# NPL 2nd Signal Processing Awareness Seminar – 23rd June 2006

## Focus on PHASE/DELAY in Filter Design

John Blakey

# INTRODUCTION

The Importance of Analogue operations for DSP

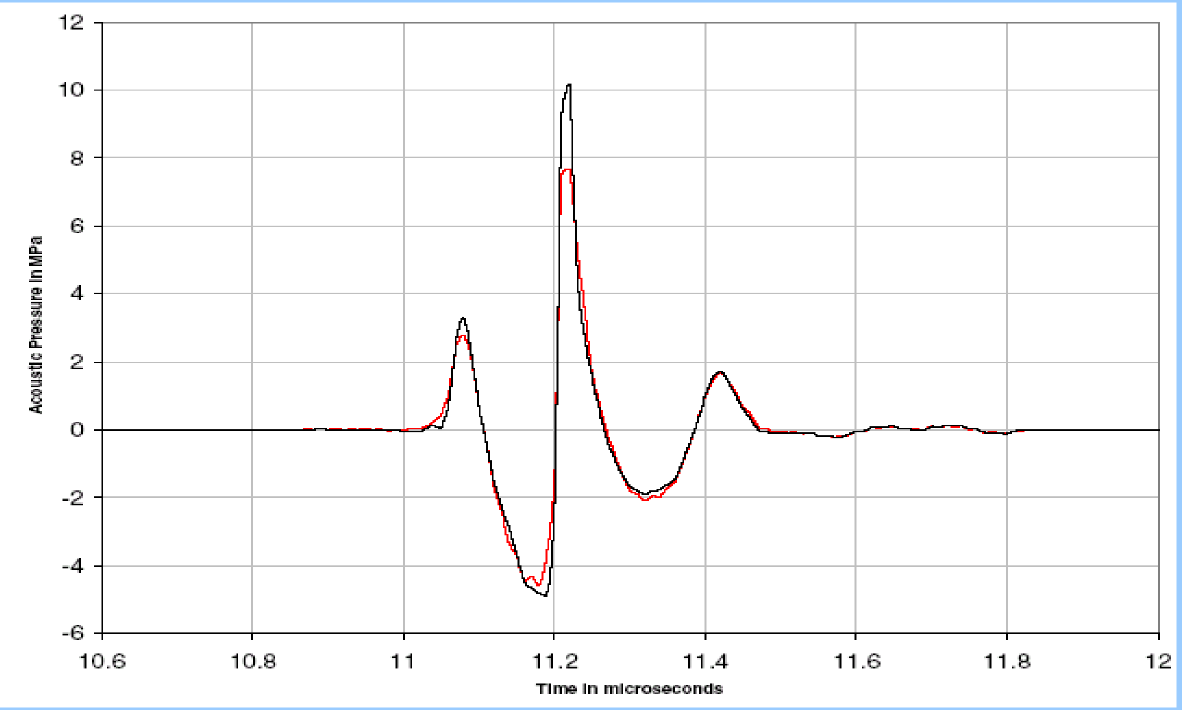
Emphasis on on analogue filters –but not exclusively

The key aspects of any filter design are:

- SPECIFCATION
- TRADE-OFFS

# The importance of Phase/Delay.

- When fidelity of signal form must be preserved
- Examples:

