National Physical Laboratory

An Introduction to the Analysis of Non-Repetitive Signals Using Joint Time-Frequency Distributions and Wavelets

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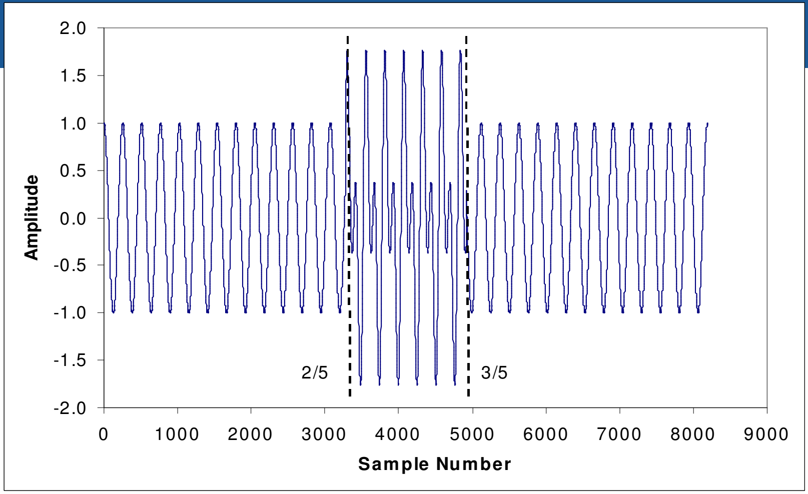
Outline

- Non-Repetitive Waveforms
- Windowing and the STFT
- Wigner-Ville Distribution
- Joint Time Frequency Distributions
- Wavelets
- Polynomial Demoduation

Fourier Transform of a Burst Signal

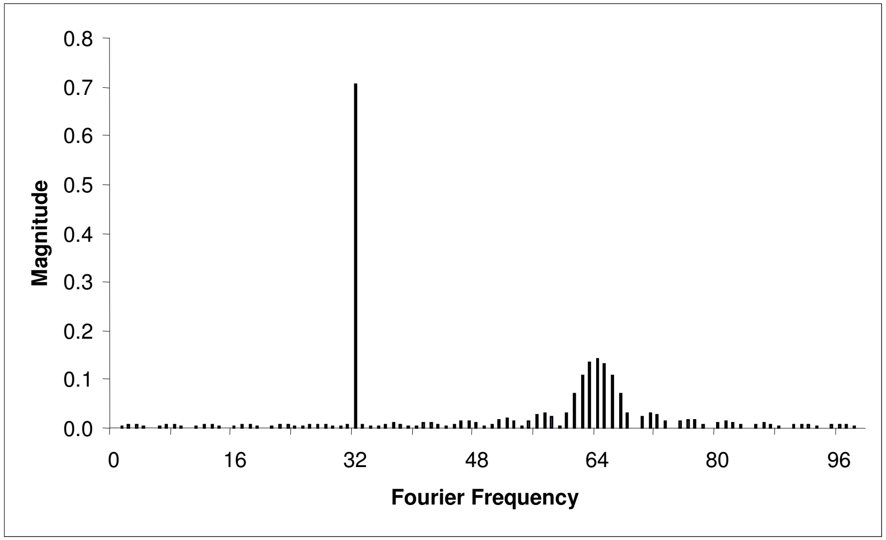


H1 and a
burst of H2

(H1 and H2 are the same amplitude)

FT

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j\,2\pi\, f\, t}\, dt$$



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Analysing Non-repetitive waveforms

$$S(f) = \int_{-\infty}^{-\infty} s(t)\, e^{-j\,2\pi\, f\, t}\, dt$$

No longer reliable!

Break the waveform up into adjacent sections

Do a DFT on each section – Now have T & F Information

Each Section Should contain an integer number of cycles

Sometime called a Short Time Fourier Transform (STFT)

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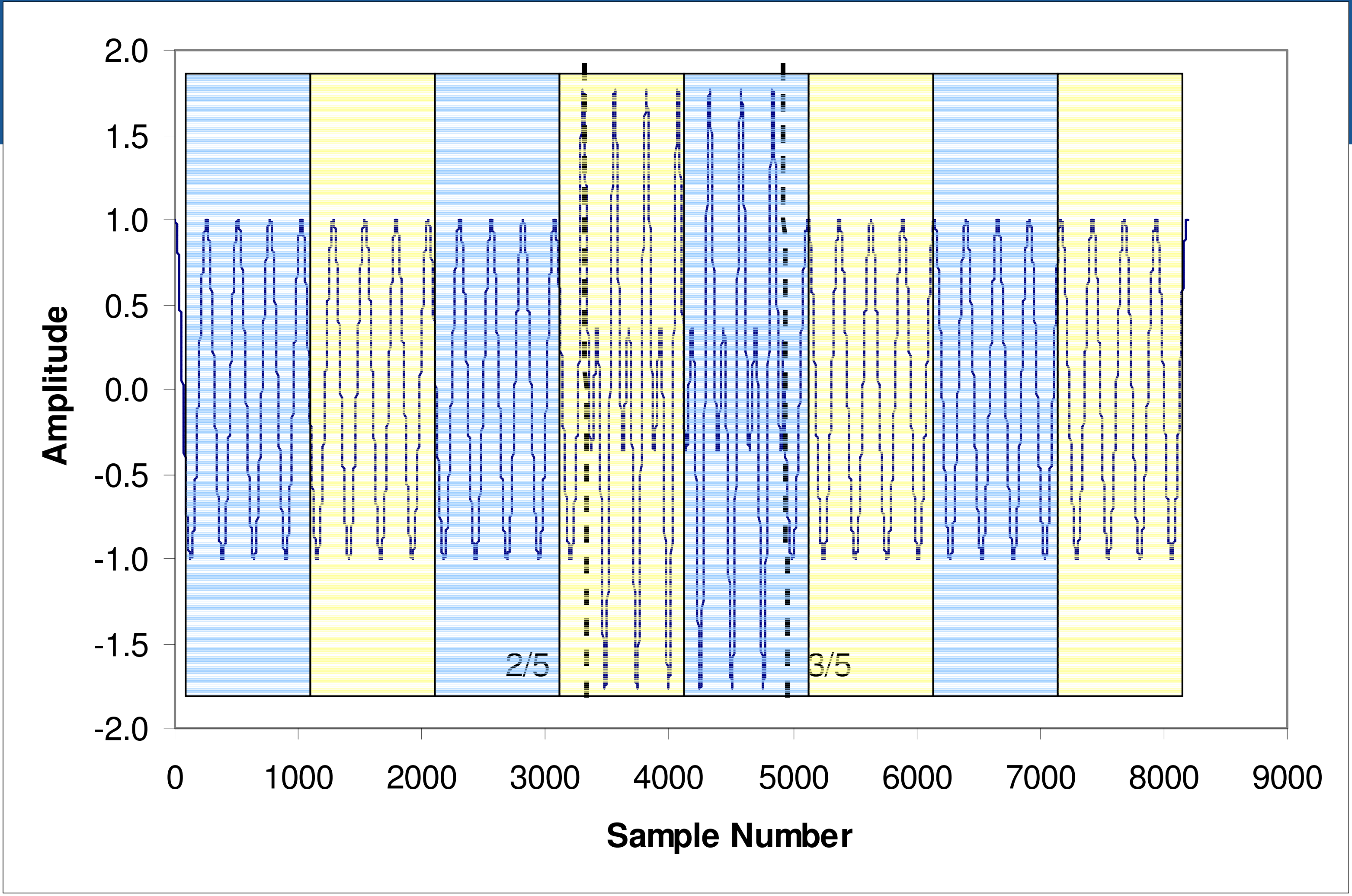
Short Time FT Using 8 Windows



