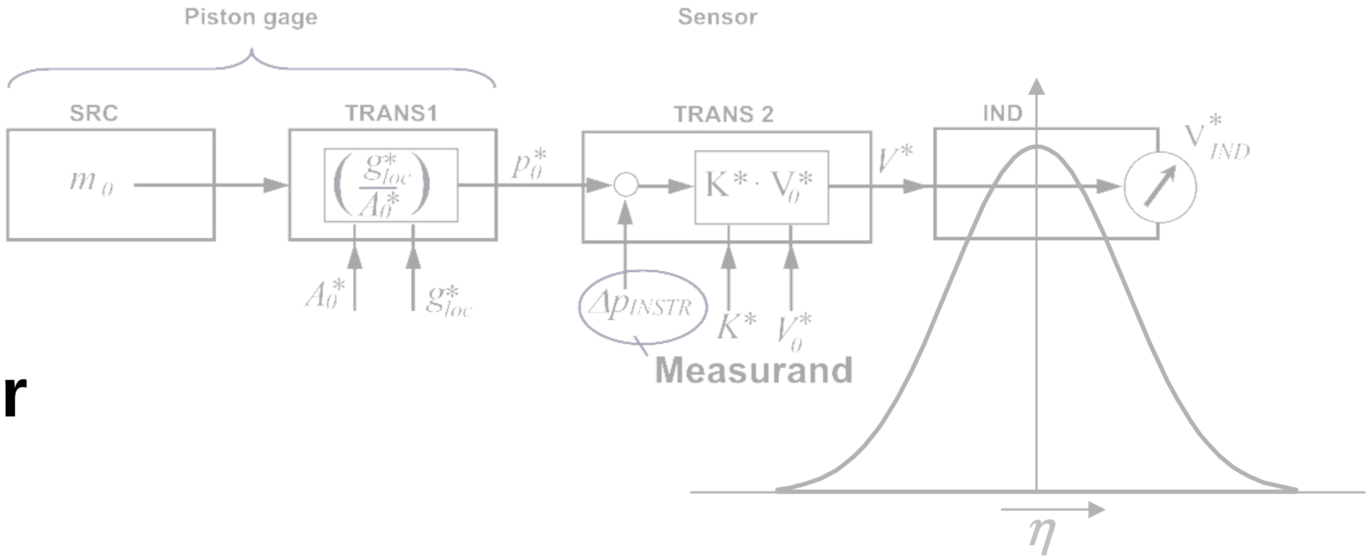
On Modelling of Measurement-Signal Processing for Uncertainty Analysis



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NPL National Physical Laboratory

2nd Signal Processing Awareness Seminar - 23rd June 2006

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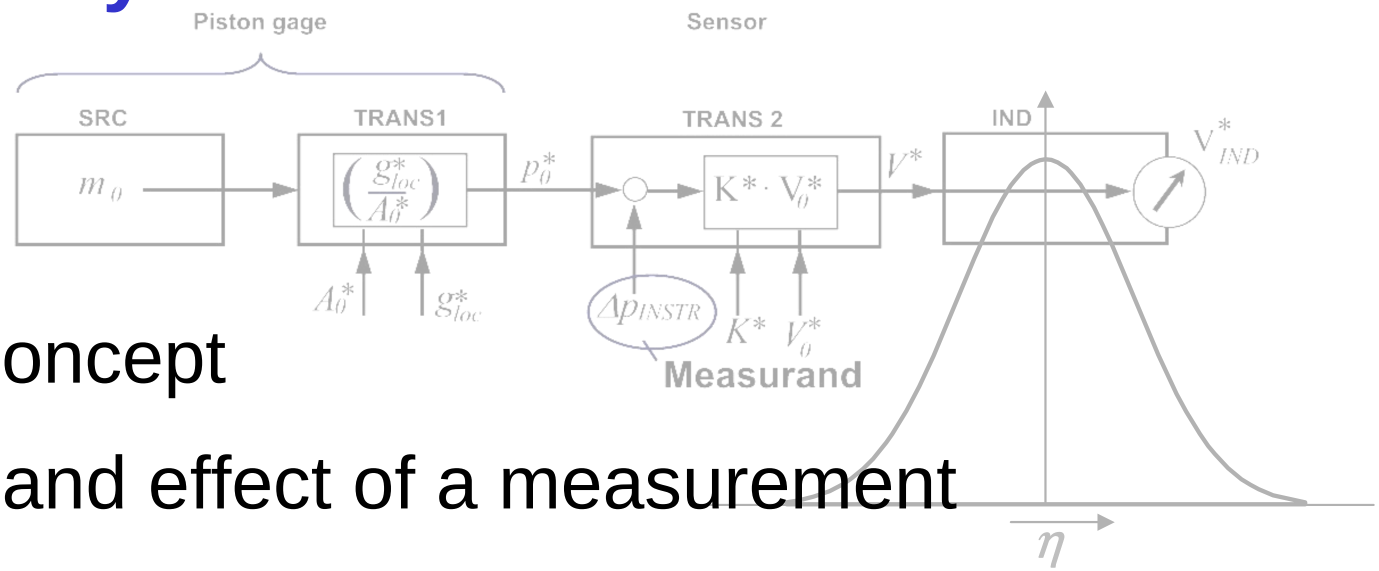


Outline:

- GUM-concept
- Cause and effect of a measurement
- Modelling concept and procedure
- Examples
- Measurement method and model structure
- Uncertainty contributions in signal processing

ISO-GUM Procedure

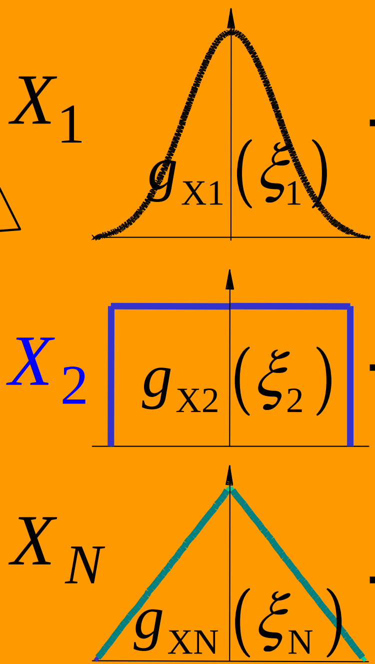
Knowledge

about the measuring process:

$$Y = f_M(X_1, X_2, \ldots, X_N)$$

(Model equation)

about the input quantities:



X_1 → x_1, u_{x1}

X_2 → x_2, u_{x2}

X_N → x_N, u_{xN}

$X_1, \ldots, X_N$	- input quantities
$x = \mathrm{E}[X]$	- expectation of X
$y = \mathrm{E}[Y]$	- expectation of Y
$u_y = \sqrt{\mathrm{Var}[Y]}$	- standard uncertainty
U	- expanded uncertainty

Gaussian uncertainty propagation

$$y = f_M(x_1, x_2, \ldots, x_N)$$

$$u_y = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f_M}{\partial x_i} \right)^2 u_{xi}^2 + \ldots}$$

Output quantity Y:

y, u_y → y, U, k_P

Propagation of Distributions (PDFs)

Draft GUM Supplement 1

K n o w l e d g e

about the measuring process:

$$Y = f_M(X_1, X_2, \dots, X_N)$$

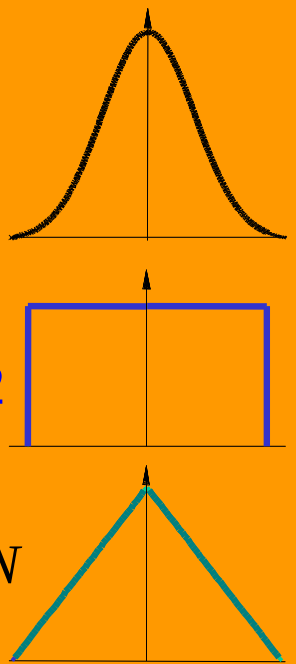
(Model equation)

about the input quantities:

X_1 — $g_{X1}(\xi_1)$

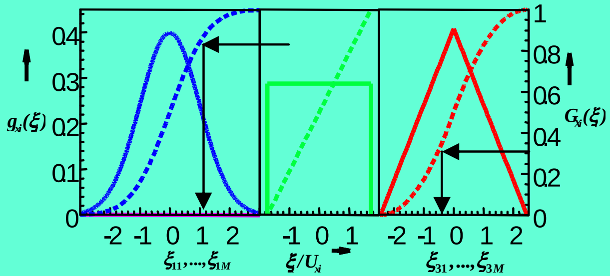
X_2 — $g_{X2}(\xi_2)$

X_N — $g_{XN}(\xi_N)$

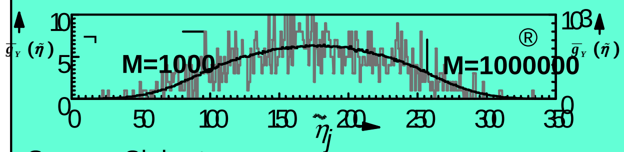


$X_1, \dots, X_N$	- input quantities
ξ_1	- possible values of a quantity X
$g_{X1}(\xi_1), \dots, g_{XN}(\xi_N)$	- PDFs for the input quantities
$g_Y(\eta)$	- PDF for the output quantity Y
η	- possible values of a quantity Y
y	- expectaion value of Y
u_y	- standard uncertainty ass. with y
U	- expanded uncertainty

Monte-Carlo method

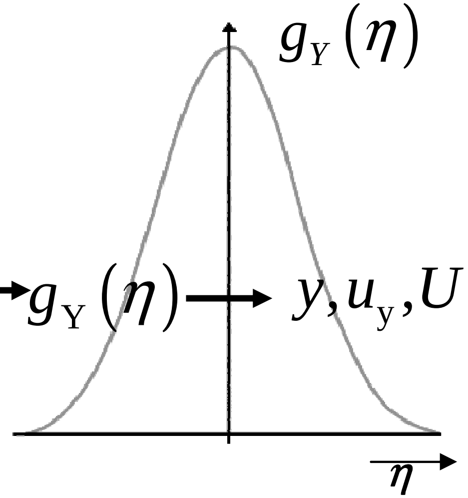


Combination: $\eta = f_Y(\xi_1, \xi_2, \xi_3) \rightarrow \eta_1, \dots \eta_M$



Source: Siebert

Output quantity Y:



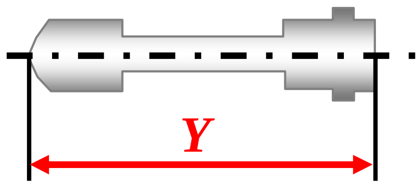
$g_Y(\eta) \rightarrow y, u_y, U$

From Cause-and-Effect to the Model Equation

Cause → **Effect**

Y, X_1, X_2, X_{N-1} X_N $\Rightarrow X_N = h(Y, X_1, X_2, ..., X_{N-1})$

(Measurement equation)



Measurand

Indication

X_1 X_2 … … X_{N-1} X_N

Model equation:

$$Y = f_M\left(X_1, X_2, ..., X_N\right)$$

… represents the mathematical relationship between the relevant (input) quantities and the measurand

Source of pictures: A. Weckenmann

Modelling Approaches

LMET

Analytical Models

- Algebraic equations for static problems
- Ordinary and partial differential equations for dynamic problems

Graphical Models

- Block diagrams
- State graphs
- Petri networks

Numerical Models

(sequences of) data with

- Value discretisation

or

- Time discretisation

What is needed in practice?

A model which can be represented both, as graphical depiction and as model equation. **It should allow for…**

- Linearization (Standard GUM)
- Decomposition of the measurement in terms of functional elements
- Using a cause-and-effect approach

It should allow for modelling of…

- Steady-state measurements, extendable to dynamic measurements
- Concentrated and distributed parameters
- Multiple input quantities

Measuring Chain (1)
Assumptions

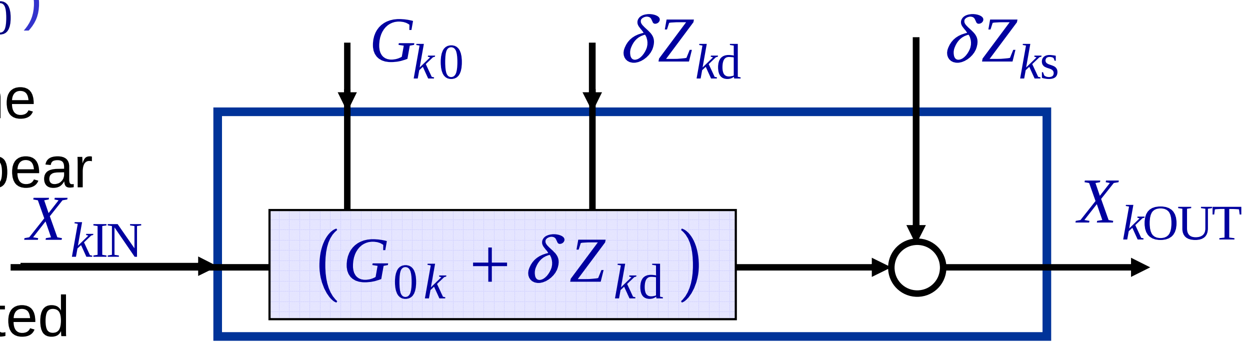
Practical assumptions:

- At least for narrow ranges, the elements may be considered to be linear (describable by first-order Taylor-series expansion)
- The steady-state response function is related to adjusted and well-known parameters and conditions (operating point $\underline{Z}_{k0}$)
- Only slight deviations $\delta \underline{Z}_k$ of the real parameters from $\underline{Z}_{k0}$ appear
- All involved quantities are treated as random quantities