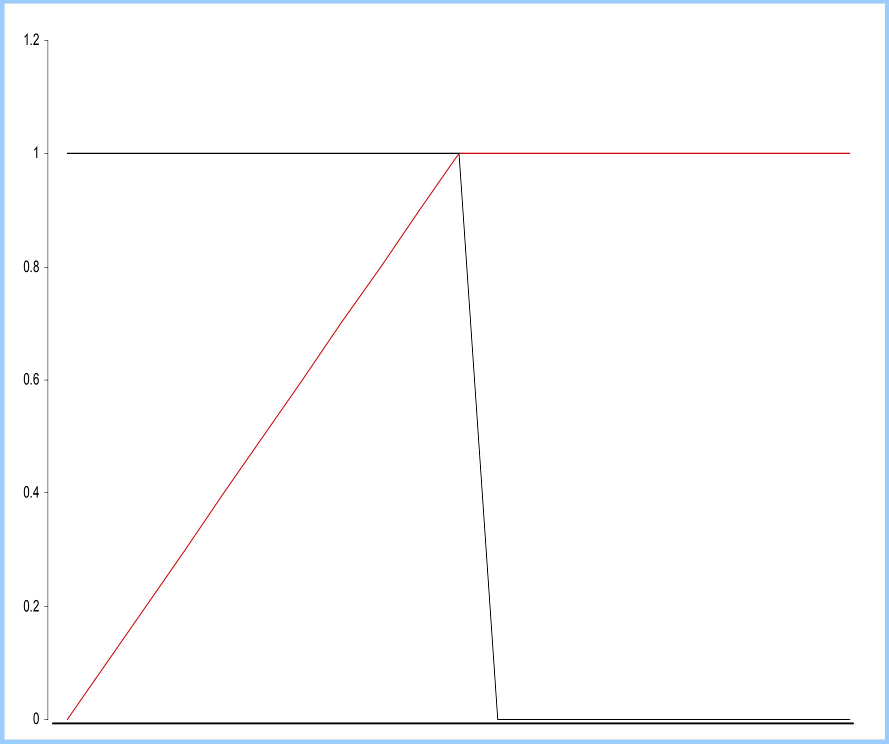


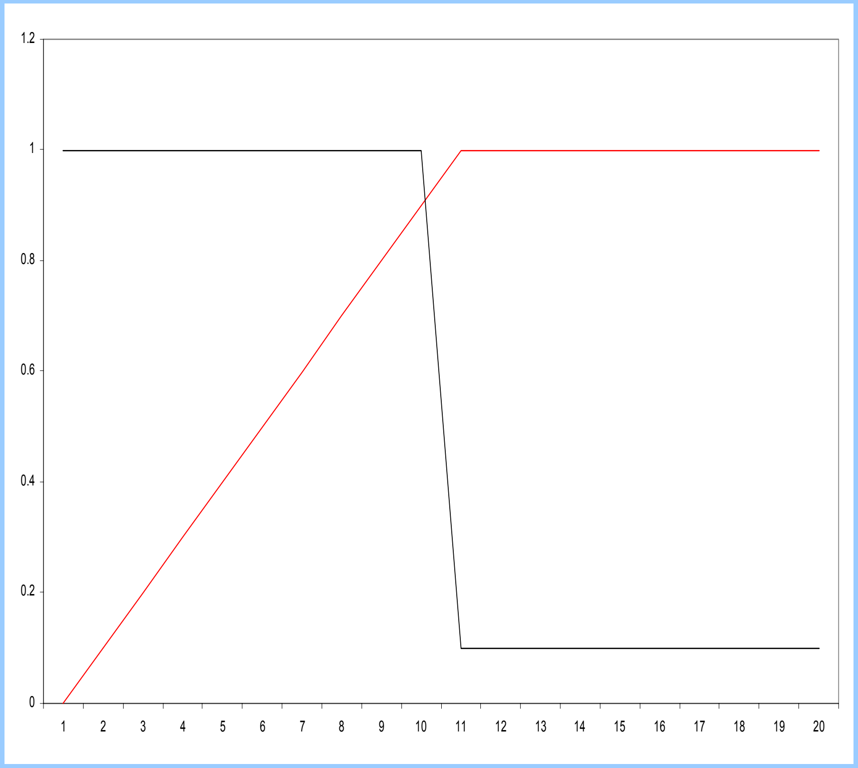
# FILTER SPECIFICATIONS

- The Ideal LP filter response
- Realistic LP Magnitude responses
- Phase/Delay responses