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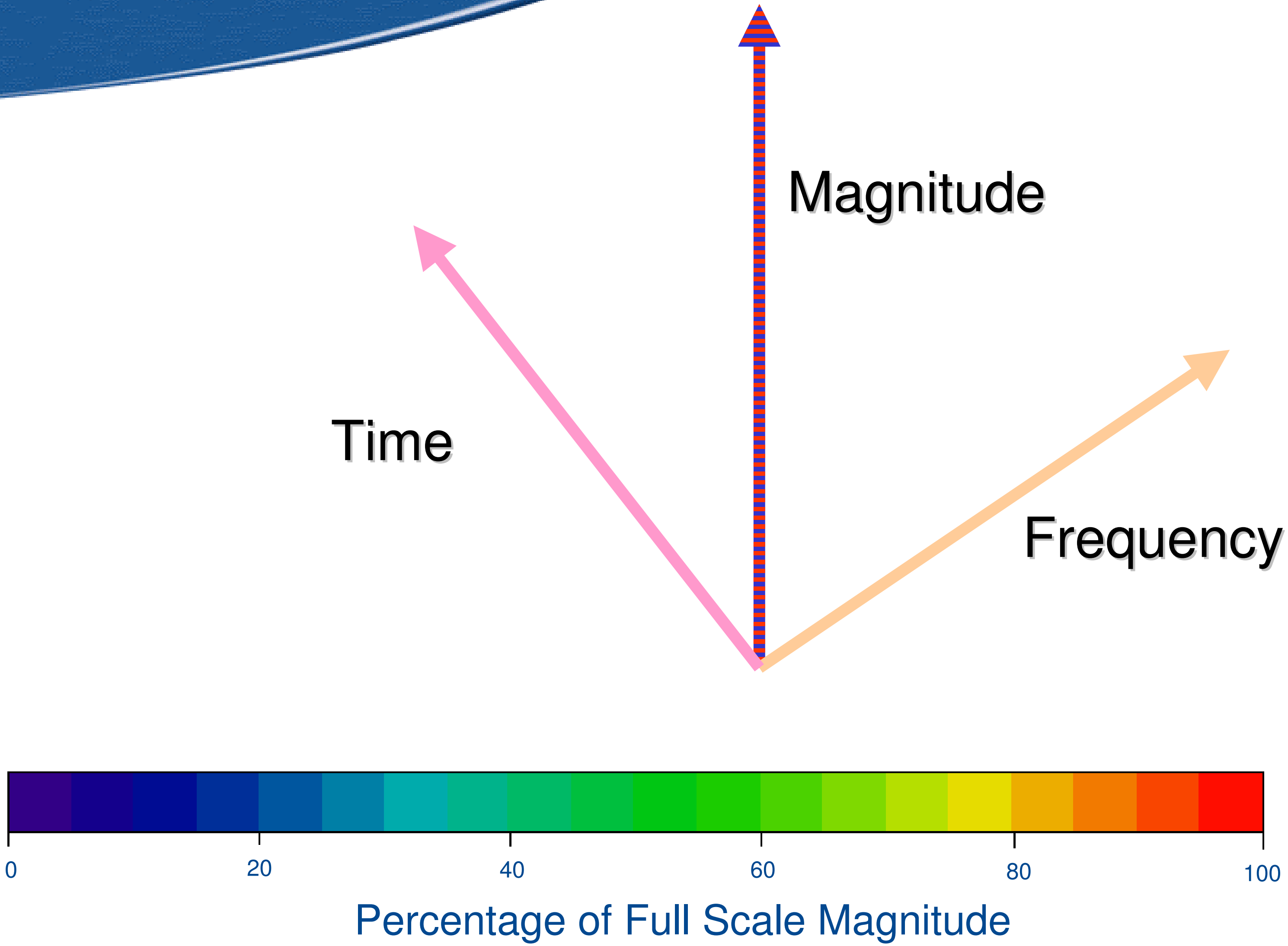
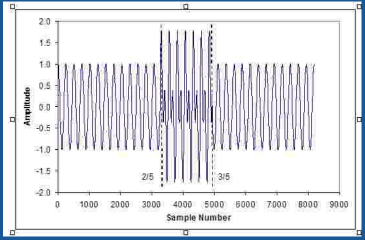
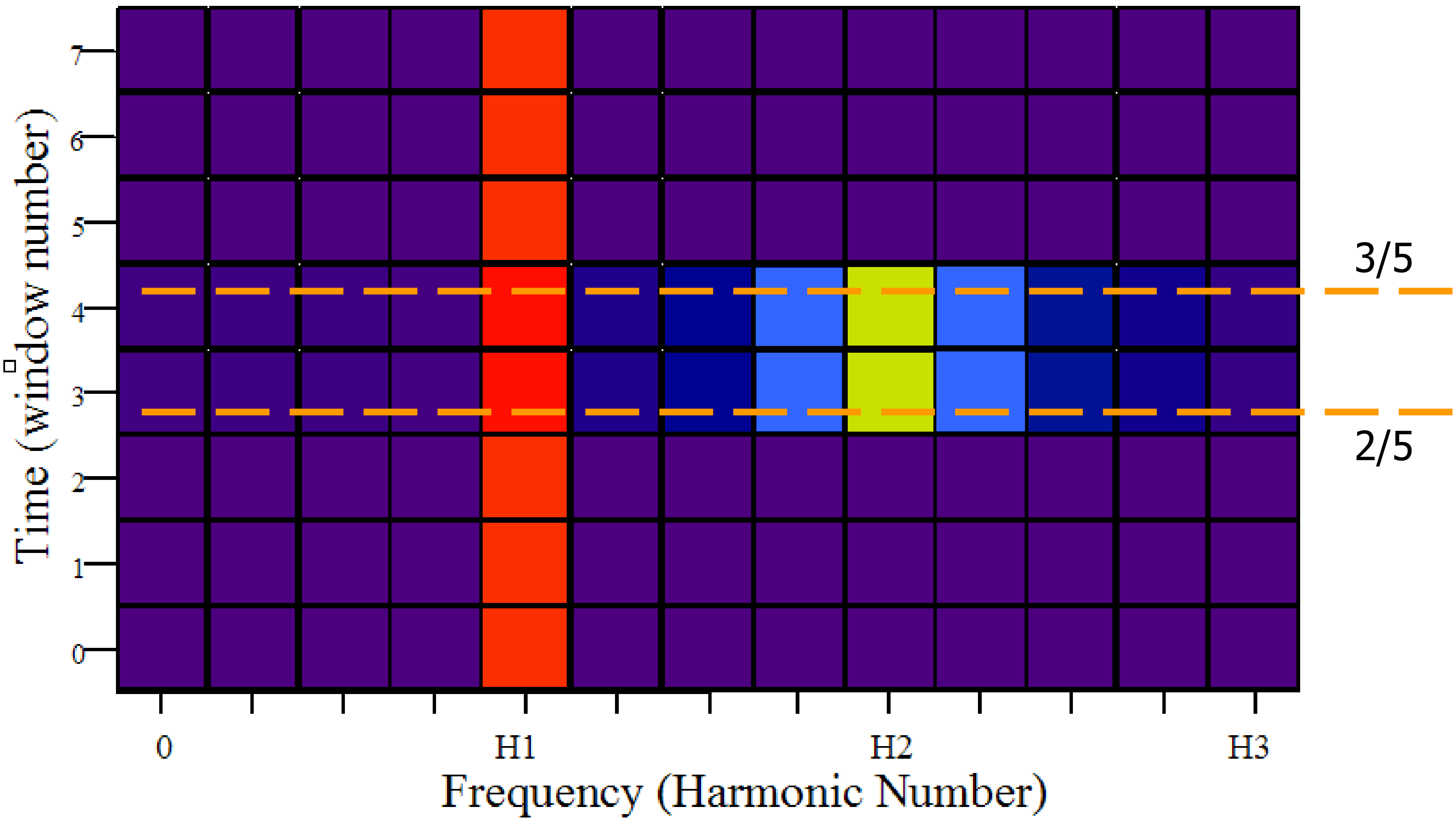
Time Frequency Distributions