Resulting simplifications:

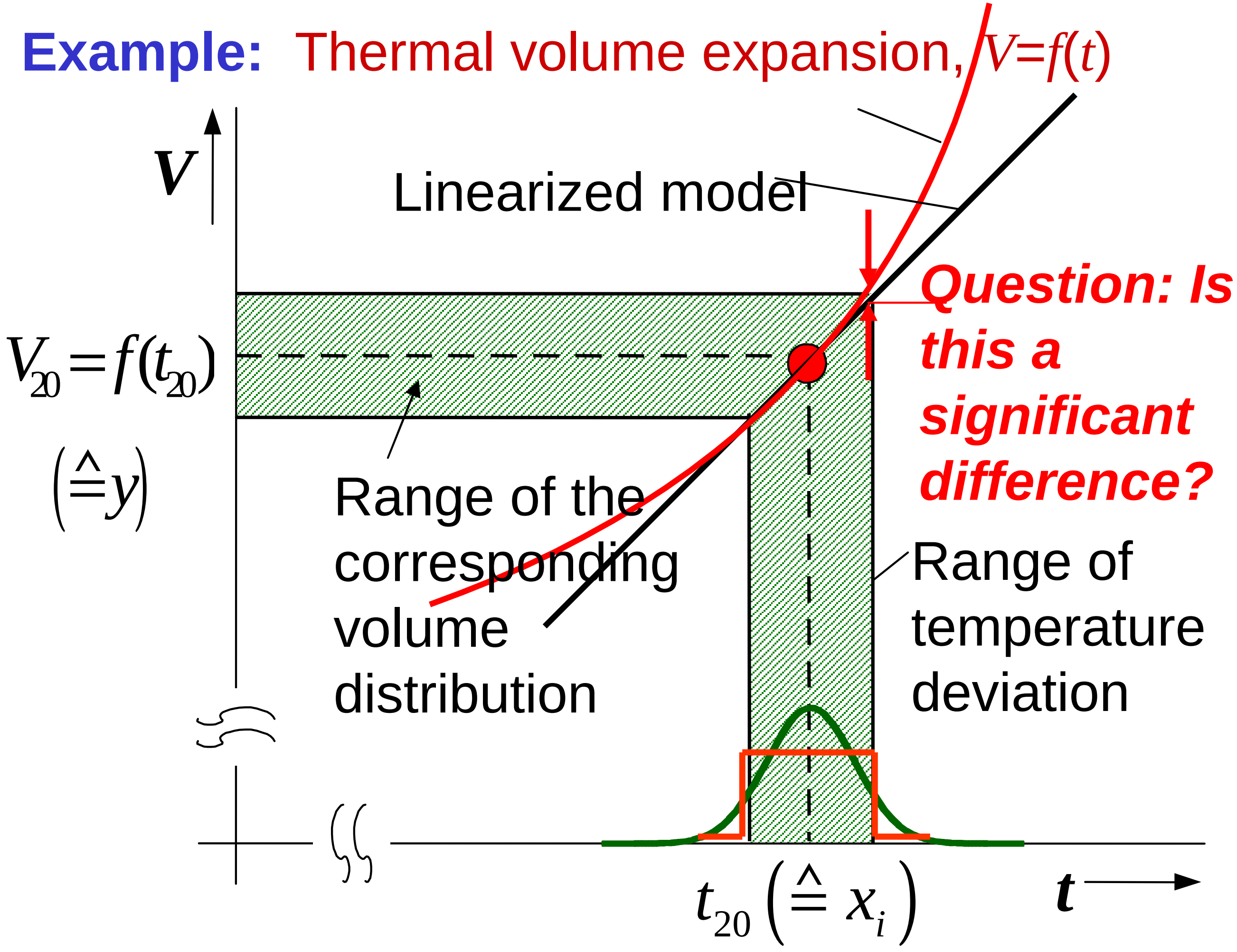
Response function h_{0k} may be expressed by

- (constant) transmission factors $\mathbf{G}_{0k}$ that are perturbed by
- superimposing and deforming deviations δZ_{ks} and δZ_{kd}



Measuring Chain (2)
Linearization

Example: Thermal volume expansion, $V=f(t)$



$$t_{20} = \mathrm{E}[t] \rightarrow V_{20} = \mathrm{E}[V] \cong f(t_{20})$$

Linearization by *Taylor* series expansion

$$V = f_\mathrm{M}(t) = f_\mathrm{M}(t_{20}) + \left.\frac{\partial f_\mathrm{M}}{\partial t}\right|_{t_{20}} \cdot \delta t \ldots$$

$$\ldots + \left.\frac{\partial^2 f_\mathrm{M}}{2\partial t^2}\right|_{t_{20}} \cdot \delta t^2 + \ldots$$

Linearization condition:

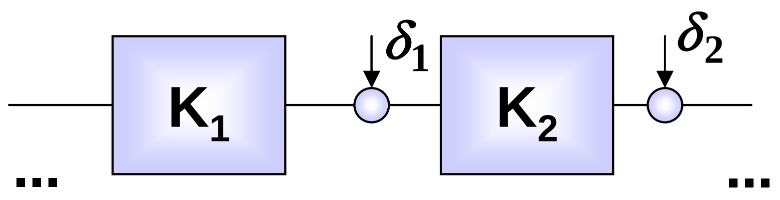
$$f_\mathrm{M}(\xi_1,\ldots,\xi_N) \cong f_\mathrm{M}(x_1,\ldots,x_N) + \sum_{i=1}^{N} \left.\frac{\partial f_\mathrm{M}}{\partial t}\right|_{t_{20}} (\xi_i - x_i)$$

Basics of the Modelling Concept

Modelling Concept
for practitioners

- …gives the (non-reactive) chaining of elements in *cause-effect direction*
- … describes disturbances and imperfections by means of **deviations**
- … determines the structure and the chaining sequence of the model

„Classical“ Measuring Chain



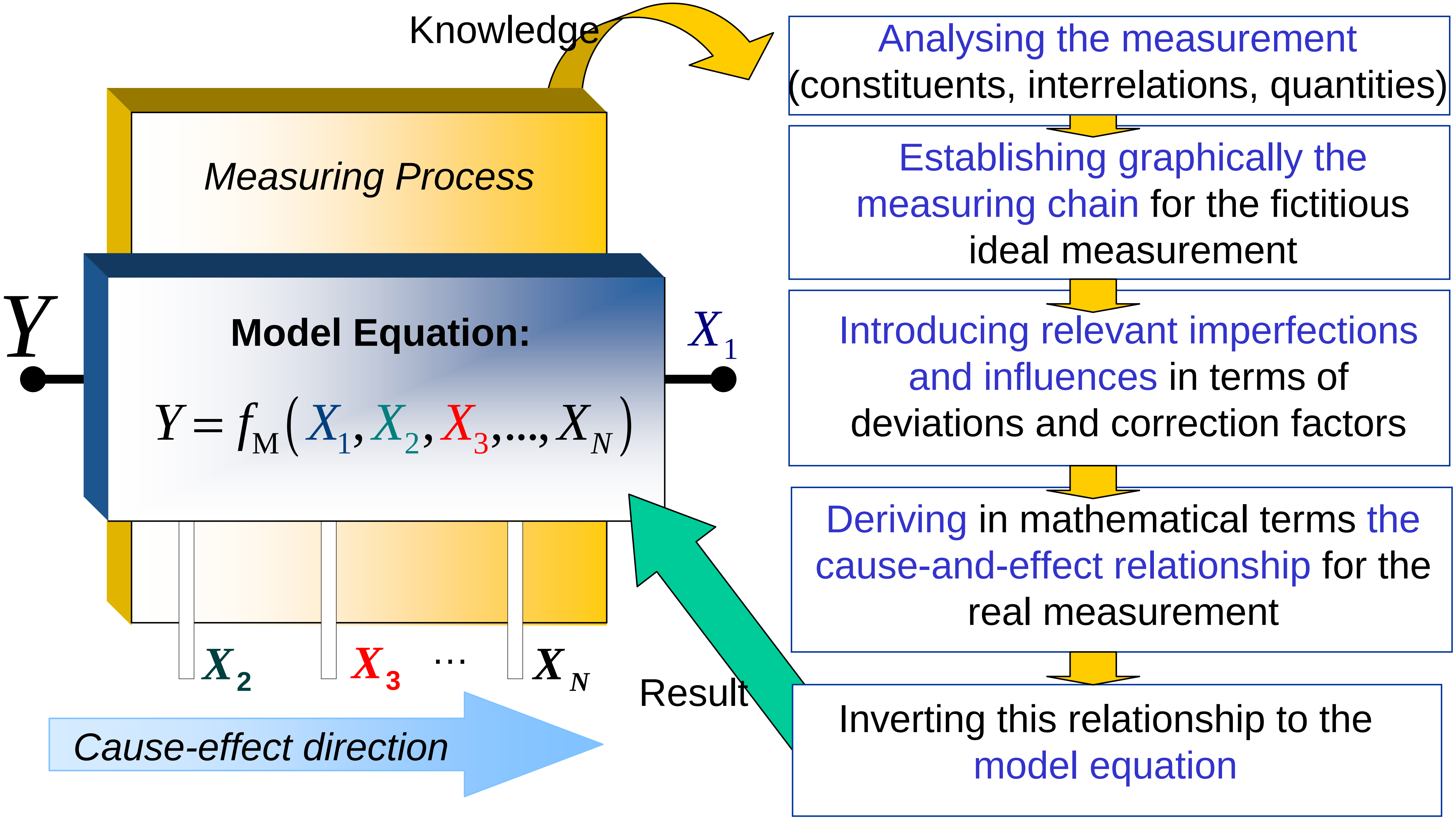
Method of Measurement

- Direct Measurement
- Substitution Method
- Difference Method

etc.

Modelling Concept

Basic Idea



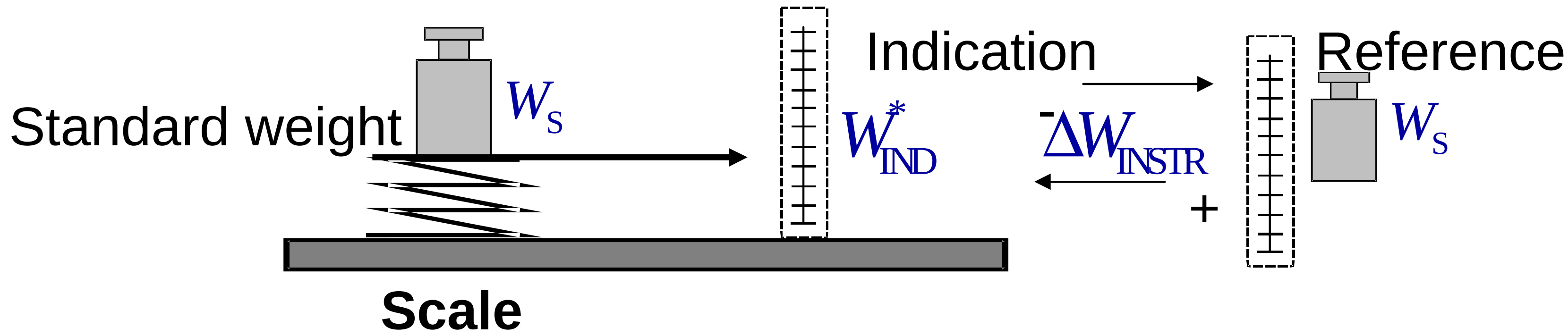
Example: Calibration of a Scale (1)

1st step

Description of the calibration:

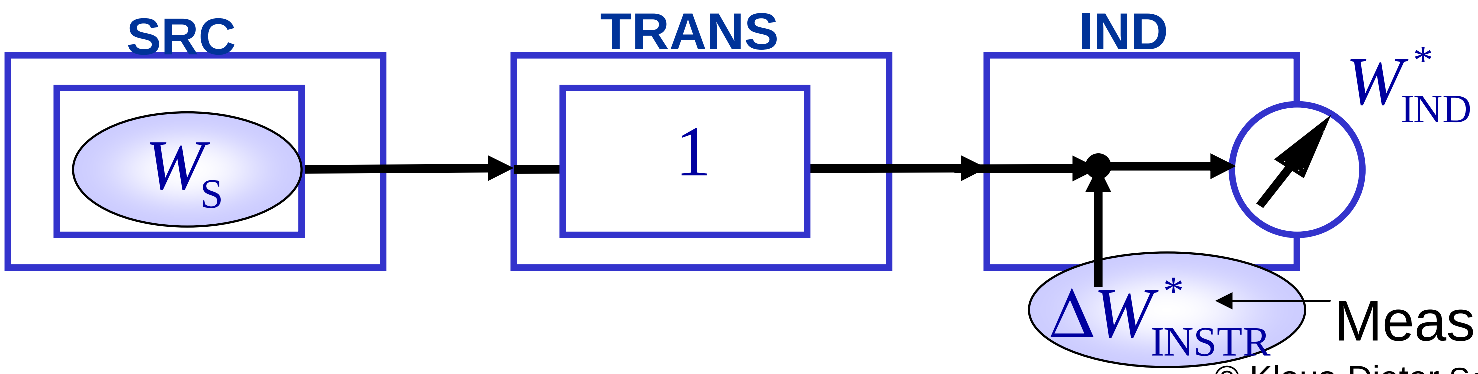
Measurand

- Determination of the instrumental error at 10 kg load
- Causal quantity: weighing value W_S
- Measurement method: direct measurement
- Constituents: standard, coupling, scale