# FILTER SPECIFICATIONS

Ideal LP filter response

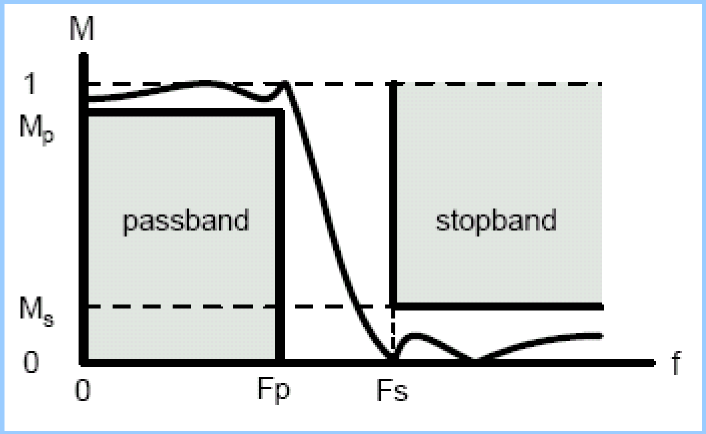


# FIILTER SPECIFICATIONS

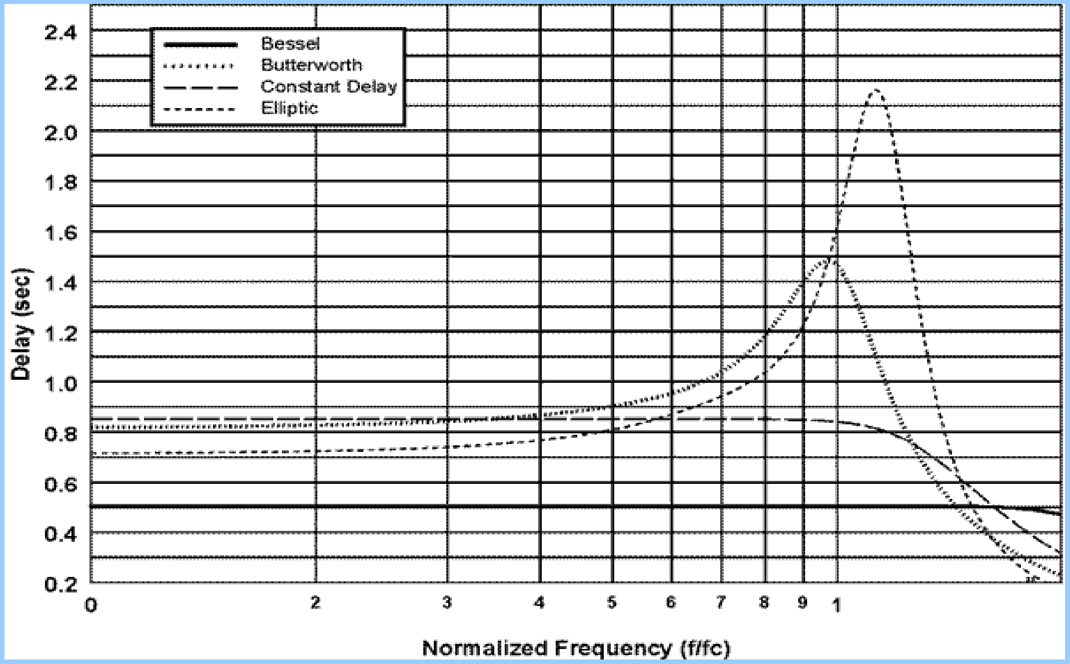
Realistic LP Magnitude responses

The ‘cookbook’ approach.



