The importance and significance of phase

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Consider linear, causal, time-invariant system with transfer characteristics $H(\omega)$ and impulse response $h(t)$. 

\[ x(t) \rightarrow h(t) \rightarrow y(t) \]
Phase and group delay

- The signal at the output of the system has a spectrum:
  \[ Y(\omega) = X(\omega)H(\omega) = X(\omega)|H(\omega)|e^{j\phi(\omega)} \]
- In the time domain, each frequency component of the output \( y(t) \) is simply a delayed and amplitude-scaled version of the corresponding input signal frequency component.
- The group delay or envelope delay of the system is:
  \[ \tau(\omega) = -d\phi(\omega)/d\omega \]
- \( \tau(\omega) \) is the time delay that a signal component of frequency \( \omega \) undergoes as it passes through the system.
The significance of phase

- Linear phase $\rightarrow$ Pure delay
- Nonlinear phase $\rightarrow$ Each frequency component of the signal is delayed by a different amount as it passes through the system
Practical example

- Hydrophone/amplifier magnitude and phase response requirements when estimating acoustic waveform pressure parameters
Ultrasonic imaging remains the most rapidly growing medical imaging modality.

The main method of measurement and characterisation of medical ultrasonic fields propagating in water is through the use of calibrated hydrophones.

NPL provides a calibration of the magnitude response over the frequency range 1-20 MHz.

*How important is hydrophone phase information when estimating acoustic waveform parameter?*
Objective

- Provide guidelines on importance of the hydrophone phase response when estimating key acoustic waveform parameters ($p^+$, $p^-$, $t_d$ and $p_i$) which will feed into international standards

- Does data have to be corrected for phase?

- If not, what uncertainty does this give rise to?
Key acoustic pressure parameters

- Peak-positive acoustic pressure $p^+$
- Peak-negative acoustic pressure $p^-$
- Pulse-pressure-squared integral $p_i$: time integral of the square of the instantaneous acoustic pressure in the pulse, integrated over the whole of the pulse
- Pulse duration $t_d$: 1.25 times the interval between the time when the time integral of the instantaneous acoustic pressure squared reaches 10% and 90% of its final value
• Marconi 25 µm film thickness, 0.5 mm element diameter bilaminar membrane hydrophone; response obtained from NPL hydrophone model [1].

• Account for uncertainties in magnitude and phase using a Monte Carlo simulation [2].

• Obtain uncertainties in acoustic pressure parameters for specified uncertainties in phase response.

[1]: Gélat PN, Preston RC and Hurrell A, “A theoretical model describing the transfer characteristics of a membrane hydrophone and validation”, Ultrasonics, article in press.

Hydrophone output voltage waveform

Time ($\mu$s) vs Voltage (V)
Schematic of deconvolution procedures

Pressure

Voltage

Single value at fundamental frequency $|H(\omega_0)|$

$P(\omega) \rightarrow H(\omega) \rightarrow V(\omega)$

$|H(\omega)|$

Zero Phase
Procedure for evaluating effect of phase response on deconvolution

- Obtain Fourier transform of output voltage of hydrophone by knowledge of input pressure $p(t)$ and hydrophone/amplifier transfer characteristics $H(\omega)$:

$$V(\omega) = H(\omega)P(\omega)$$

- Pressure waveform can be estimated as follows:

$$\tilde{p}_1(t) = F^{-1}\left(\frac{V(\omega)}{H(\omega)}\right)$$

$$\tilde{p}_2(t) = F^{-1}\left(\frac{V(\omega)}{|H(\omega_0)|}\right)$$

$$\tilde{p}_3(t) = F^{-1}\left(\frac{V(\omega)}{|H(\omega)|}\right)$$
Sample result of Monte Carlo simulation

- $p_+$
  - $2 \times 10^6$
  - $3 \times 10^6$
  - $4 \times 10^6$

- $p_-$
  - $-12 \times 10^{-6}$
  - $-10 \times 10^{-6}$
  - $-8 \times 10^{-6}$

- $p_i$
  - $3 \times 10^5$
  - $4 \times 10^5$
  - $5 \times 10^5$

- $p_d$
  - $4 \times 10^7$
  - $5 \times 10^7$
  - $6 \times 10^7$