STFT Using 8 Windows





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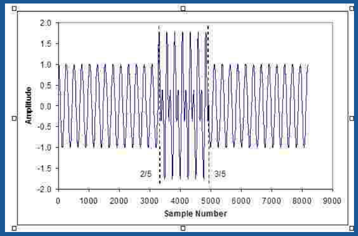
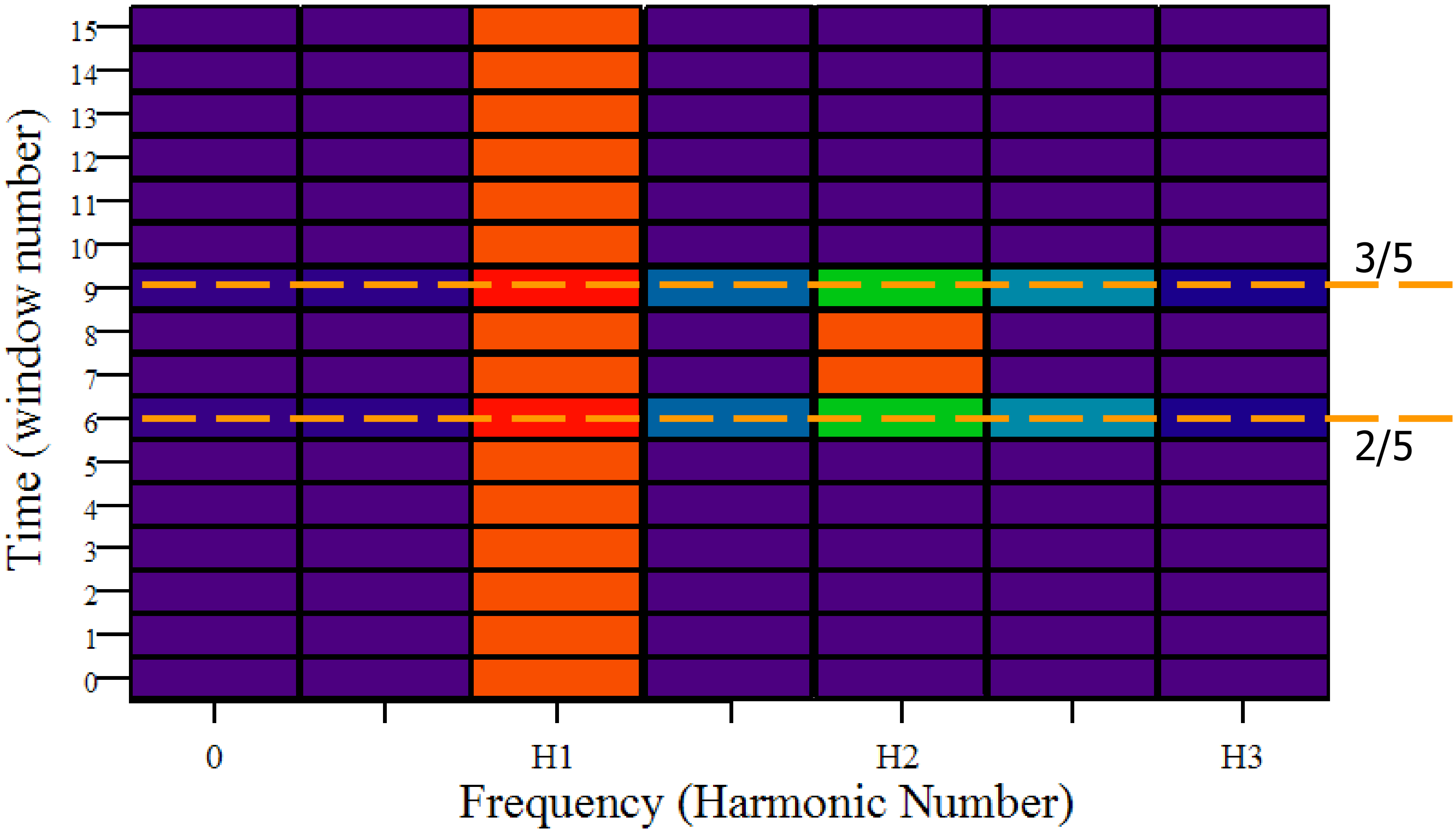
STFT Using 16 Windows





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STFT Results with Various Windows

Number of Windows	Fundamental Cycles/window	Frequency Resolution	2nd Harmonic Error
Zero	32	1/32	-55.5 %
4	8	1/8	-37.1 %
8	4	1/4	-11.1 %
16	2	1/2	-8.0 %
32	1	1	-2.2 %

The Wigner-Ville Distribution

Auto-correlation function of signal *s(t)*:

$$R(t,\tau) = \overline{s(t - \tfrac{1}{2}\tau)} s(t + \tfrac{1}{2}\tau)$$

Wigner Distribution of signal *s(t)*:

$$W(t,\omega) = FT_{\tau}(R(t,\tau))$$

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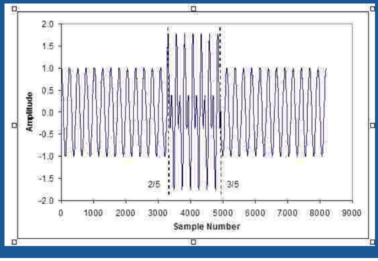
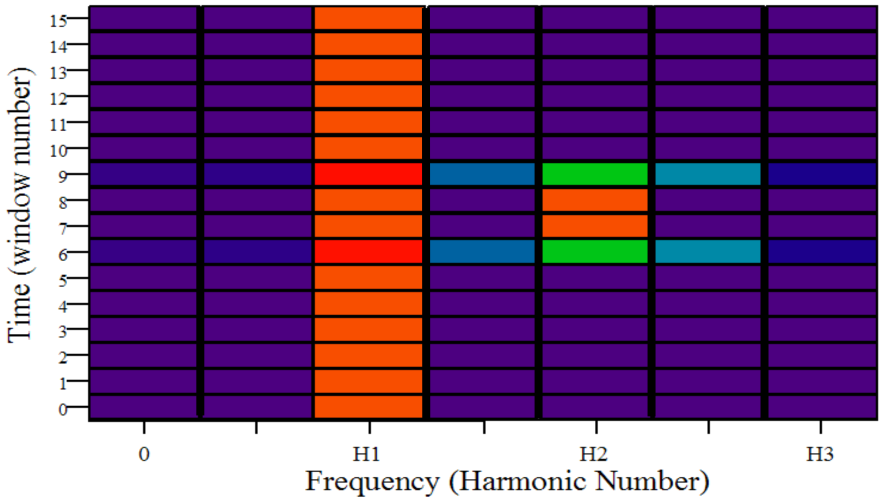
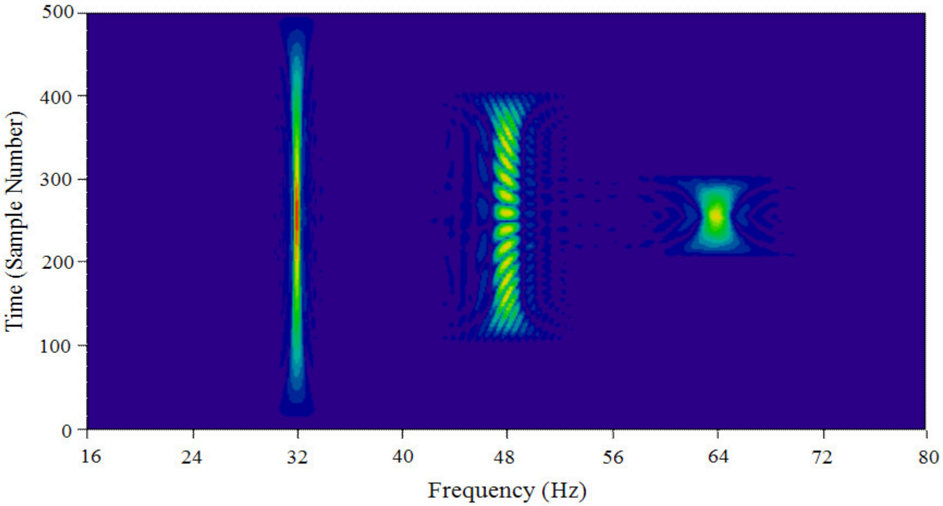
Analysis of a Burst