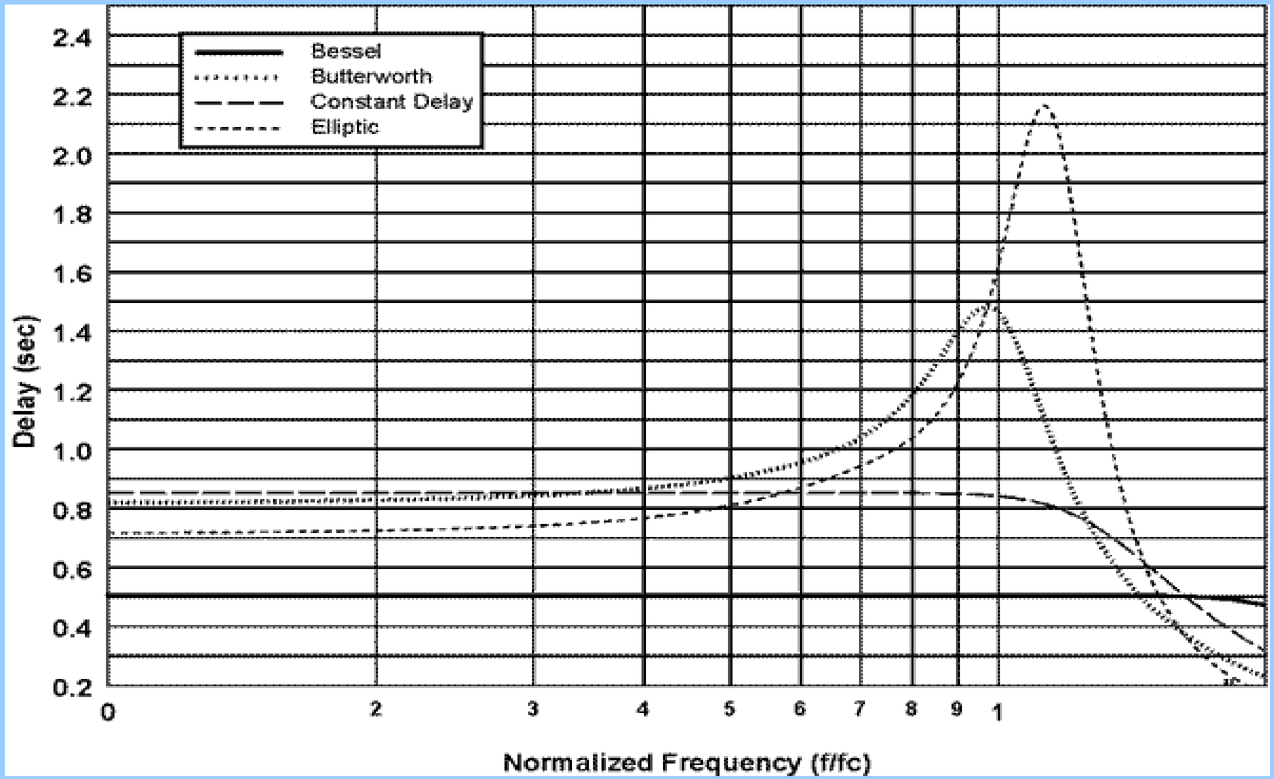
# FIILTER SPECIFICATIONS

Some realistic LP Magnitude responses



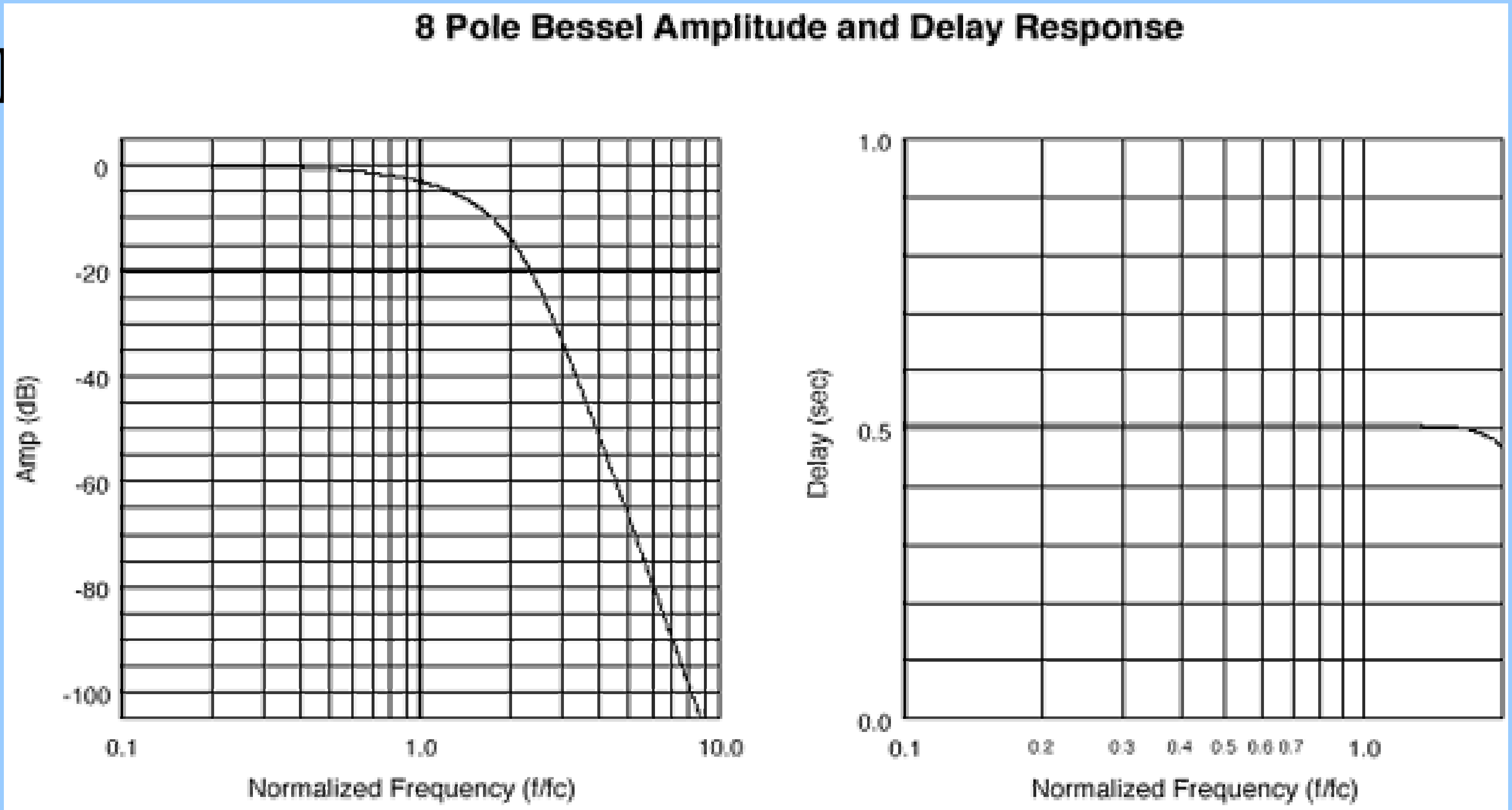
# FILTER SPECIFICATIONS

Phase/Delay response



# CONTROLLING THE PHASE RESPONSE

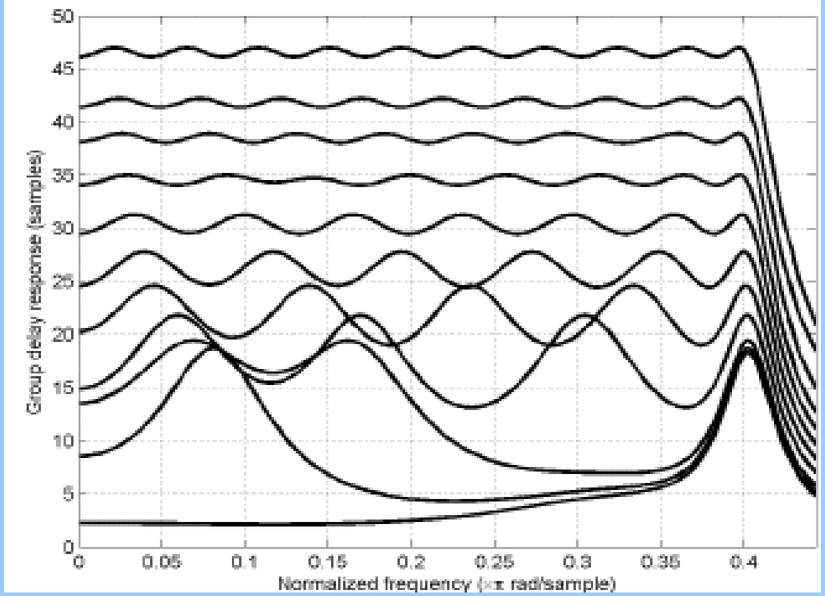
- CONVENTIONAL METHODS



# CONTROLLING THE PHASE RESPONSE

- CONVENTIONAL METHODS

Correction/compensation – All-pass filters



# CONTEMPORARY DESIGN APPROACHES

1. Optimisation methods

2. Generalised Sampling Theory