2nd step

Cause-and-effect relationship of the fictitious ideal measurement:

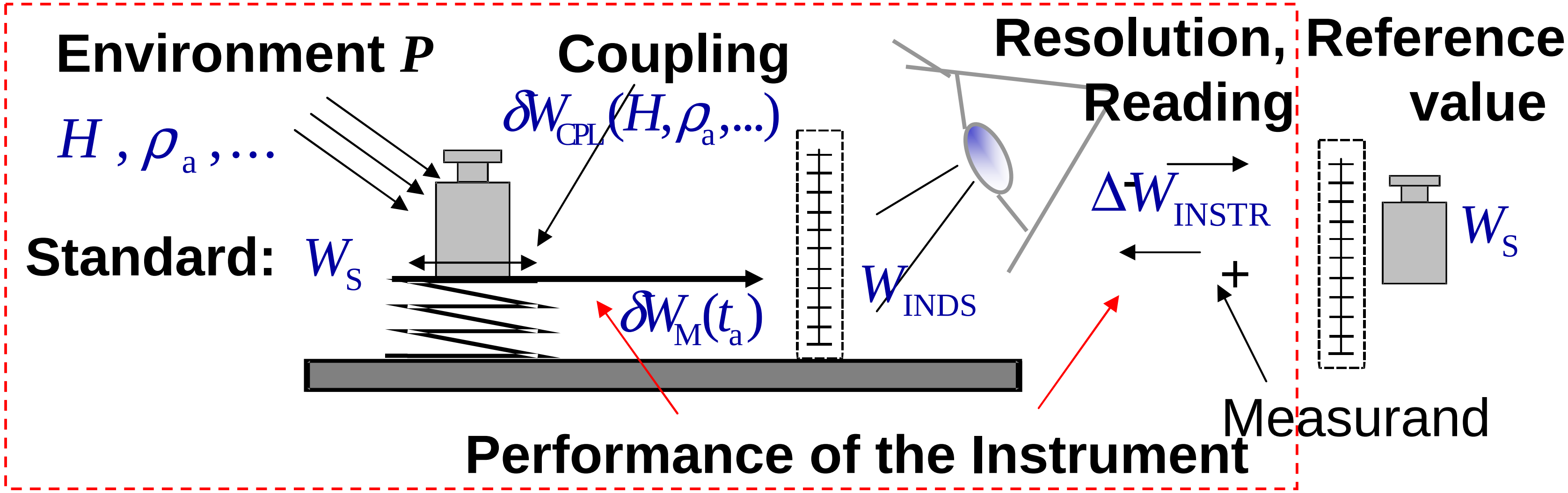


Example: Calibration of a Scale (2)

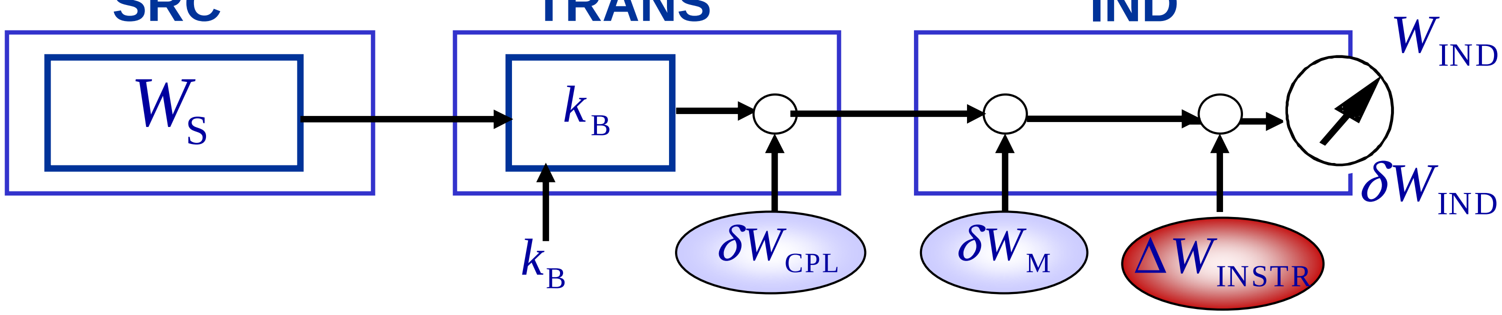
3rd step

Cause-and-effect relationship for the real calibration:

a) Illustration:
(simplified)



b) Block diagram:
(simplified)

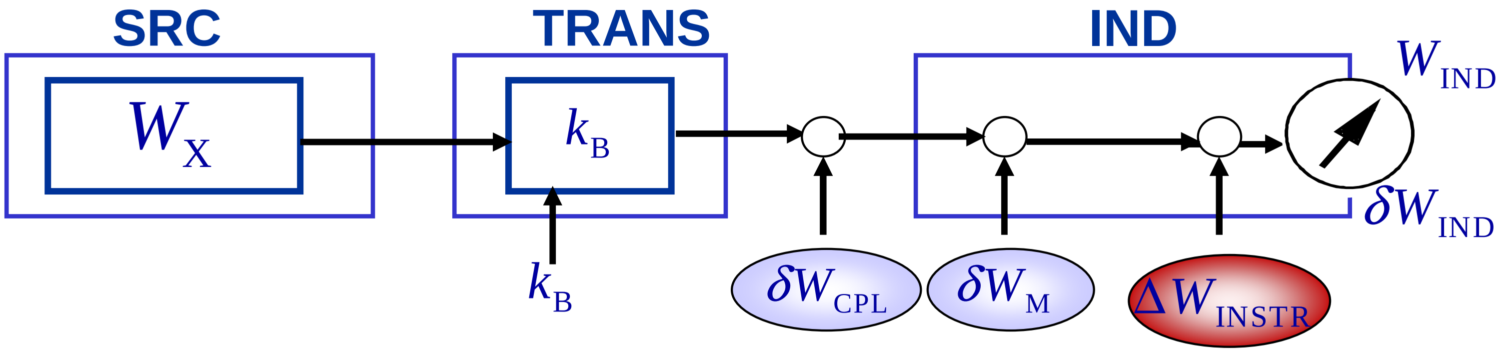


$$k_B = \left(1 - \rho_a \rho_X^{-1}\right)\left(1 - \rho_{1,2} \rho_{8000}^{-1}\right)$$

δW_{CPL} - deviation due to excentric loading, magnetic…

Example: Calibration of a Scale (3)

b) Block diagram:
(simplified)



c) *Cause-and-effect relationship* expressed in mathematical terms:

$$W_{IND} = W_S \cdot k_B + \delta W_{CPL} + \delta W_M + \Delta W_{INSTR} + \delta W_{IND}$$

5th step

Inversion to the model equation:

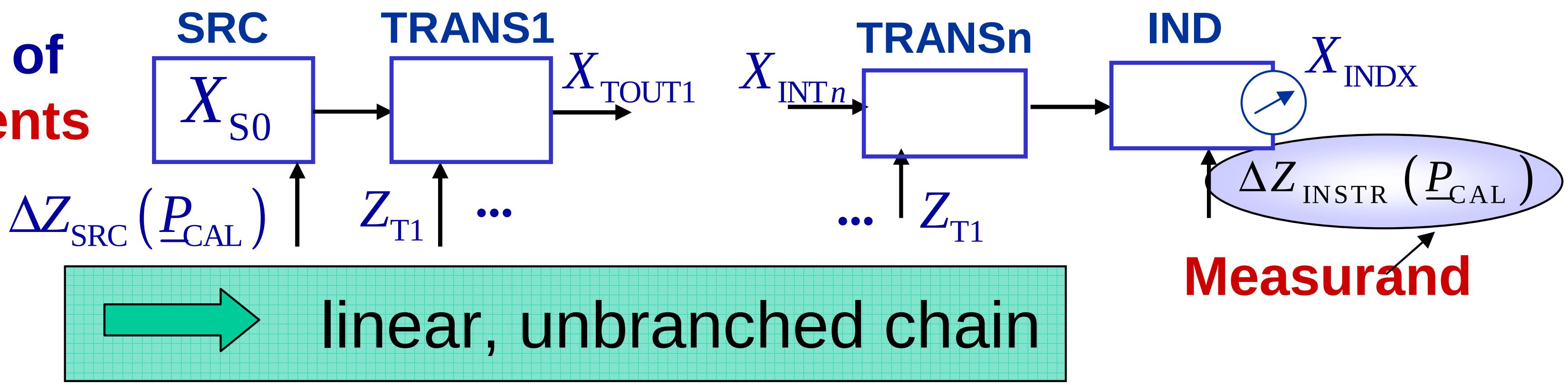
$$\Delta W_{INSTR} = W_{IND} - W_S \cdot k_B - \delta W_{CPL} - \delta W_M - \delta W_{IND}$$

… and the (unavoidably) incomplete knowledge about these quantities is to be evaluated by assigning appropriate probability density function (pdfs) to them

Role of the Measurement Method (1)

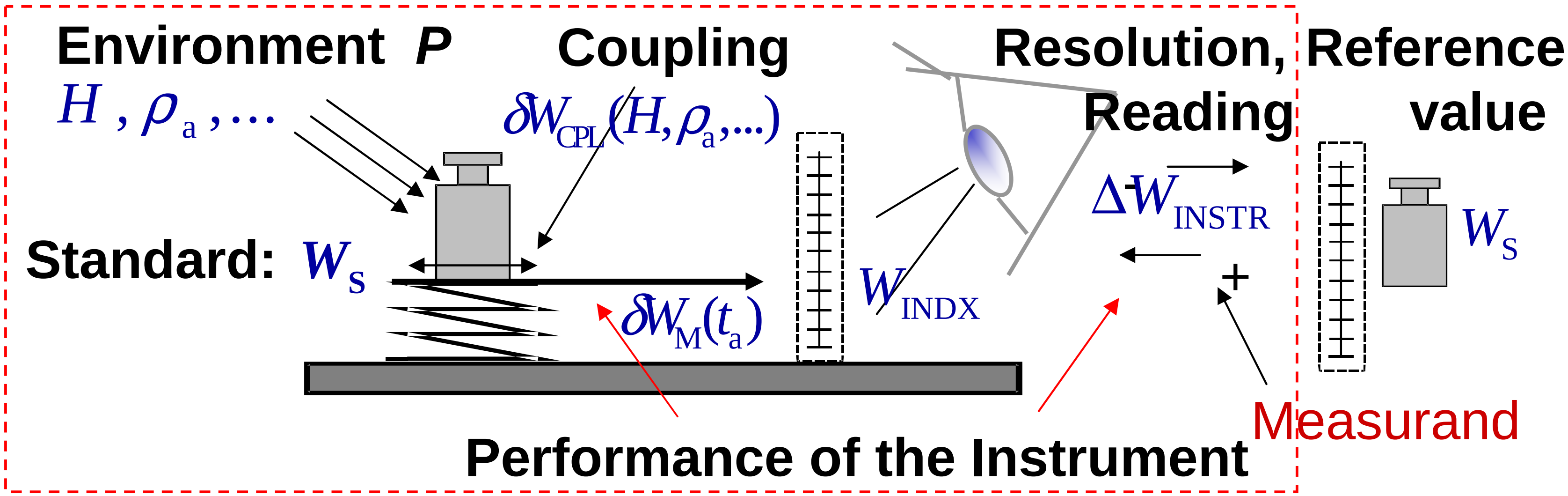
Structure and chaining sequence are determined by the method used

Generic structure of direct measurements of standards:



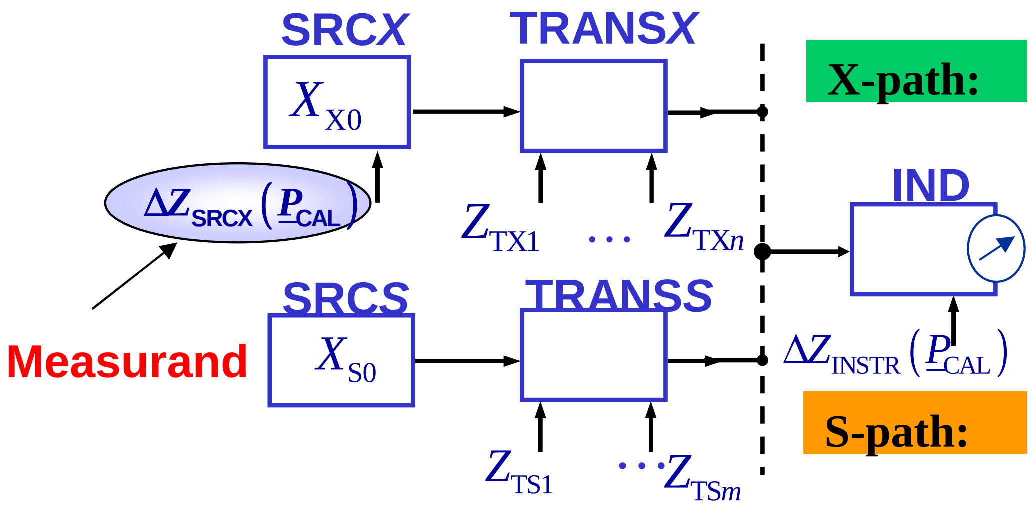
Example:

Calibration of a scale



Role of the Measurement Method (2)