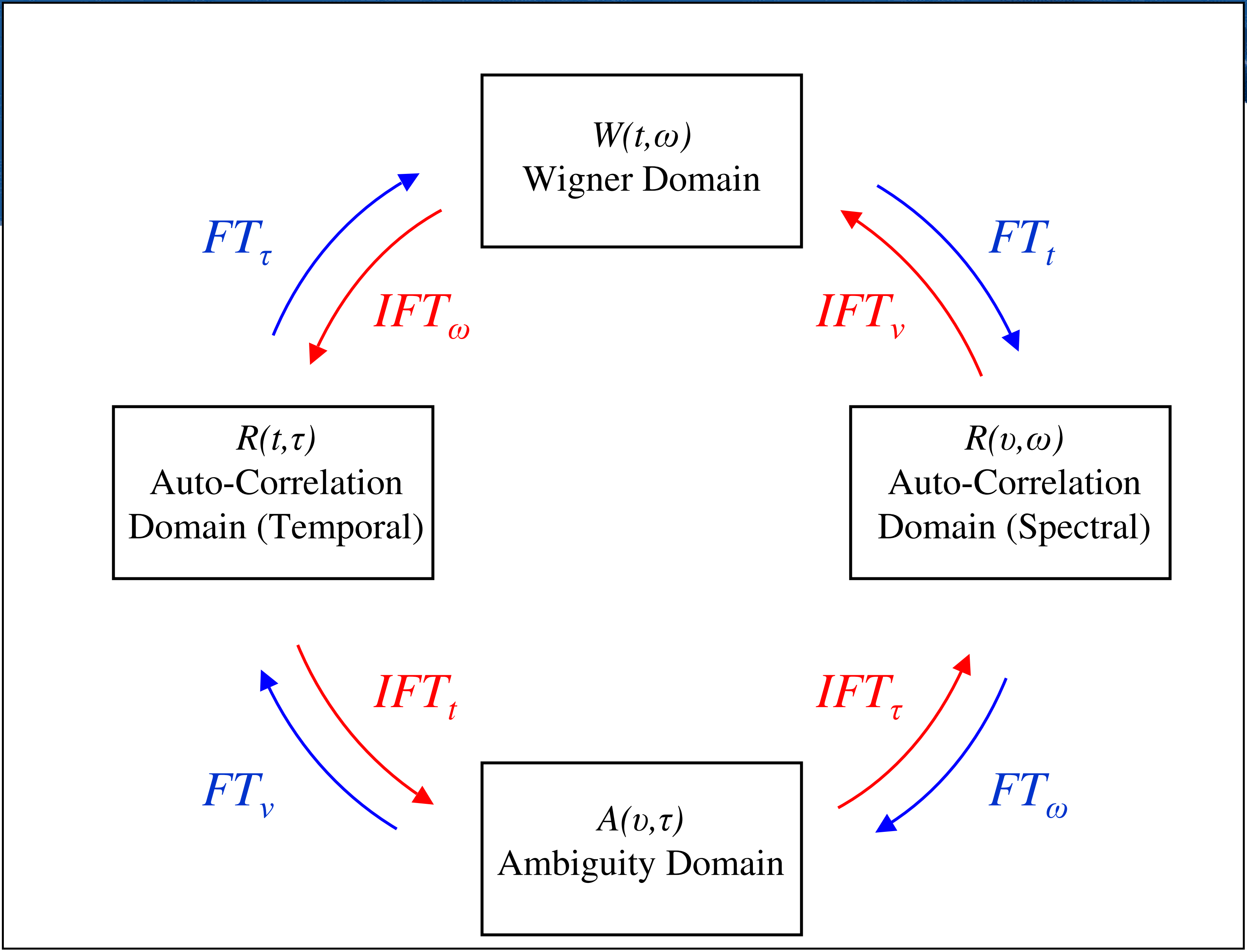




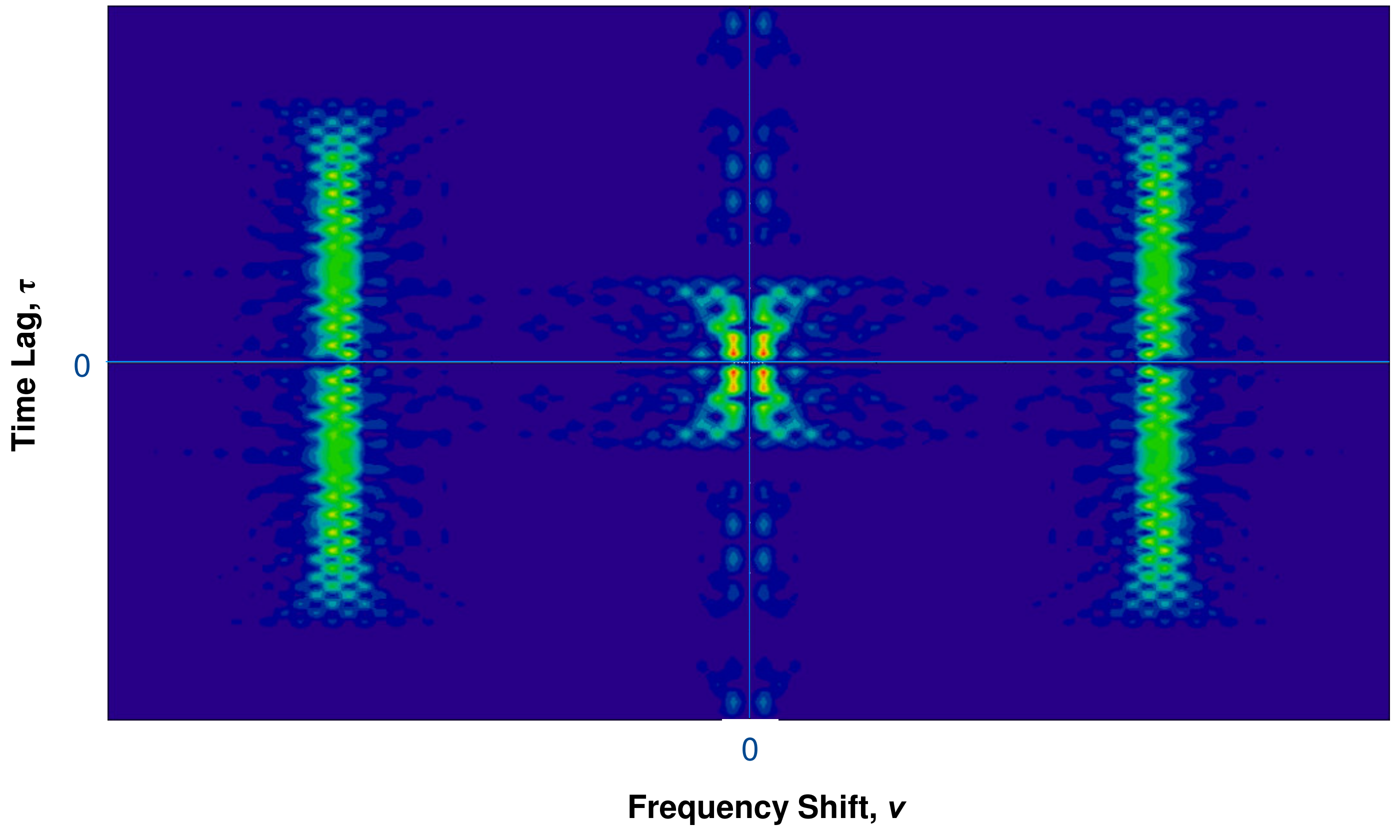
16 Window STFT

Wigner Ville Distribution

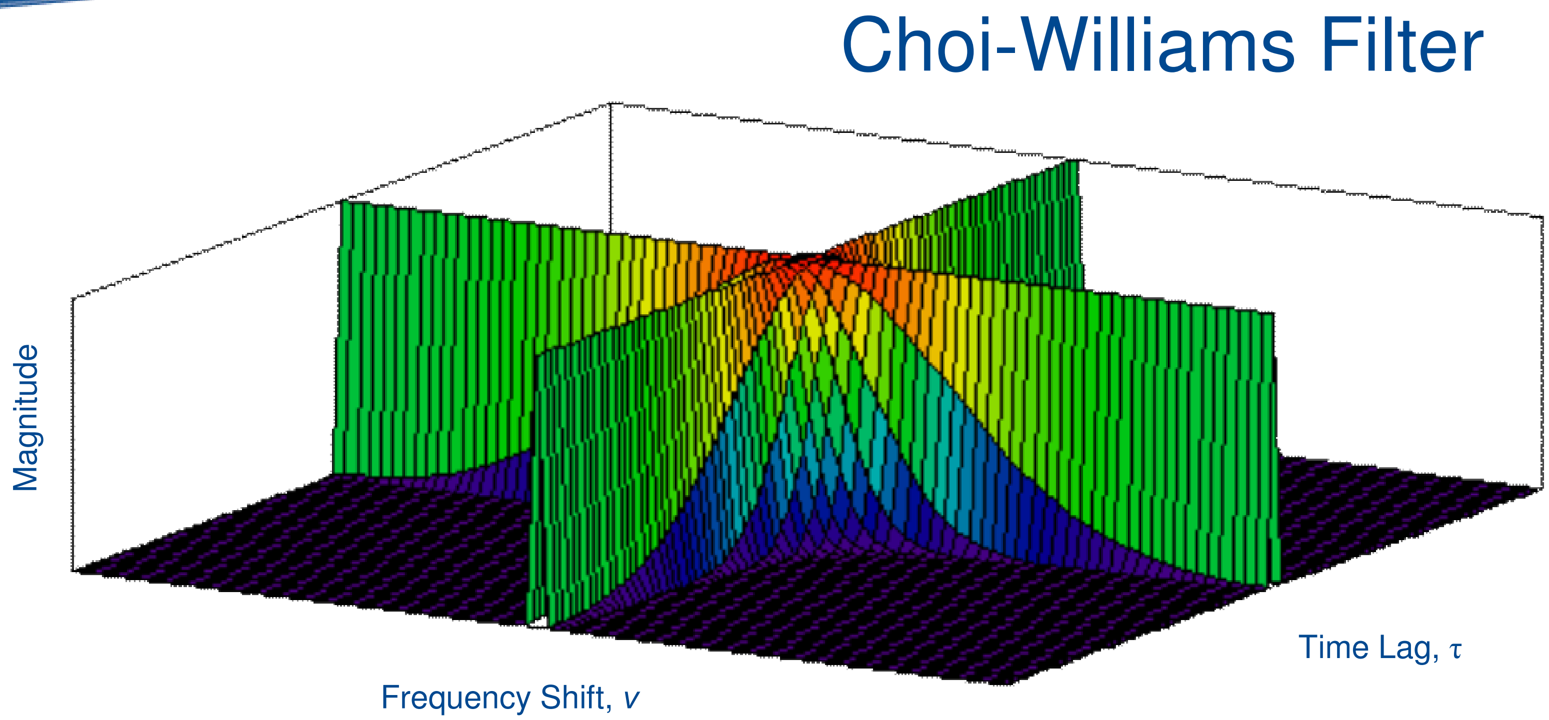



$W(t,\omega)$
Wigner Domain
$R(t,\tau)$
Auto-Correlation
Domain (Temporal)
$R(\upsilon,\omega)$
Auto-Correlation
Domain (Spectral)