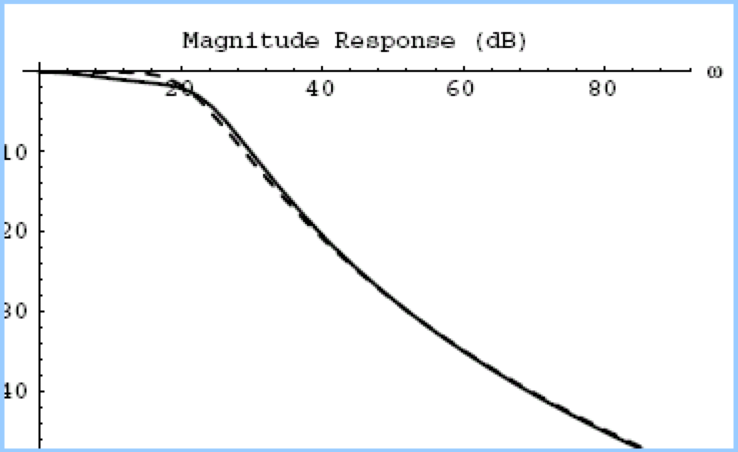
# Optimisation methods

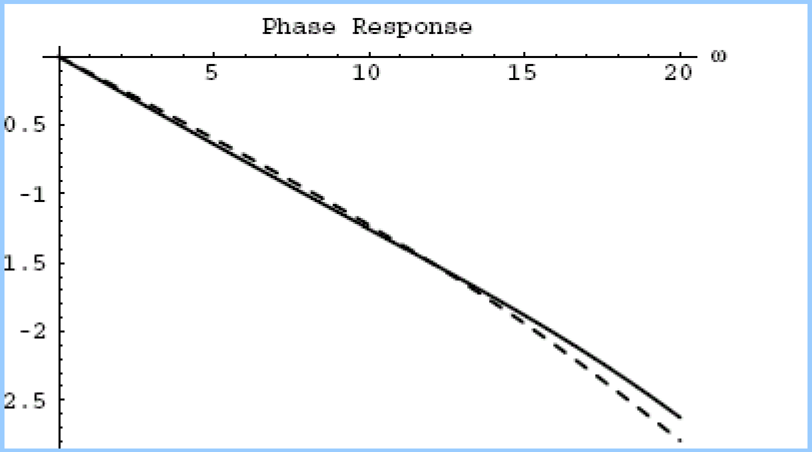
Constrained specifications & Design Space

- Hard and soft constraints
  - Time and frequency specifications

# Optimisation methods

- Design space & constrained designs



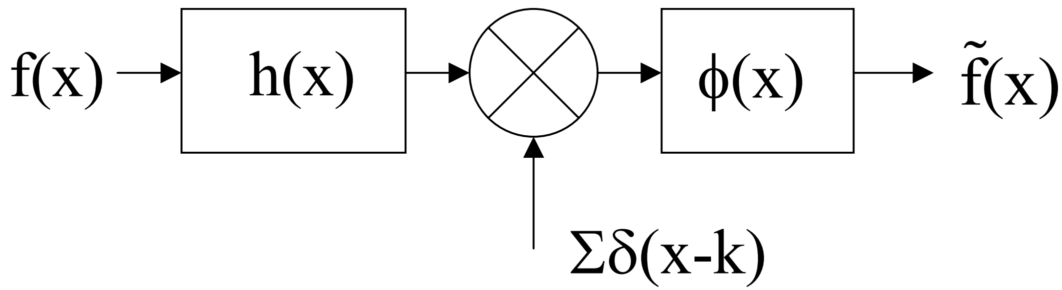



# Optimisation methods

- Evolutionary design (Genetic Programming).

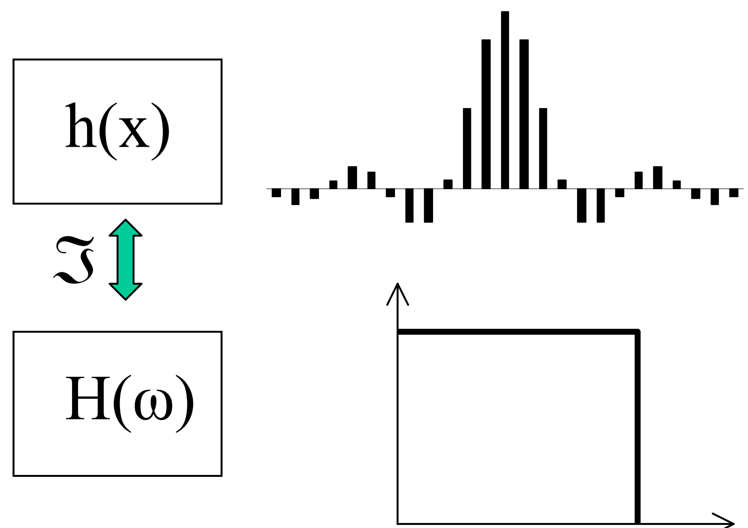
# Generalised Sampling Theory

Classical Shannon sampling system.




**Interpolation formula**

$$f(x) = \sum_{n} f(nT)\,\mathrm{sinc}(x/T - n)$$



# Generalised Sampling Theory

- Application of APPROXIMATION THEORY
- Replaces the $\varphi(x)$= sinc x function with more general functions, $\varphi'(x)$

$$\mathbf{f(x)} = \sum_{\mathbf{n}} \mathbf{f(nT)sinc(x/T - n)} \quad \Rightarrow \sum_{\mathbf{n}} \mathbf{c(n)}\varphi\mathbf{'(x - n)}$$

- What are these functions?

# Generalised Sampling Theory

The functions of choice are based on the

B-Spline.