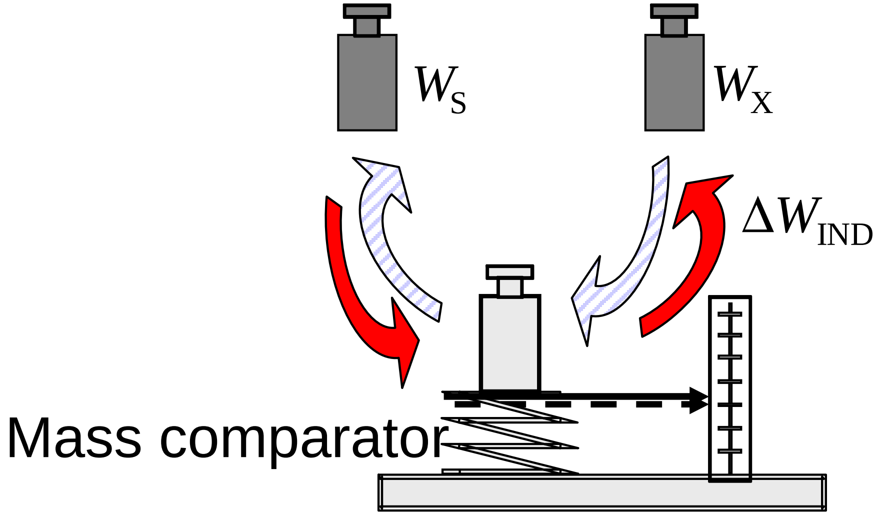
Generic structure
of the substitution method:



conjoining chains

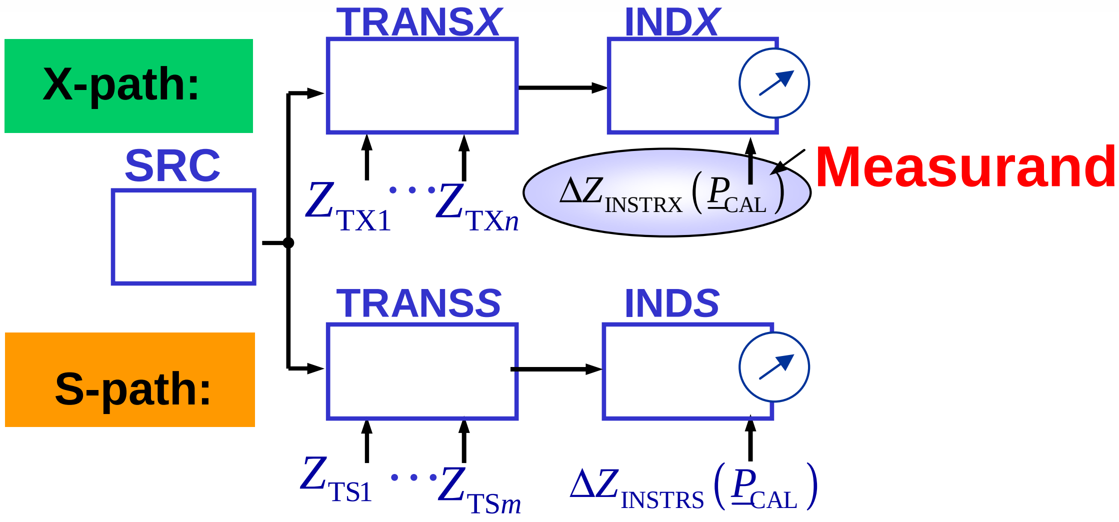
Example:

Calibration of a weight piece by means of a mass comparator



Role of the Measurement Method (3)

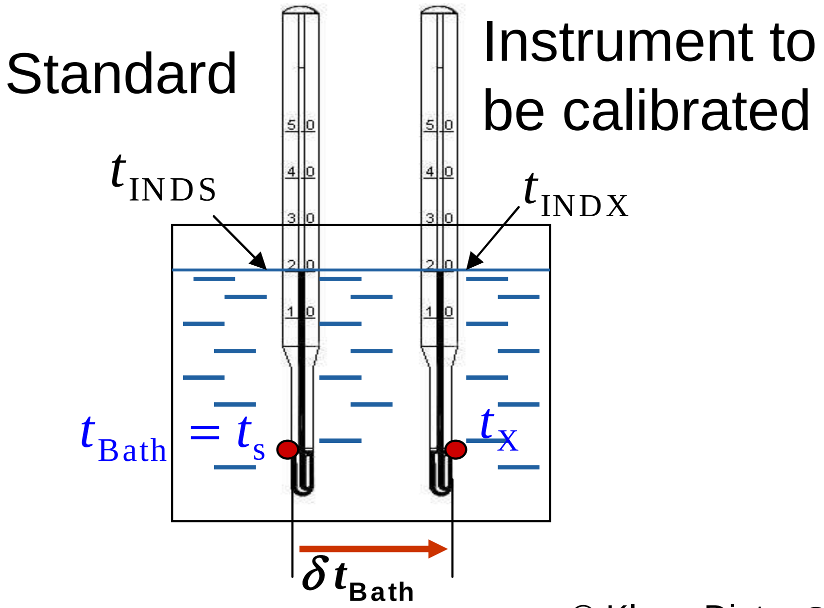
Generic structure
of direkt comparison
of indicating instruments:



→ forking chain

Example:

Calibration of a liquid-in-glass thermometer

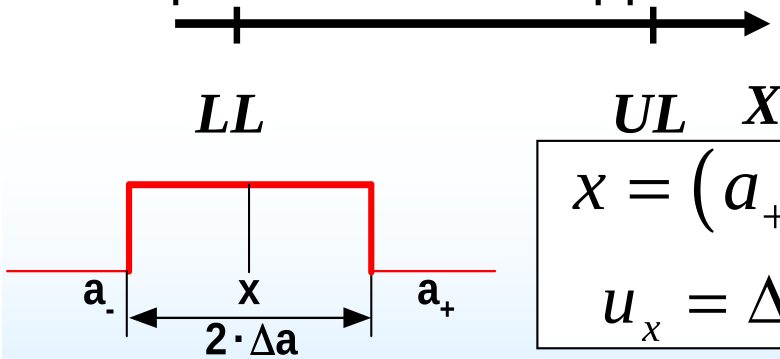


Practical Evaluation of the Quantities (GUM)

Knowledge about the Quantities

B non-statistical information

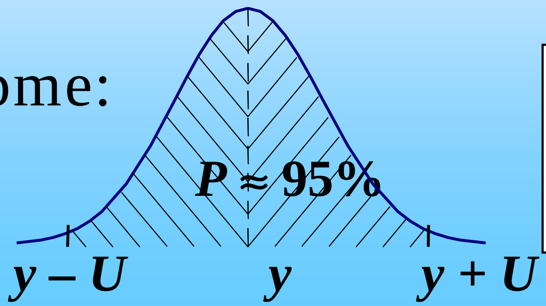
Example: lower and upper limits



$$x = (a_+ + a_-)/2$$
$$u_x = \Delta a / \sqrt{3}$$

Example:
statement in a calibration certificate

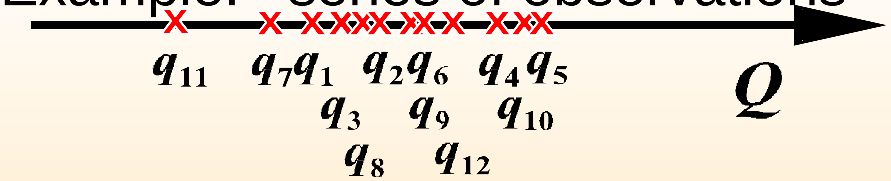
pme:



$$x = y \approx \mu$$
$$u_x = U/k \approx \sigma$$

A statistical information

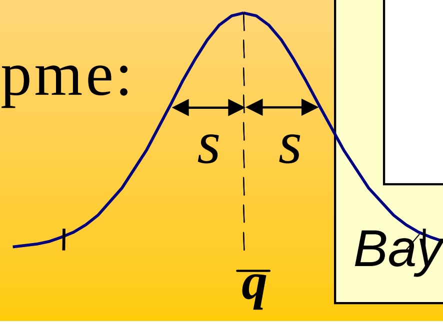
Example: series of observations



statistical analysis:

$$\bar{q} = \frac{1}{n}\sum_{k=1}^{n} q_k; \quad s(q_k) = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(q_k - \bar{q})^2}$$

pme:



$$x = \bar{q}$$
$$u_x = s(q_k)/\sqrt{n}$$

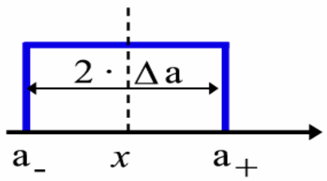
Bayesian approximation

Survey About Selected *a-priori* Distributions (PDFs)
(Type-B Evaluation)

Knowledge About the quantity | **pdf** | **Expectation value** | **Standard uncertainty**

rectangular

Possible value are Contained in an interval

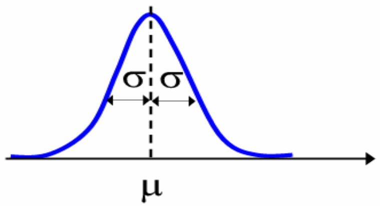


Expectation value: $$x = \frac{(a_+ + a_-)}{2}$$

Standard uncertainty: $$u_x = \frac{\Delta a}{\sqrt{3}}$$

Gaussian

Best estimate μ and standard deviation s are know

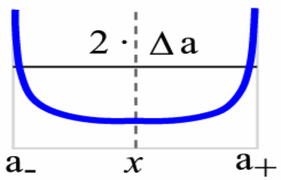


Expectation value: $$x = \mu$$

Standard uncertainty: $$u_x = \sigma$$

U-shaped

Quantity is described as function $X = \Delta a \cdot \sin\Phi$ where F is unknown

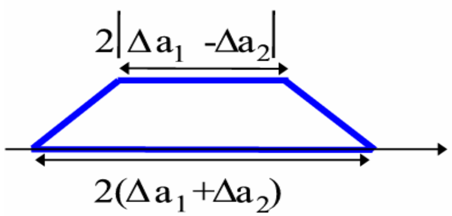


Expectation value: $$x = 0$$

Standard uncertainty: $$u_x = \frac{\Delta a}{\sqrt{2}}$$

trapezoidal

Quantity is a sum/ difference $\boldsymbol{X_1 \pm X_2}$; knowledge about X1 and X2: rectangular distributed

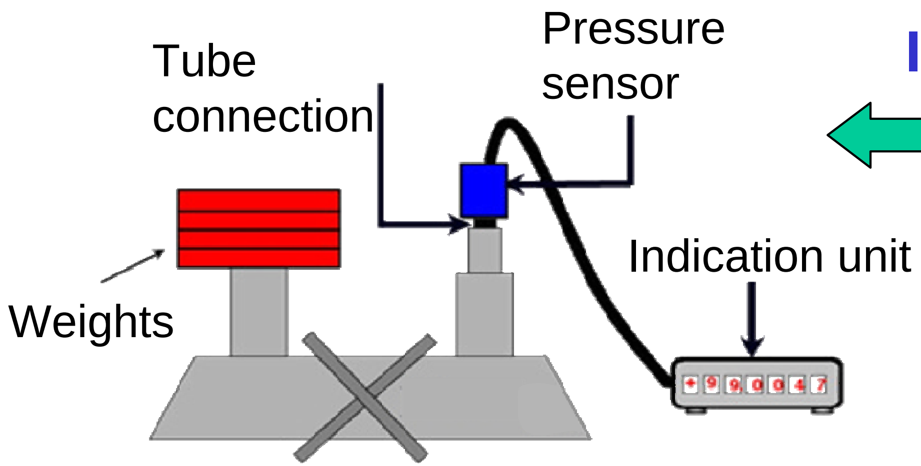


Expectation value: $$x = x_1 \pm x_2$$ $$\Delta a = \Delta a_1 + \Delta a_2$$

Standard uncertainty: $$u_x = \frac{\Delta a}{\sqrt{6}}\sqrt{1+\beta^2}$$

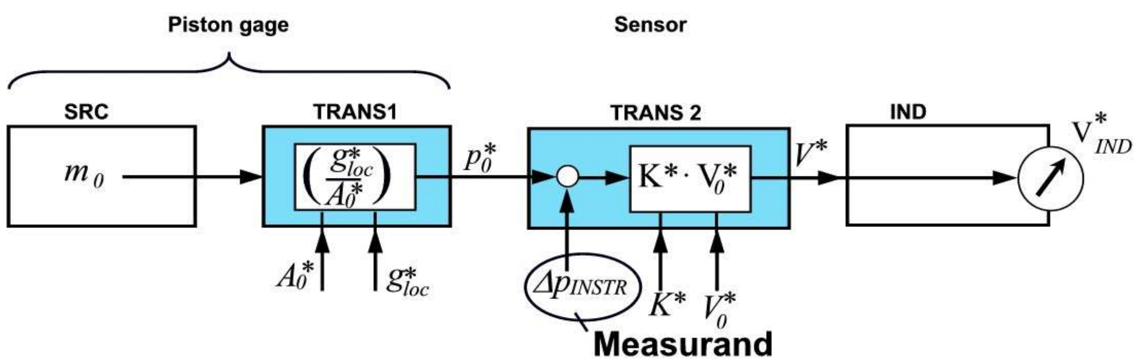
Measuring Chain in Digital Signal Processing