$A(\upsilon,\tau)$
Ambiguity Domain
FT_τ
IFT_ω
FT_t
IFT_v
IFT_t
FT_v
IFT_τ
FT_ω

Ambiguity Domain

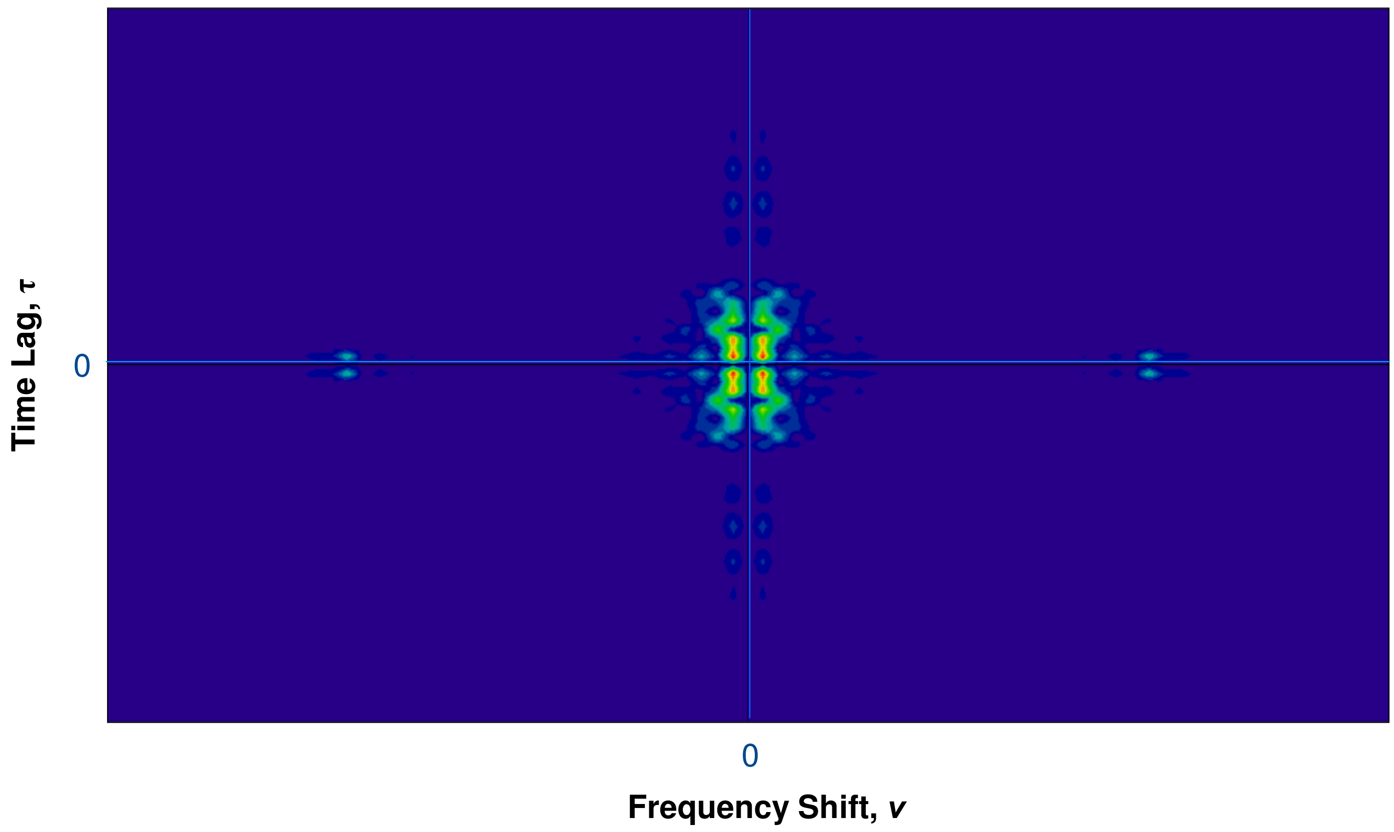


Kernel Filter Response

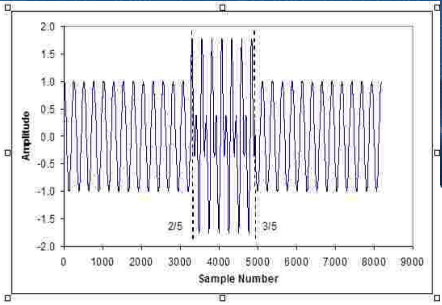


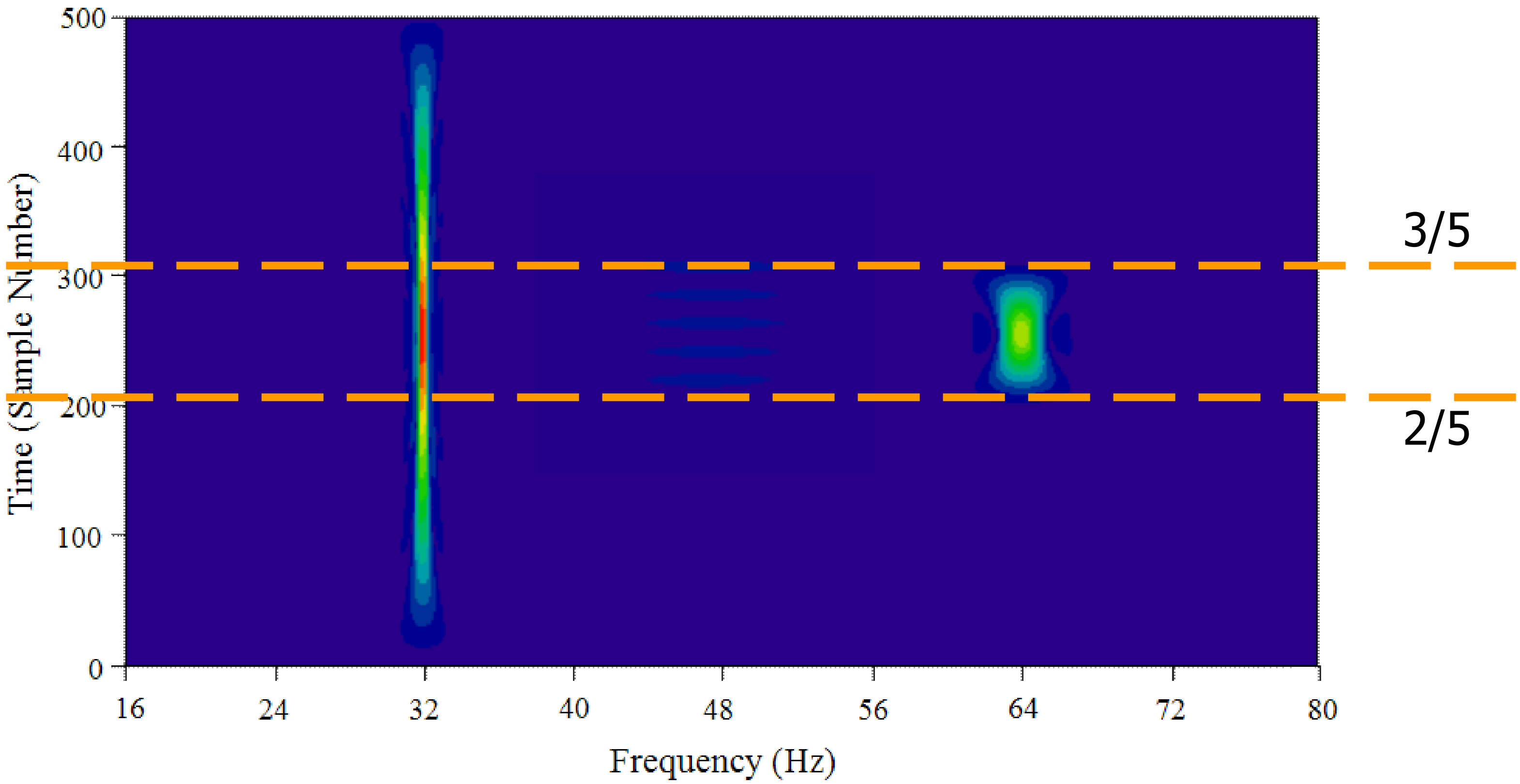
Design Filter Kernels in the Ambiguity Domain