- The spline family is generated from the zero$^{th}$ degree spline

$$\beta^0(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ \frac{1}{2}, & |x| = \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}; \quad \tilde{\beta}^0 = \left( \frac{\sin(\omega/2)}{\omega/2} \right)$$

by repeated convolutions

$$\beta^n(x) = \beta^0 * \beta^0 * \ldots * \beta^0(x); \quad \tilde{\beta}^n(\omega) = \left( \frac{\sin(\omega/2)}{\omega/2} \right)^{n+1}$$

# Generalised Sampling Theory

- The required reconstruction spline interpolator ή$^{n}$(x), analogous to sinc(x), is generated from the B-spline of the required degree.
- The Fourier transform of ή$^{n}$(x) gives the required frequency response of the reconstruction filter.

$$\Phi^{n}(\omega)=\left(\frac{\sin(\omega/2)}{(\omega/2)}\right)^{n+1}\frac{1}{B^{n}(e^{j\omega})}$$

Where $B^{n}(e^{j\omega})$ is the Fourier transform of the discrete B-spline of degree n.

# Generalised Sampling Theory

The choice of reconstruction filter determines the required response of the pre-filter.

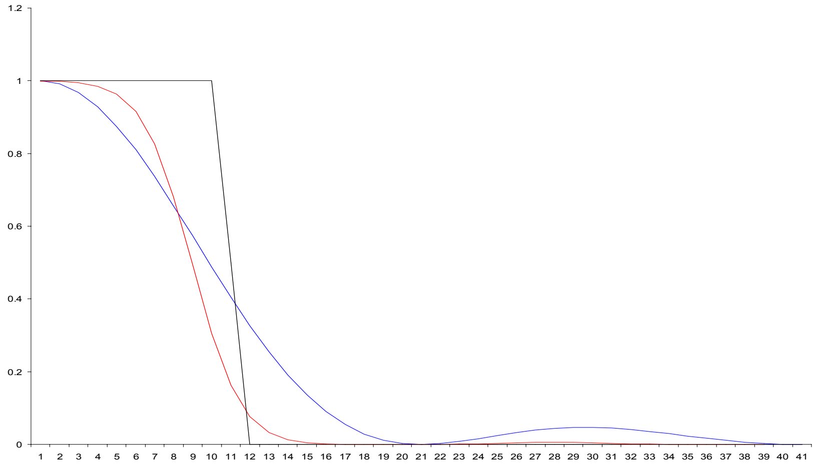
Given the post–filter response $\varphi'(x)$,the optimal response for the pre-filter $h'(x)$, is derived from the biorthogonality condition

i.e. $\langle h'(x),\varphi'(x)\rangle = \delta(k-l)$.

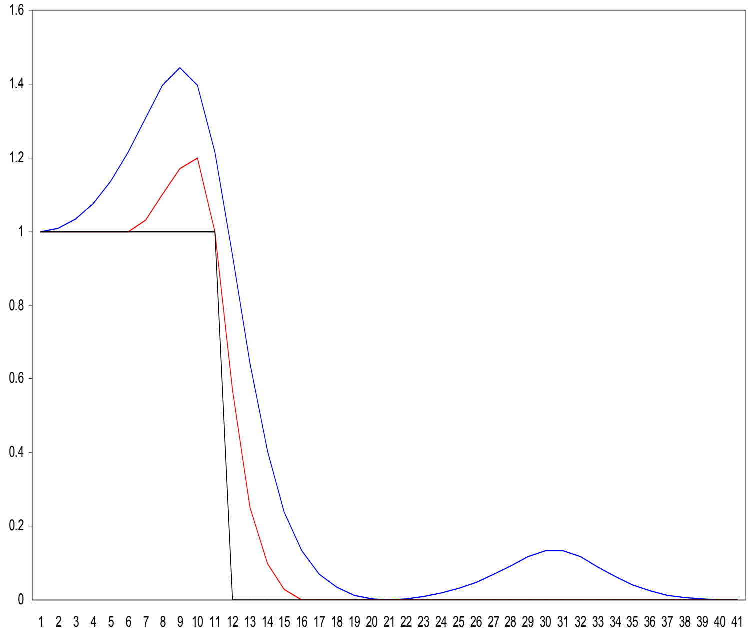
This provides a minimum error solution (in the least squares sense). The pre-filter frequency response is then:

# Generalised Sampling Theory

- Post filter responses




- Pre-filter responses



# THE END?