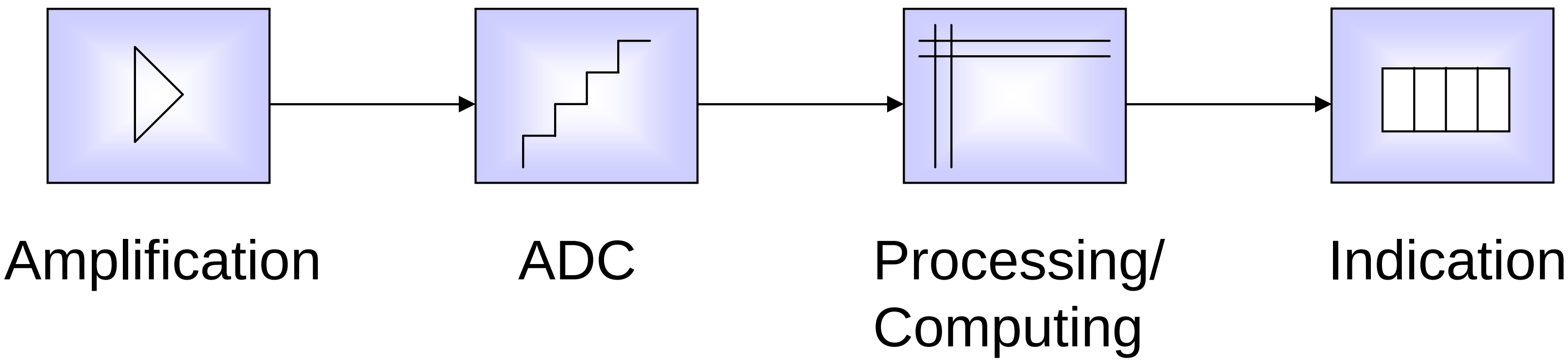
Illustration



Source: Janovsky

Cause-effect principle of the fictitious ideal measuring





Possible Imperfections in Digital Signal Processing

Digital measurements:

- Counting errors
- Quantisation errors (error due to limited/digital resolution)
- Slope errors in analogue-digital conversion (ADC)
- Averaging errors in ADC
- Errors due to imperfect sampling
- Errors due to non-ideal software (e.g. rounding error)
- Errors due to synchronisation (jitter)

Dynamic error contributions:

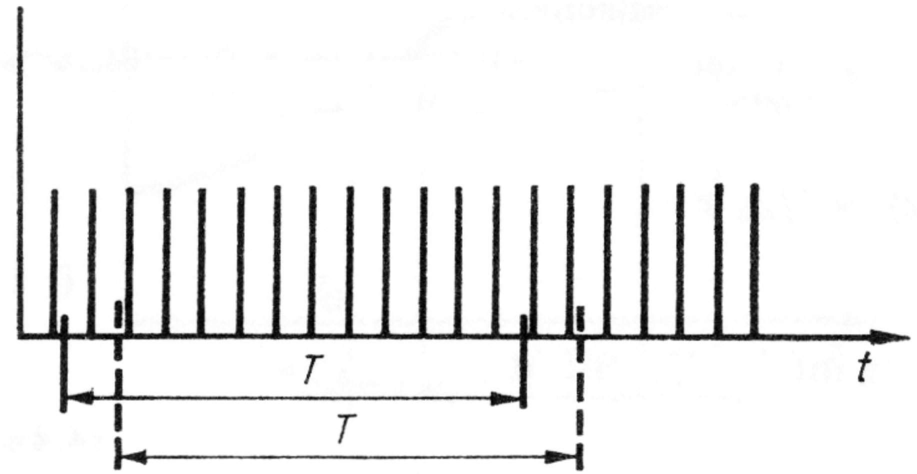
- System disturbance caused by interfering fields
- Transient effects
- Drift effects

Digital Signal Processing:
Counting Errors

- **Application of electronic counters:**
  - frequency measurement
  - period and time-interval measurements

- **Relative error of time measurement**

$$\frac{\Delta T}{T} = \frac{1}{n} = \frac{1}{f_i \cdot T}$$



f_i – frequency of pulse signal

T – period of measurement signal

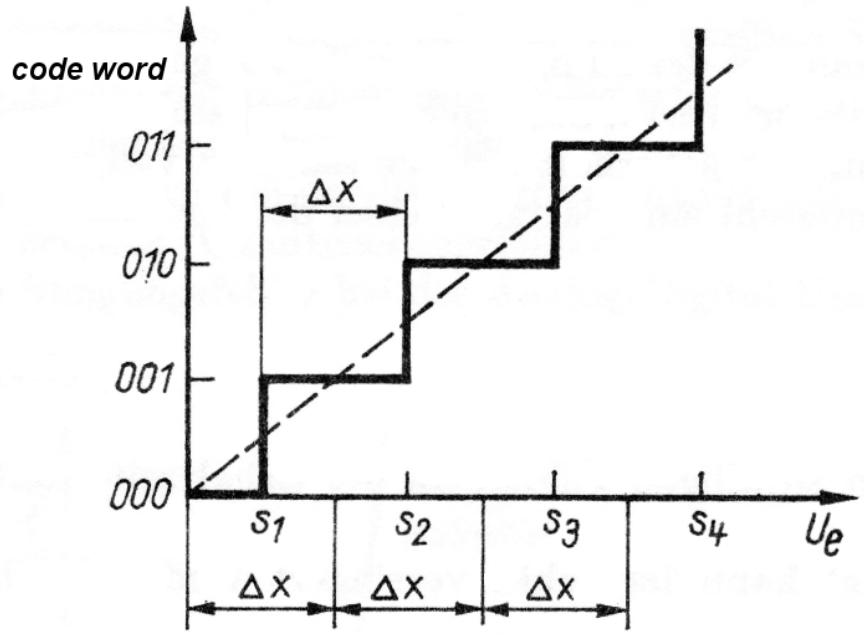
n – number of pulses per period

- Maximum counting error due to quantisation of time: **± 1 digit**
- pdf for error values: **rectangular**

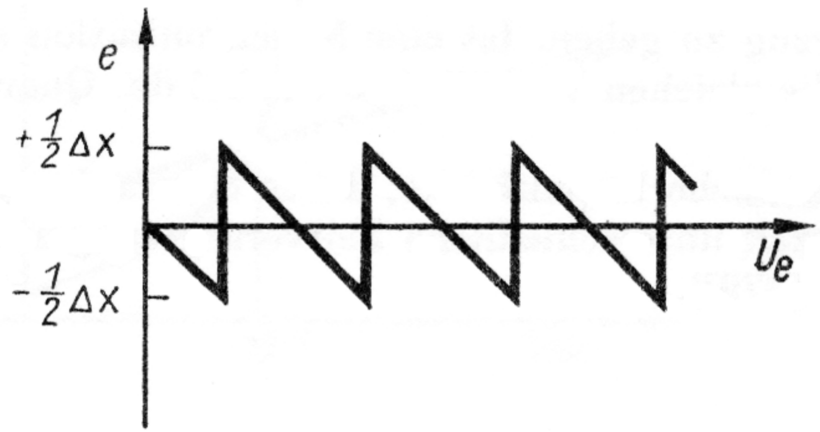
Source of pictures: H. Hart

Digital Signal Processing:
Quantisation Errors / Errors Due to Limited Resolution

Steplike transmission characteristic

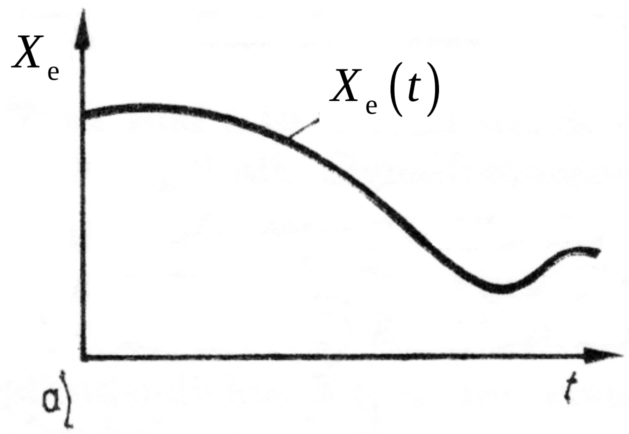


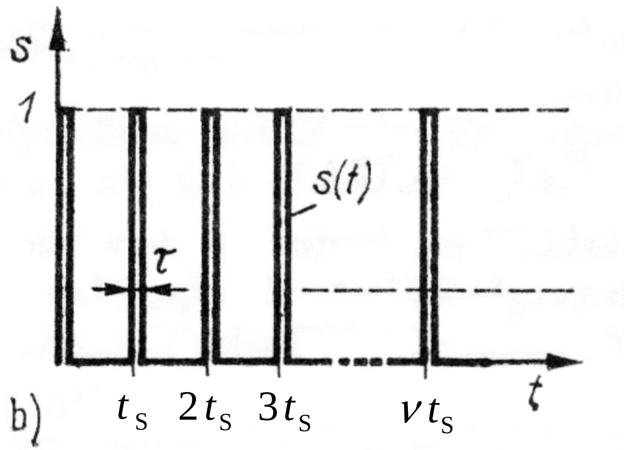
- Maximum error: **± 1/2 digit**
- pdf for error values: **rectangular**
  (for sufficiently large numbers of quantisation steps)



Source of pictures: H. Hart

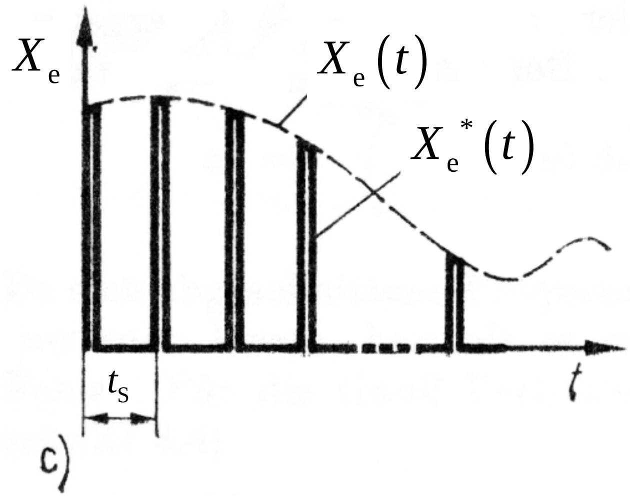
Digital Signal Processing:
Errors Due to Imperfect Sampling (1)





Represented in time domain

a) Input signal $X_e(t)$
b) Pulse sequence $s(t)$
c) Sampled signal $X_e{}^*(t)$



- Sampling is the reduction of a continuous signal to a discrete signal by multiplication with a pulse sequence

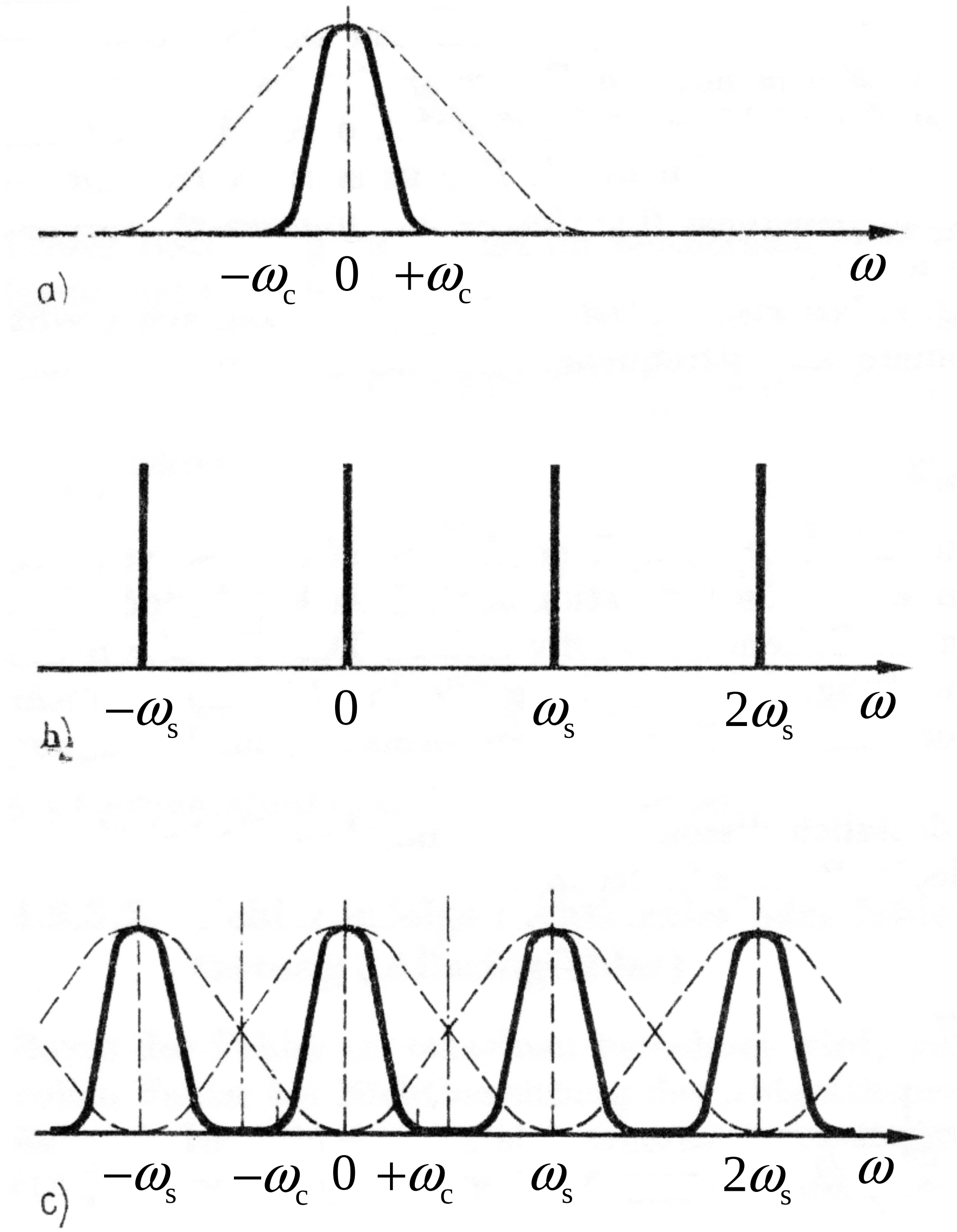
$$s(t)=\sum_{r=0}^{\infty}\delta(t-rt_s)$$