Filtered Ambiguity Function



Choi-Williams Distribution





Wavelets

Fourier Transform

$$F(\omega) = correlation[f(t), \psi(t,\omega)]$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t).\psi(t,\omega)dt$$

Wavelet Transform

$$W_{ab}(t) = correlation[f(t), \psi_{ab}(t)]$$

$$W_{ab}(t) = \int_{-\infty}^{\infty} f(t).\psi_{ab}(t)dt$$

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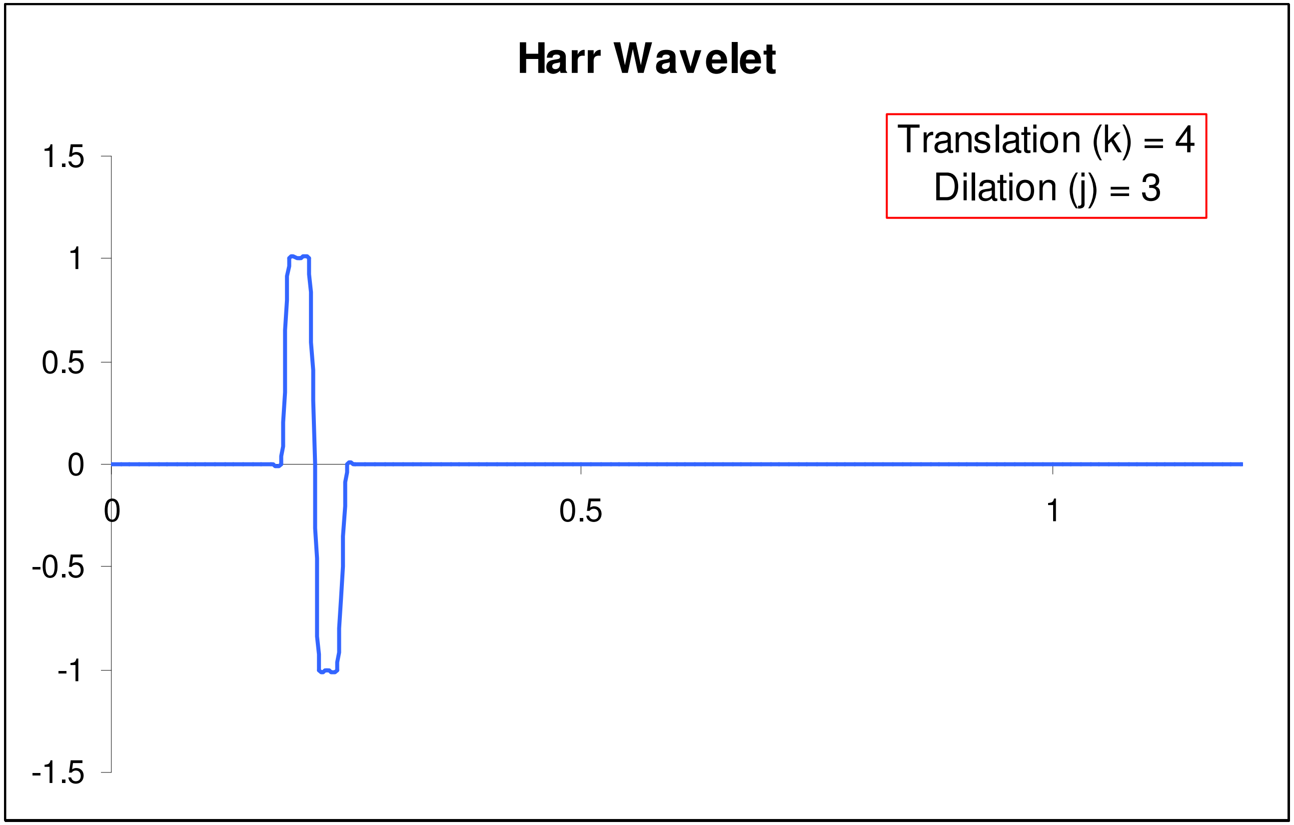
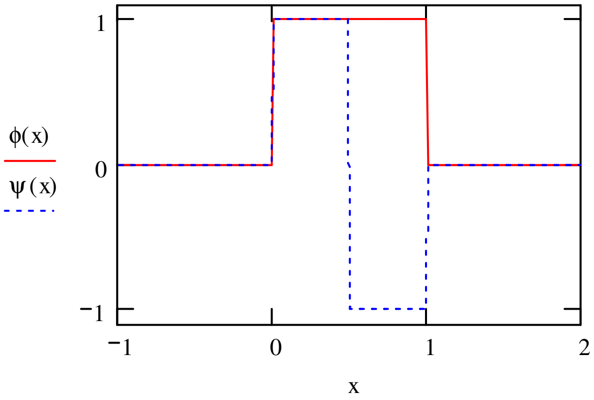
Translation and Dilation

$$\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k)$$

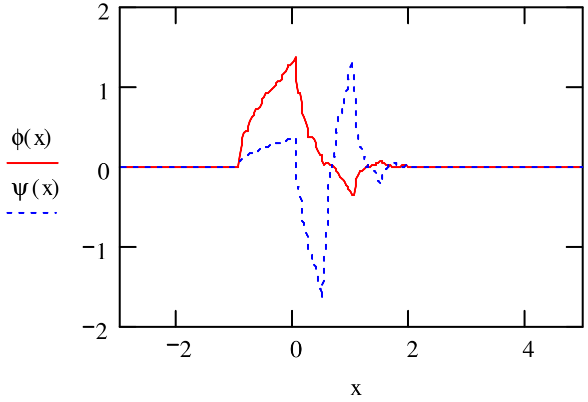
Where the integer *j* is the dilation parameter and integer *k* is used for translation



