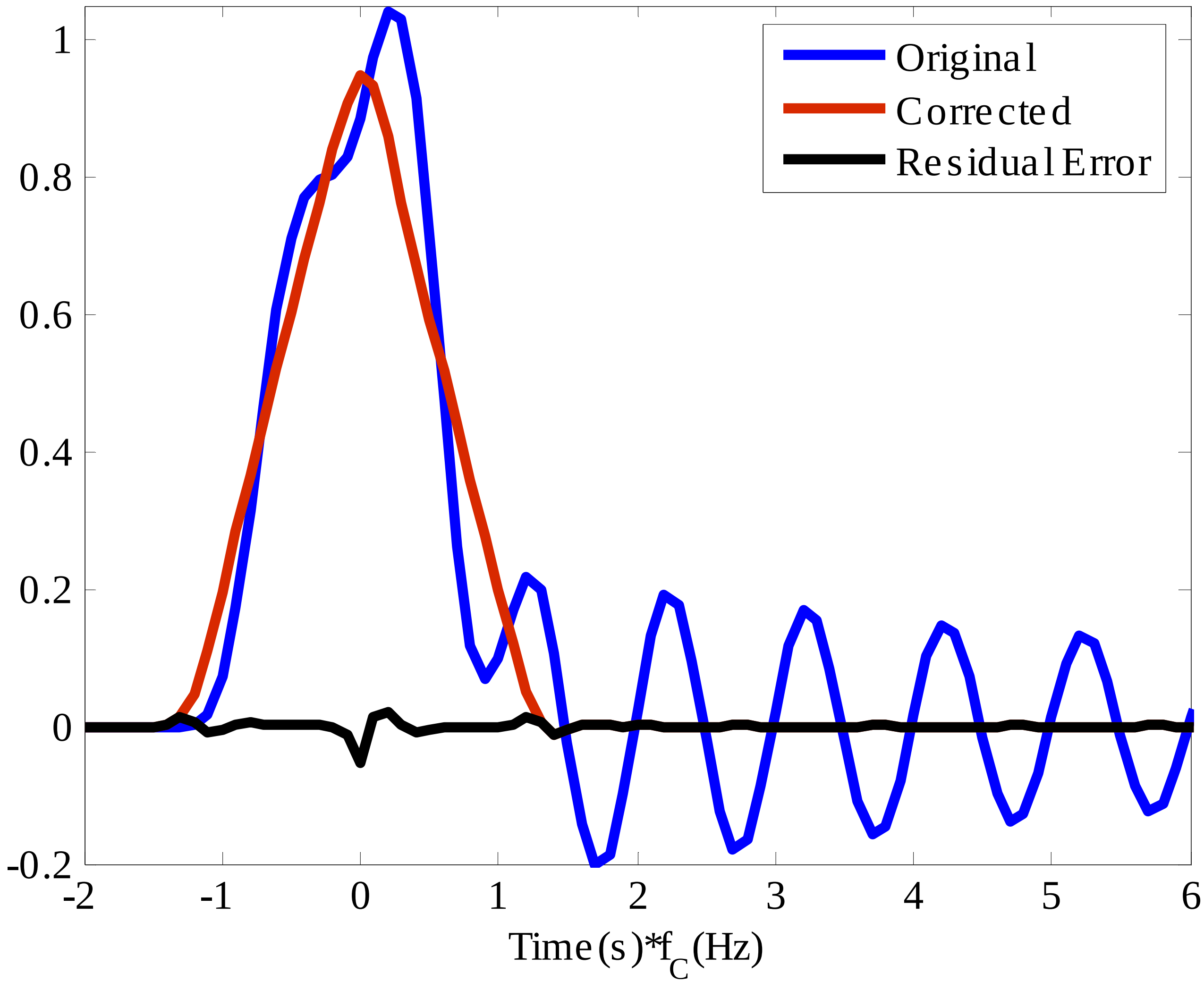
DT Poles and zeros of correction filter



Filter coefficients (5 digits!)

k	Input b_K	Output a_K
-6	8.23	0
-5	-2.33	0
-4	-25.36	0
-3	17.43	0
-2	30.36	0
-1	-31.44	0
0	-12.91	1.0
1	22.92	-0.39
2	-1.70	0.53
3	-6.04	-0.08
4	1.91	0.02

Verification: Transducer system



Conclusions

Digital filters for correcting measured signals

(Application)

Synthesis

Result - Propose synthesis method

Requires sufficiently high sampling frequency

General – all linear subsystems (mech., electr. etc.)

Robust – no optimization of annihilating parameters

Direct/straight-forward recipe

Practical limitations considered!

- High frequency noise amplification of correction methods
- Parameter/model uncertainties of characterization
- Numerical cancellation of filter implementation

Thanks for Your attention!

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