- Results in sampled signal (r – counting variable)

$$X^*(t)=X(t)\cdot s(t)=\sum_{r=0}^{\infty}X(rt_s)\cdot\delta(t-rt_s)$$

Source of pictures: H. Hart

Digital Signal Processing:
Errors Due to Imperfect Sampling (2)



Represented in frequency domain

a) Spectrum of input signal $X_e(t)$
b) Spectrum of pulse sequence $s(t)$
c) Spectrum of sampled signal $X_e{}^*(t)$

- Sampling leads to a periodic continuation of the baseband in frequency domain
- For accurate reconstruction of the original signal the spectrum must have a cut-off frequency (f_c):

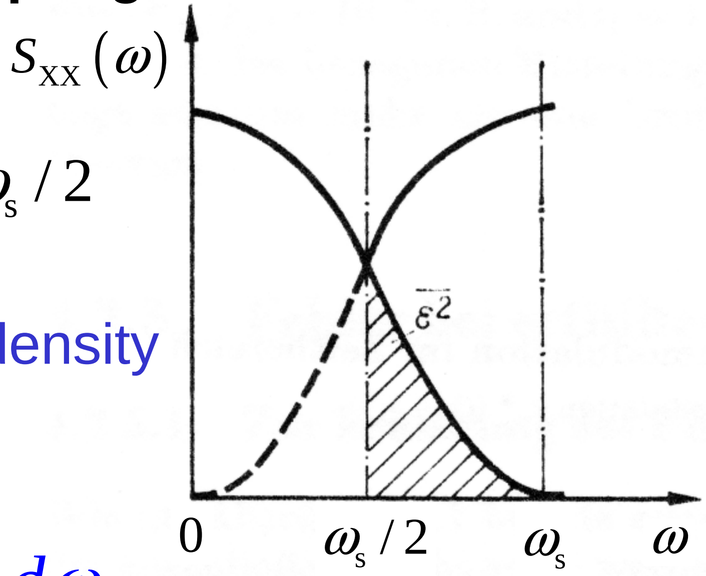
$$\omega_c \leq \frac{\omega_s}{2}$$

(*Sampling theorem*)

Source of pictures: H. Hart

Digital Signal Processing:
Errors Due to Imperfect Sampling (3)

- **Error in case of ideal low-pass filtering before sampling:** (anti-aliasing filter)
  - Ideal frequency reduction for input signal to $\omega_c = \omega_s / 2$
  - Therefore no overlay in spectrum of sampled data
  - Mean-square error described by power spectrum density $S_{XX}(\omega)$



$$\overline{\varepsilon^2} = \int_{-\infty}^{-\omega_s/2} S_{xx}(\omega)\,d\omega + \int_{\omega_s/2}^{\infty} S_{xx}(\omega)\,d\omega = 2\int_{\omega_s/2}^{\infty} S_{xx}(\omega)\,d\omega$$

- **Error in case of non-ideal or nonexisting low-pass filtering (aliasing)**
  - Overlay in spectrum of sampled data leads to error contribution in the frequency range $-\omega_s / 2 < \omega < \omega_s / 2$

$$\overline{\varepsilon_a^2} = 2\int_{-\omega_s/2}^{\omega_s/2} S_{xx}(\omega_s + \omega)\,d\omega = 2\int_{\omega_s/2}^{3\omega_s/2} S_{xx}(\omega)\,d\omega$$

- **pdf for error values: pme: Gaussian**

Source of pictures: H. Hart

Digital Signal Processing:
Errors Due to Non-Ideal Software

- **Error depends on the mathematical algorithms used**; Example:
  Numerical integration: rectangular or trapezoidal approximation

- Computers with higher clock speed allow for better approximations and therefore smaller error values (dynamic effects)

- Rounding errors due to limited word size:
  Additionally it depends on whether fixed point or floating point operations are used

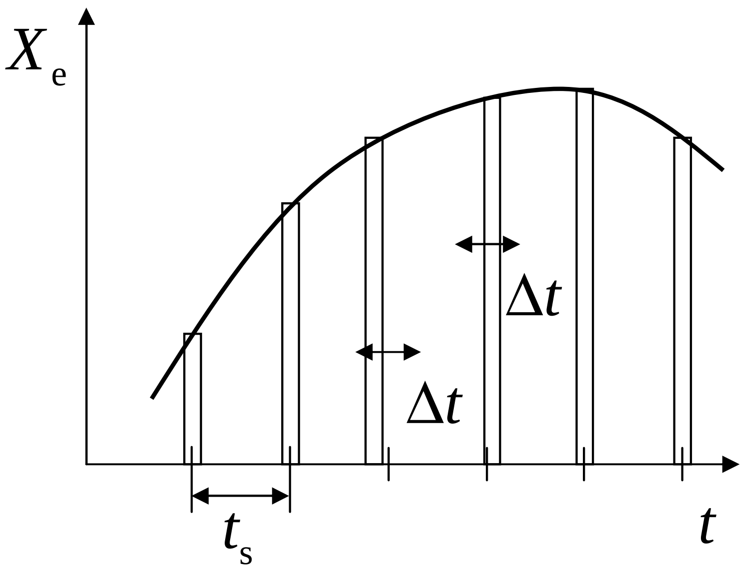
- **pdf for error values: rectangular or to be determined by test procedures (parameters: to be determined by test procedures)**

Digital Signal Processing:
Errors Due to Imperfect Synchronisation (Jitter)

- **Time error Δt due to deviations from periodic sampling** caused by perturbed synchronisation

- **Resulting error:**

$$\Delta X_{\mathrm{e}} \approx \frac{\partial X_{\mathrm{e}}}{\partial t} \Delta t$$



- **pdf for error values: approximately Gaussian**

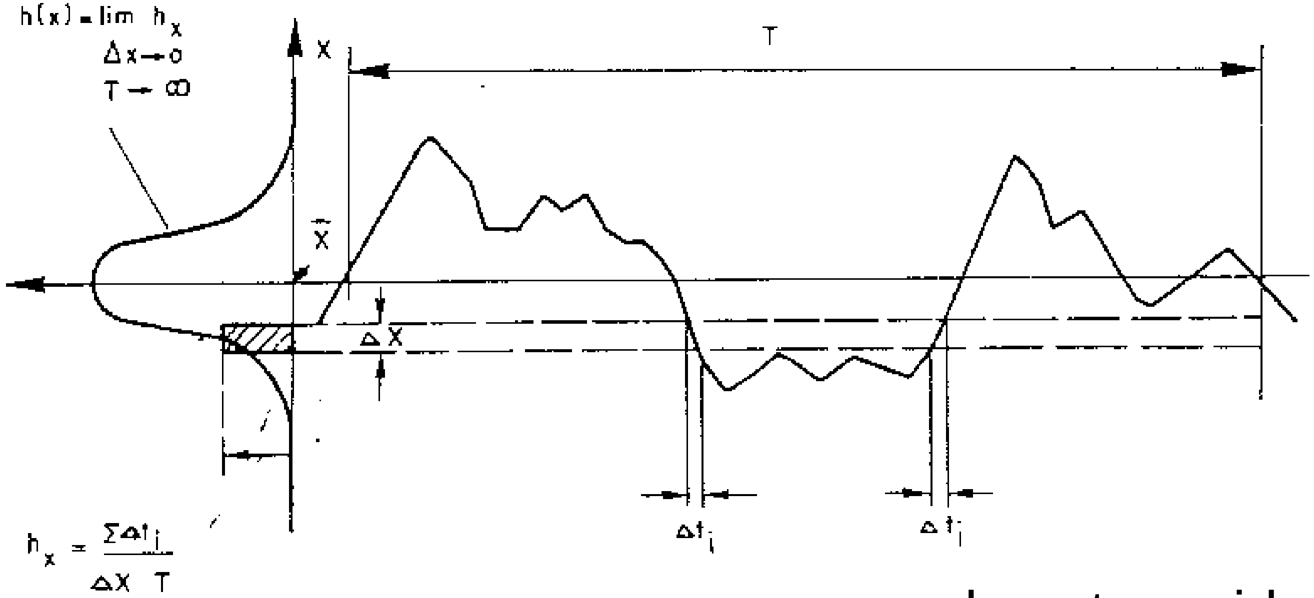
  (parameters: slope, maximum deviation $\frac{\partial X_{\mathrm{e}}}{\partial t}; \ \Delta t$)

Dynamic Uncertainty Contributions

Description of Stochastic Signals in the Time Domain (1)

- Amplitude density
  (of a stochastic signal):

$$h(X) = \lim_{\substack{\Delta X \to 0 \\ T \to \infty}} \frac{1}{\Delta X \cdot T} \sum_{i=1}^{n} \Delta t_i$$



- **Mean value of the signal:**

$$\overline{X} = \frac{1}{T}\int_0^T X(t)\,dt = \int_{-\infty}^{+\infty} h(X)\,X \cdot dX$$

- **Variance/dispersion of the signal:**

$$\sigma_x^2 = \frac{1}{T}\int_0^T \left[X(t) - \overline{X}\right]^2 dt = \overline{X^2} - \overline{X}^2 = \int_{-\infty}^{+\infty} h(X)\left(X - \overline{X}\right)^2 \cdot dX$$

... do not provide information about the conservation tendency of the signal

Source of pictures: P. Profos

Dynamic Uncertainty Contributions
Description of Stochastic Signals in the Time Domain (2)

(Auto-)Correlation function

$$\Phi_{xx}(\tau) = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{+\frac{T}{2}} X(t)\cdot X(t+\tau)\,dt$$

... represents the intrinsic coherence of the signal

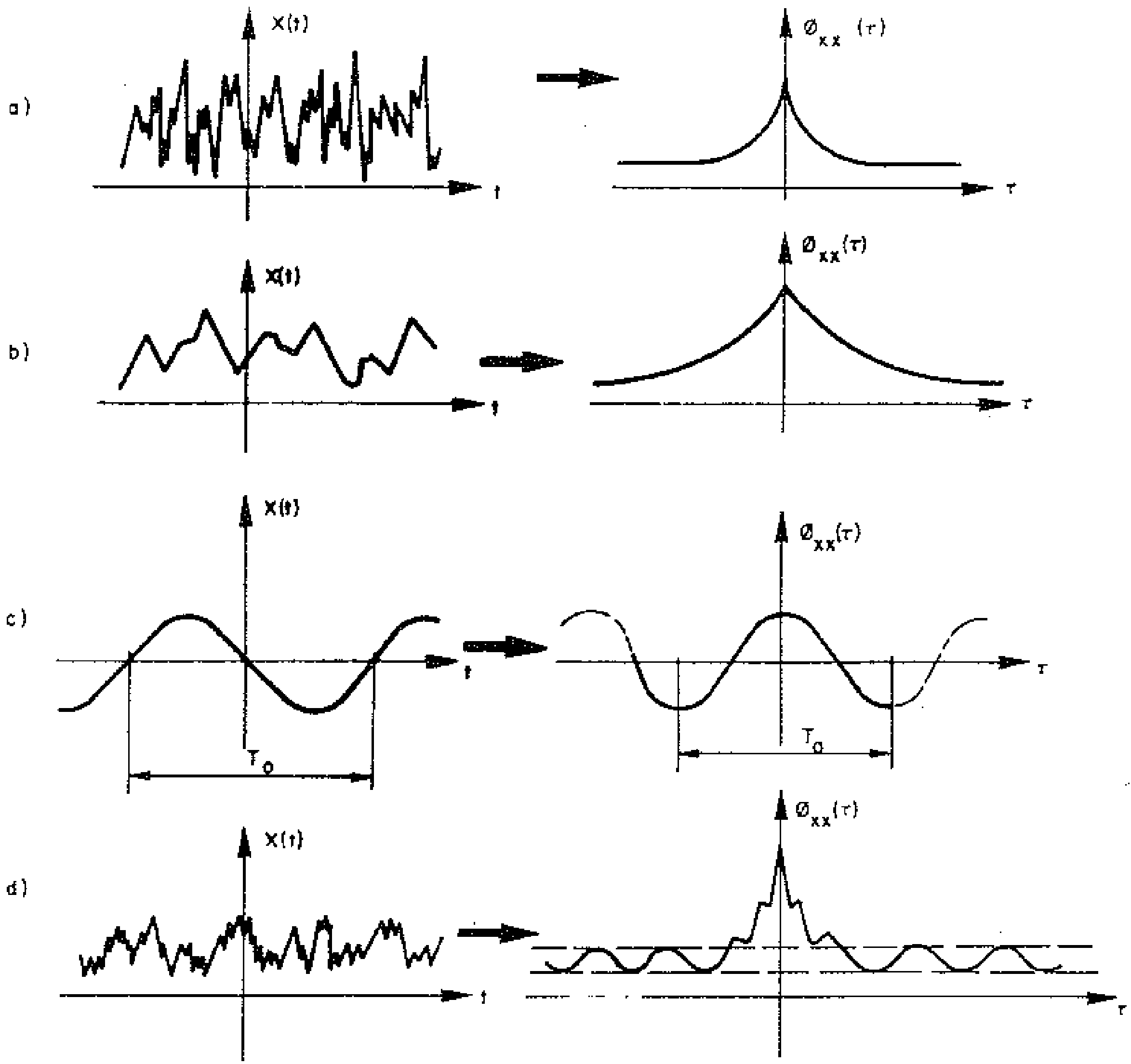
It can be demonstrated that ...