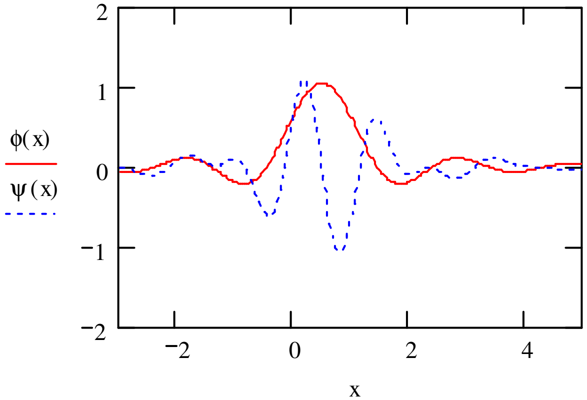
Wavelet Families



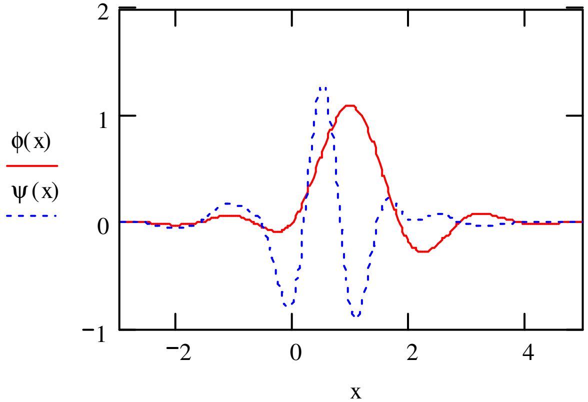
Haar



Daubechies 4

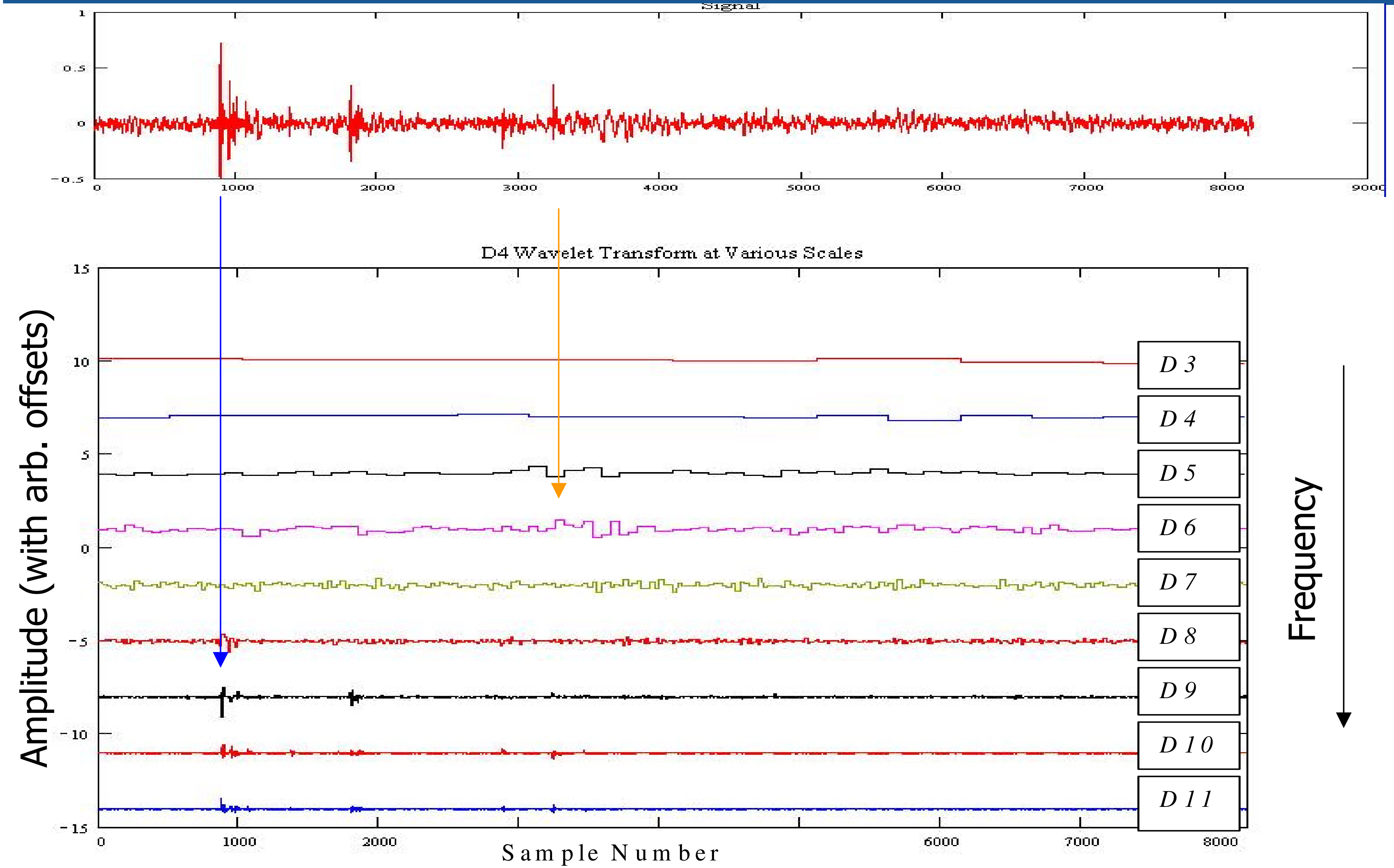


Battle-Lemarie 6



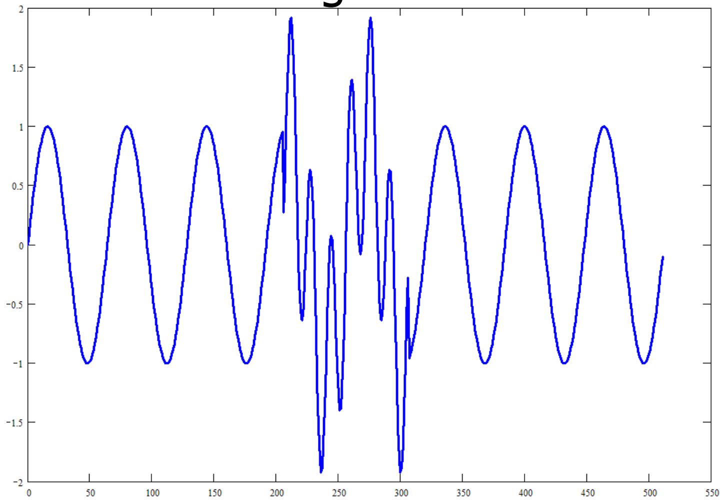
Symmlet 20

Vibration Analysis Using Wavelets



Wavelet Data/Image Compression

Signal



Reject all wavelet coefficients below a given Threshold, then invert what's left….

75% data compression in this example.

Compressed Data Signal

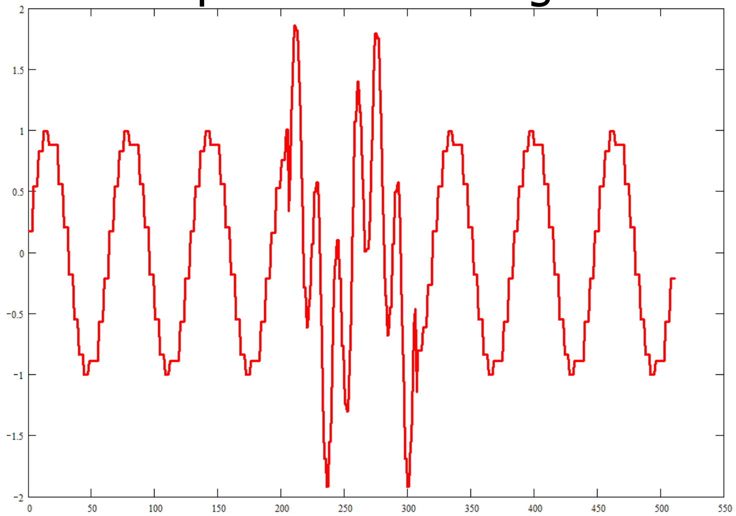


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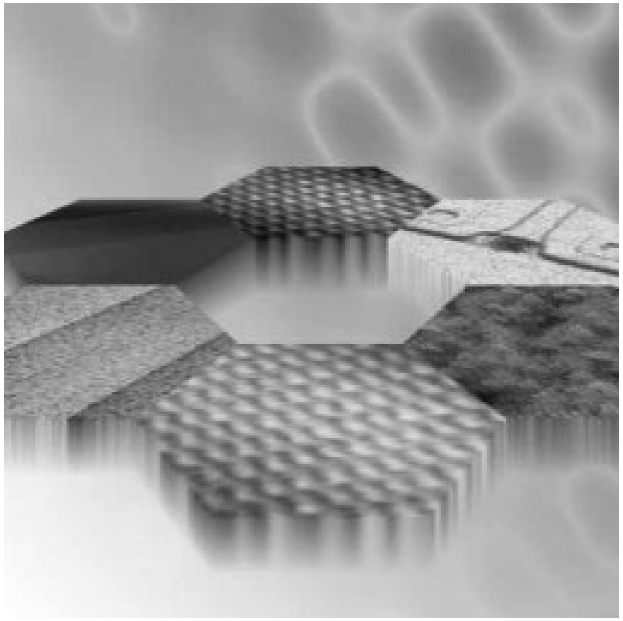
Image Compression