$$\Phi_{xx}(\tau) \le \Phi_{xx}(0)$$

$$\Phi_{xx}(0) = \overline{X^2}$$

$$\Phi_{xx}(\infty) = \overline{X}^2$$

$$\Rightarrow \sigma_x^2 = \Phi_{xx}(0) - \Phi_{xx}(\infty)$$



Source of pictures: P. Profos

Dynamic Uncertainty Contributions

Description of Stochastic Signals in the Frequency Domain

Fourier transformation of $\Phi_{xx}(\tau)$ yields the so-called

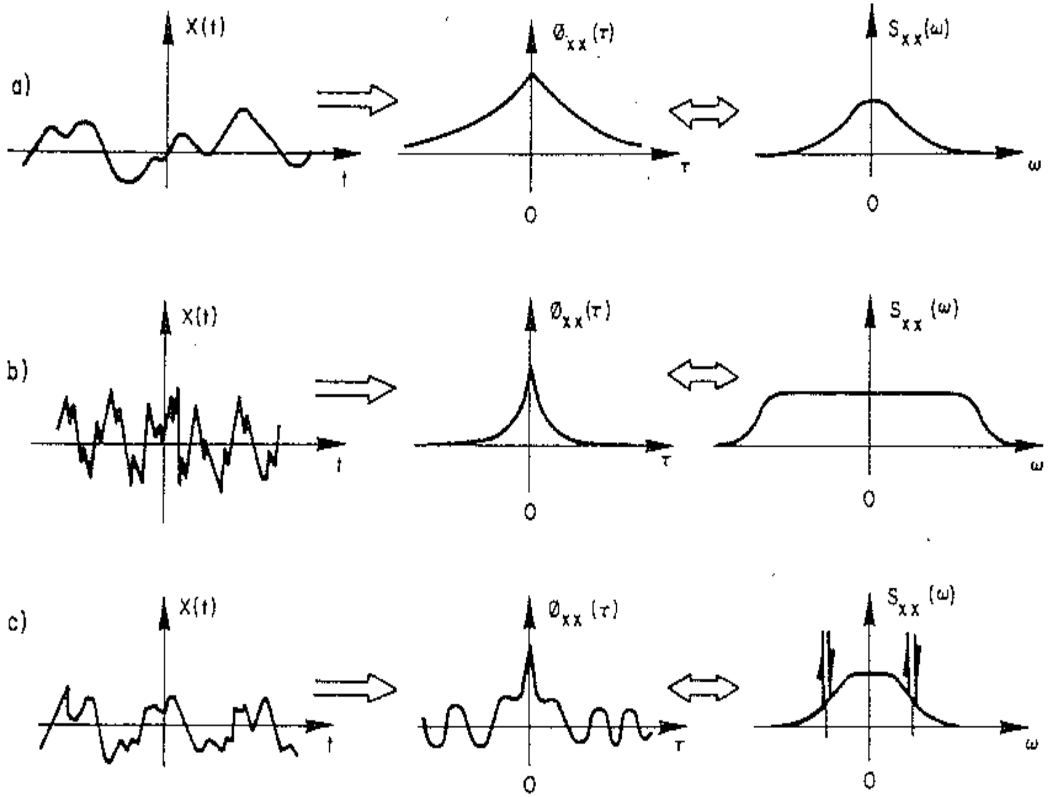
- **(Auto) Power spectrum density** S_{xx}

$$S_{xx}(\omega) = \int_{-\infty}^{+\infty} \Phi_{xx}(\tau) \cdot e^{-j\omega\tau} d\tau$$

... and reverse transformation yields

$$\Phi_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega) \cdot e^{j\omega\tau} d\omega$$

(Wiener-Chinchine Theorem)



Source of pictures: P. Profos

Dynamic Uncertainty Contributions

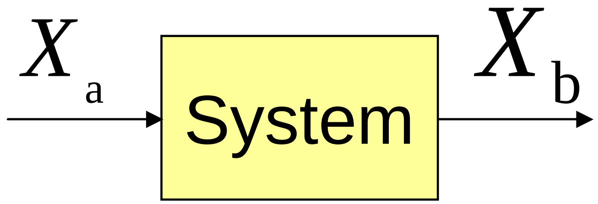
Transmission Behaviour of Systems in the Time Domain

Prerequisite: Linear(ized) systems

Description: by means of ordinary differential equations

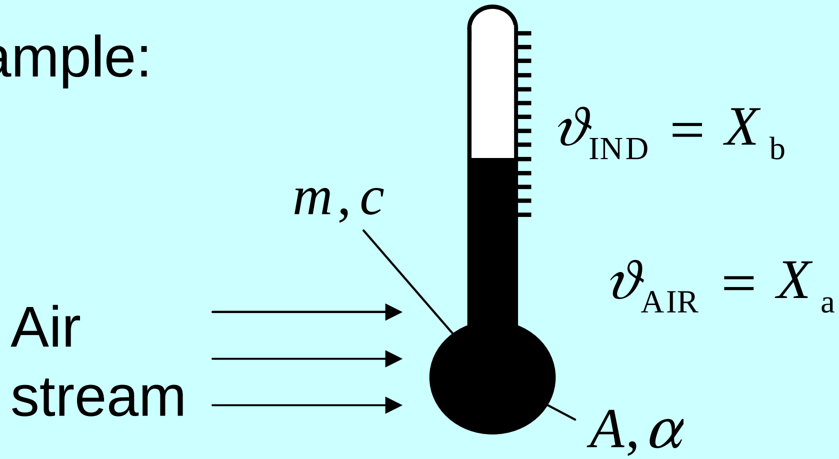
General expression:

$$a_0 X_a + a_1 X_a{}' + \ldots + a_m X_a^{(m)} = b_0 X_b + b_1 X_b{}' + \ldots + b_n X_b^{(n)}$$



where $X' = \dfrac{dX}{dt}$

Example:



$$\alpha \cdot A \cdot \left(\vartheta_{\mathrm{AIR}} - \vartheta_{\mathrm{IND}}\right) = m \cdot c \cdot \frac{d\vartheta_{\mathrm{IND}}}{dt}$$

or

$$\vartheta_{\mathrm{AIR}} = \vartheta_{\mathrm{IND}} + T \cdot \frac{d\vartheta_{\mathrm{IND}}}{dt}; \; T = \frac{m \cdot c}{\alpha \cdot A}$$

Source: H. Hart

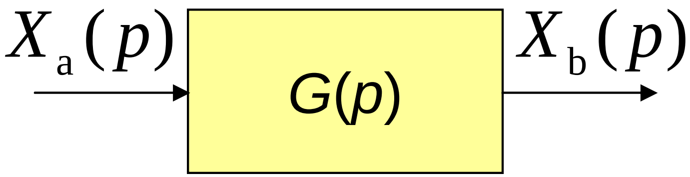
Dynamic Uncertainty Contributions

Transmission Behaviour of Systems - Frequency Domain

Description of the transmission behaviour by *transfer functions*:

$\mathfrak{L}$ - Laplace operator:

$$\mathfrak{L}\left\{X_{\mathrm{b}}(t)\right\} = \mathfrak{L}\left\{X_{\mathrm{a}}(t)\right\} \cdot G(p)$$



$$\Rightarrow X_{\mathrm{b}}(p) = X_{\mathrm{a}}(p) \cdot G(p)$$

Dynamic Uncertainty Contributions

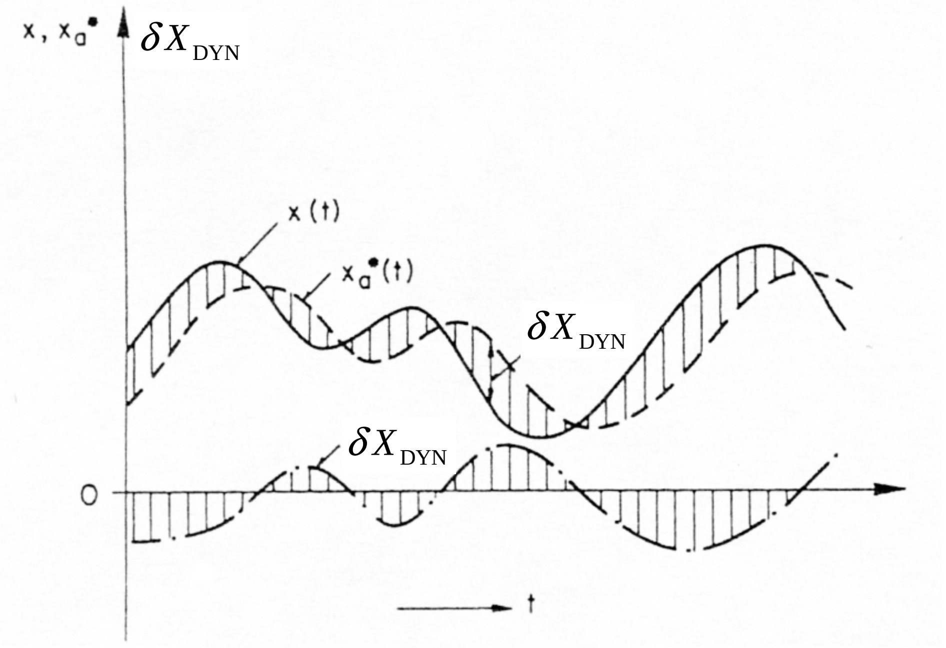
Definition of a Dynamic Error (Source: Profos)

- **Instantaneous dynamic error:**

$$\delta X_{\mathrm{DYN}}(t) = X_{\mathrm{a}}^{*}(t) - X_{\mathrm{a}}(t)$$

$X_{\mathrm{a}}^{*}(t)$ - unperturbed characteristic

$X_{\mathrm{a}}(t)$ - dynamically affected characteristic



- **In practice,** the instantaneous dynamic error is not very useful. Therefore, **mean values are utilized**, e.g. the following ***quality criterion***:

$$E_{\mathrm{Q,DYN}} = K\int_0^{\infty}\left(X_{\mathrm{a}}^{*} - X_{\mathrm{a}}\right)^2 dt = K\int_0^{\infty}\delta X_{\mathrm{DYN}}^2 dt$$

or the ***mean-square dynamic error*** **:**

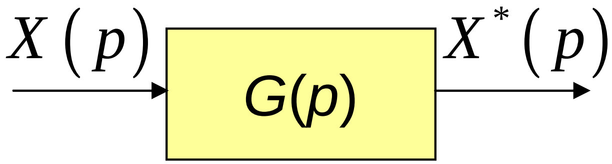
$$\overline{\delta X_{\mathrm{DYN}}^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^{T}\left(X_{\mathrm{a}}^{*} - X_{\mathrm{a}}\right)^2 dt$$

Dynamic Uncertainty Contributions

Evaluation of the Dynamic Error (Source: Profos)

- For linear systems holds:

$$X^{*}(p) = X(p) \cdot G(p)$$



- If, **in case of deterministic signals**, e.g. a step function, two out of the three quantities are known, the error is knowable:

$$\delta X_{\mathrm{DYN}}(p) = X(p)\left[\frac{X^{*}(p)}{X(p)} - 1\right] = X(p)\left[G(p) - 1\right] \quad \text{and}$$

$$\delta X_{\mathrm{DYN}}(t) = {}^{-1}\mathfrak{L}\left\{X(p)\left[G(p) - 1\right]\right\} \quad \text{respectively}$$

- **In case of stochastic quantities/signals**, the transfer relationship is to be expressed by means of the power spectrum density:

$$S_{\mathrm{xx}}(\omega) \cdot \left|G(j\omega)\right|^{2} = S_{\mathrm{x^{*}x^{*}}}(\omega)$$

Dynamic Uncertainty Contributions

Evaluation of the Dynamic Error (Source: Profos)

- **In case of (steady-state) stochastic signals**, an instantaneous dynamic error cannot be given, but the **dynamic mean-square error**:

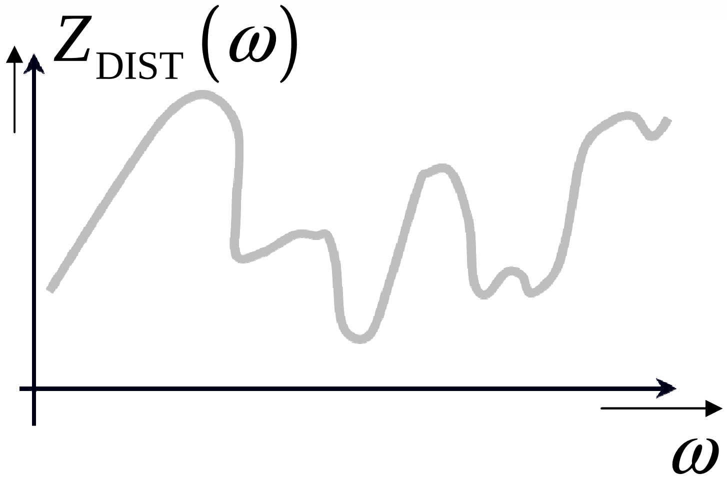
$$\overline{\delta X^2_{\mathrm{DYN}}} = \frac{1}{\pi}\int_0^{\infty} S_{\mathrm{xx}}(\omega)\cdot\left|G(j\omega)-1\right|^2 d\omega$$

or

$$\overline{\delta X^2_{\mathrm{DYN}}} = \frac{1}{\pi}\int_0^{\infty} S_{\mathrm{x^*x^*}}(\omega)\cdot\left|1-\frac{1}{G(\omega)}\right|^2 d\omega$$

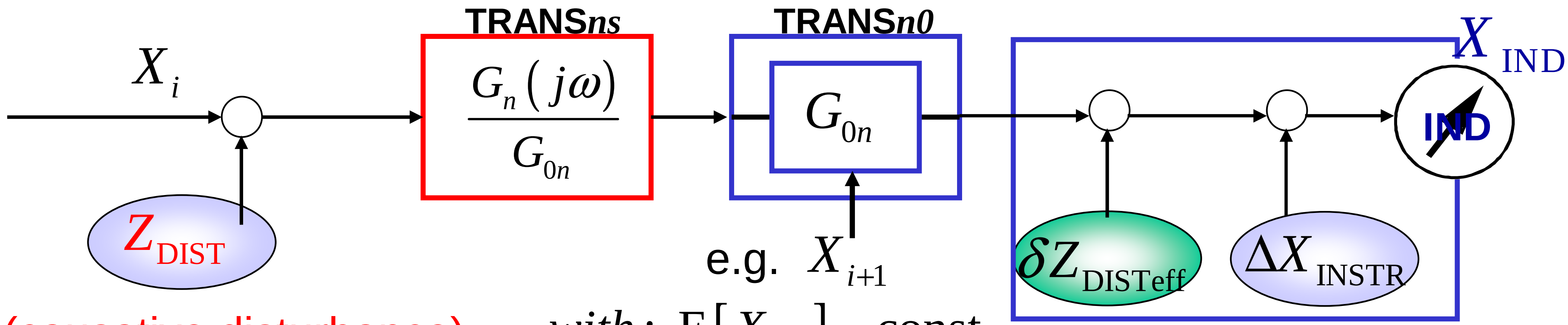
Example: External Stochastic Disturbances (1)

Special Modelling



Possible treatment:

- Separate treatment of static and stochastic errors
- Perpetuation of the model structure
- Linearization of the model (at Z_{k0}) and introducing frequency-dependent elements $G_n(j\omega)/G_{0n}$



(causative disturbance)

$with:\ \mathrm{E}[X_{i+1}] = \mathrm{const.}$

Indication-effectual contribution:

$$\delta Z_{\mathrm{DISTeff}}(j\omega) = Z_{\mathrm{DIST}}(j\omega)\cdot\left[\left(G_n/G_{0n}\right)\cdot G_{0n}\right]$$

Possible course of action:

- Evaluating the dynamic mean-square error of the surrogate quantity $\delta Z_{\mathrm{DISTeff}}$

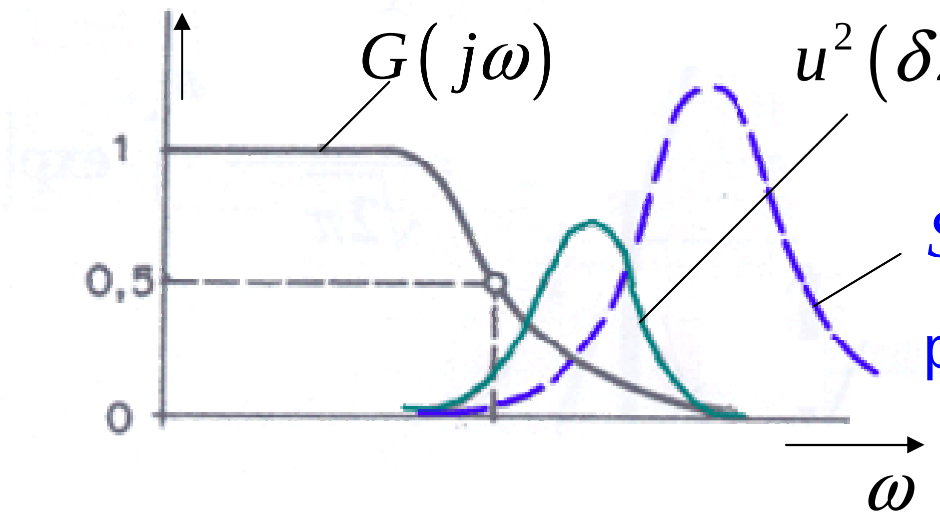
$$\overline{\delta Z^2_{\mathrm{DISTeff}}} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \delta Z^2_{\mathrm{DISTeff}}(t)\,dt \overset{!}{=} \overline{u^2(\delta Z_{\mathrm{DISTeff}})}$$

(taken as variance for this quantity)

- pdf: Gaussian (pme, power-limited)

$$\mathrm{E}[\delta Z_{\mathrm{DISTeff}}] = 0; \quad \overline{u^2(\delta Z_{\mathrm{DISTeff}})} = \frac{1}{\pi}\int_0^\infty S_{ZZ}(\omega)\cdot\left|G(j\omega)^{-1}\right|^2 d\omega$$

- Meaning:



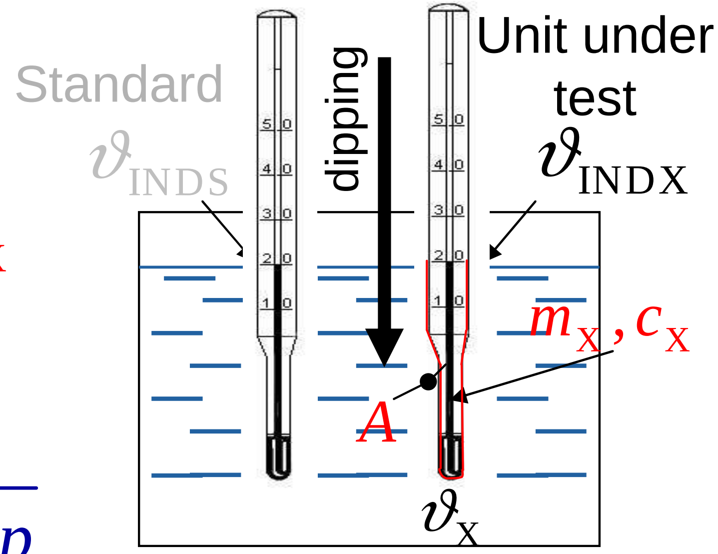
Source of pictures: P. Profos

Example: Transient Transmission Behaviour (1)

- Problem: Measurement or calibration setup with transients, storage effects and the like

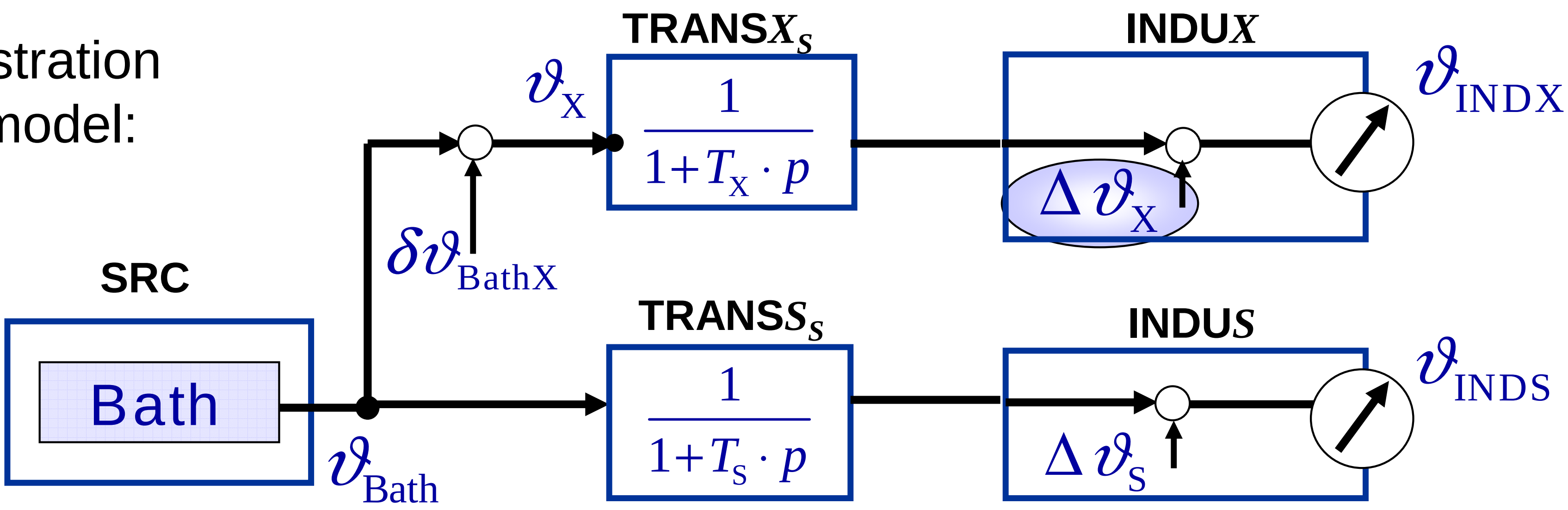
- Example: $\alpha A\left(\vartheta_{\mathrm{X}} - \delta\vartheta_{\mathrm{INDX}}\right) = m_{\mathrm{X}} \cdot c_{\mathrm{X}} \left.\frac{\partial \vartheta_{\mathrm{INDX}}}{\partial t}\right|_{\vartheta \mathrm{INDX}}$

time constant: $m_{\mathrm{X}} \cdot c_{\mathrm{X}} \left(\alpha \cdot A\right)^{-1}$



$$\Rightarrow \quad \delta\vartheta_{\mathrm{INDX}}(p) = \delta\vartheta_{\mathrm{X}}(p) \cdot \frac{1}{1 + T_{\mathrm{X}} \cdot p}$$

Illustration of model:



Example: Transient Transmission Behaviour (2)

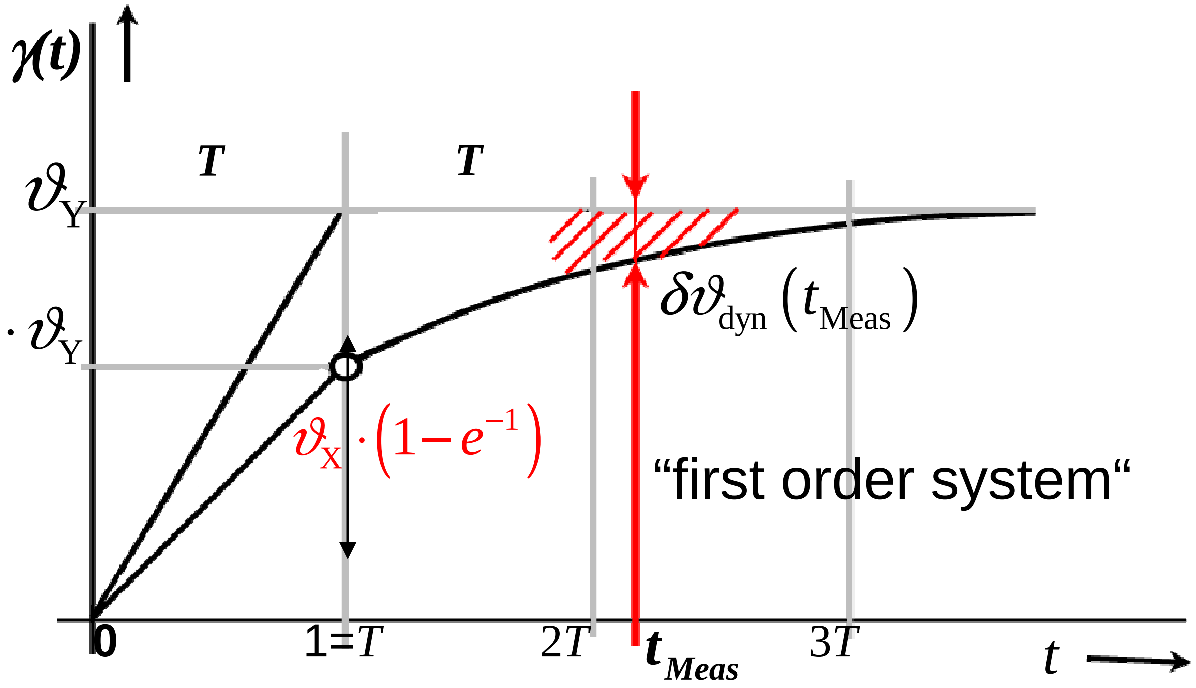
Course of action for consideration of dynamic measurement deviation

- Usual course of action:

  wait for the steady state

  $$\left(t >> 100 \cdot T\right)$$

  $$\Rightarrow \lim_{p \to 0} G(p)/G_0 = 1$$



- Alternative:

  shorter waiting $\left(t > 20 \cdot T\right)$

  and consideration of “remaining dynamic deviation“

- Evaluating the remaining dynamic deviation:

  PDF: rectangular (pme, amplitude-limited)

  Estimation value: $E\left[\delta\vartheta_{dyn}\right] = -0{,}5 \cdot$

  Standard uncertainty: $u_{\delta\vartheta dyn} = \frac{1}{\sqrt{12}}$

  } for maximum allowable deviation

Conclusion

- The modelling procedure presented is **applicable to most areas of uncertainty evaluation** of measurements and calibrations

- … is clearly structured in five elementary steps
- … is applicable to complex problems
- … is applicable to the modelling of dynamic system behaviour
- … in describing dynamic system behaviour, the dynamic error preferably should be expressed by a mean-square value

Systematic modelling is possible

Systematic modelling constitutes an important improvement of uncertainty evaluation