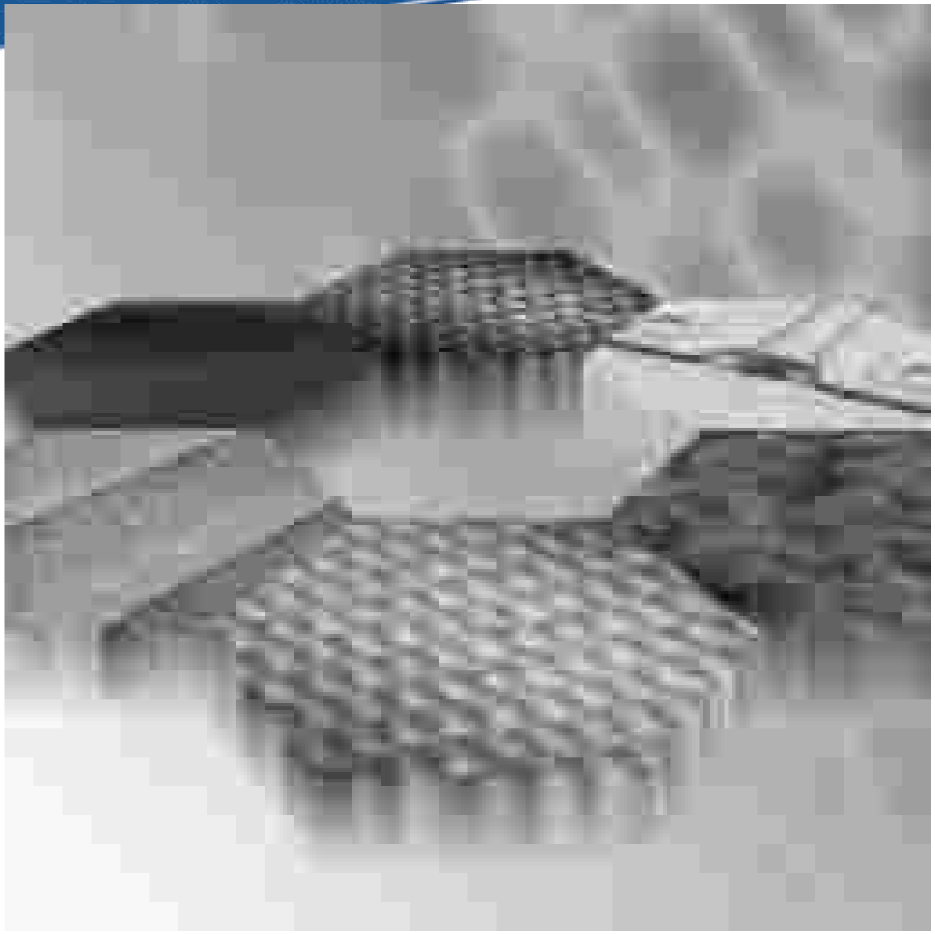
Original Image



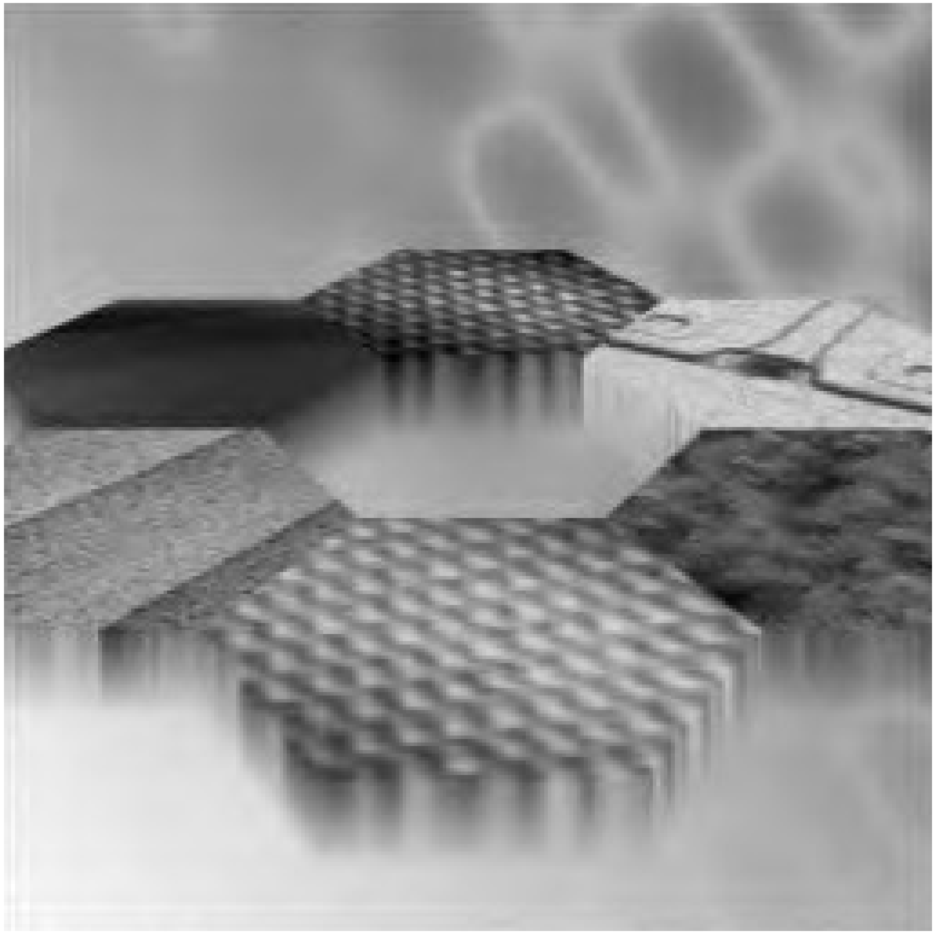
Reconstructed Image – 100:1 compression

Image Compression

JPEG Compression



Wavelet Compression



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Wavelet Data/Image Fusion

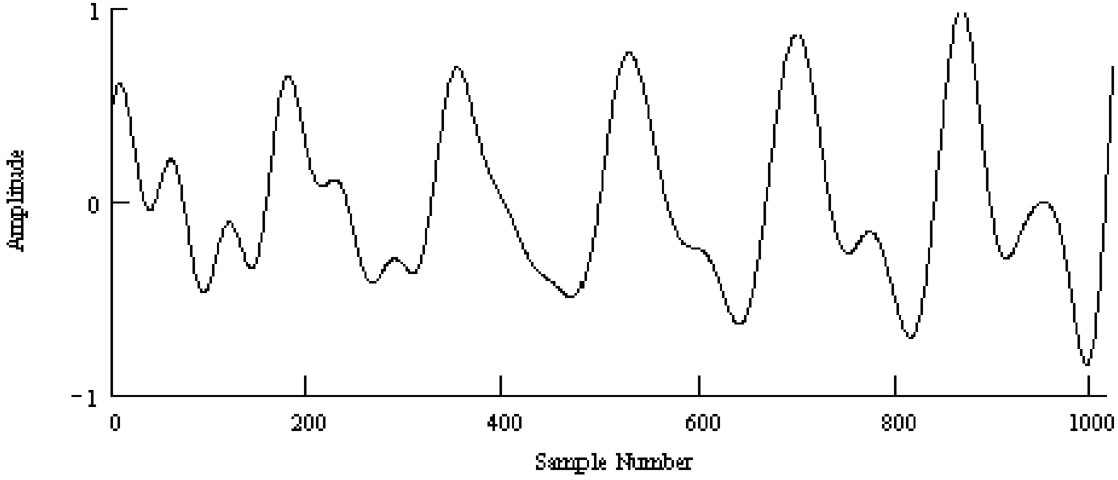
Fine Spatial Texture Image

Fused Image

Coarse Spatial Spectral Image

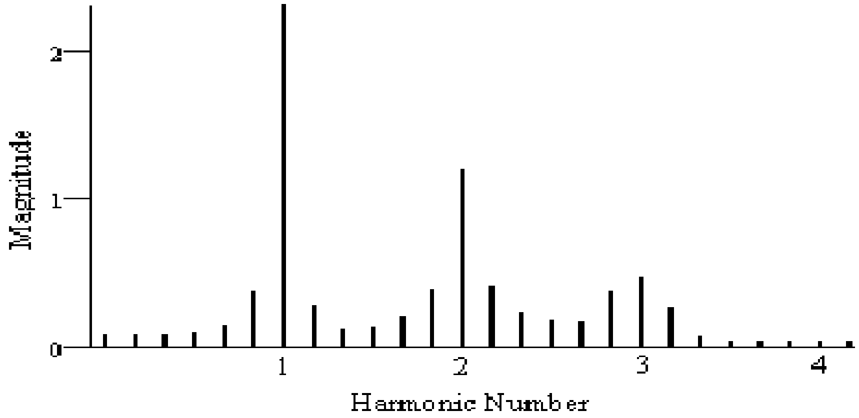
(NB this is illustrative!)

Smooth Modulation



Made of H1, H2 and H3

But…



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A Harmonic Modulation Model

A signal *x(t)* with N non-modulated harmonics:

$$x(t) = p_0 + \sum_{n=1}^{N} p_n \cos(n\omega t) + q_0 + \sum_{n=1}^{N} q_n \sin(n\omega t)$$

For modulated harmonics, use modulation functions *m(t)* for the *p*'s & *q*'s:

$$p_n(t) = a_{n,0} m^{<0>} + a_{n,1} m^{<1>} + \dots a_{n,K} m^{<K>}$$

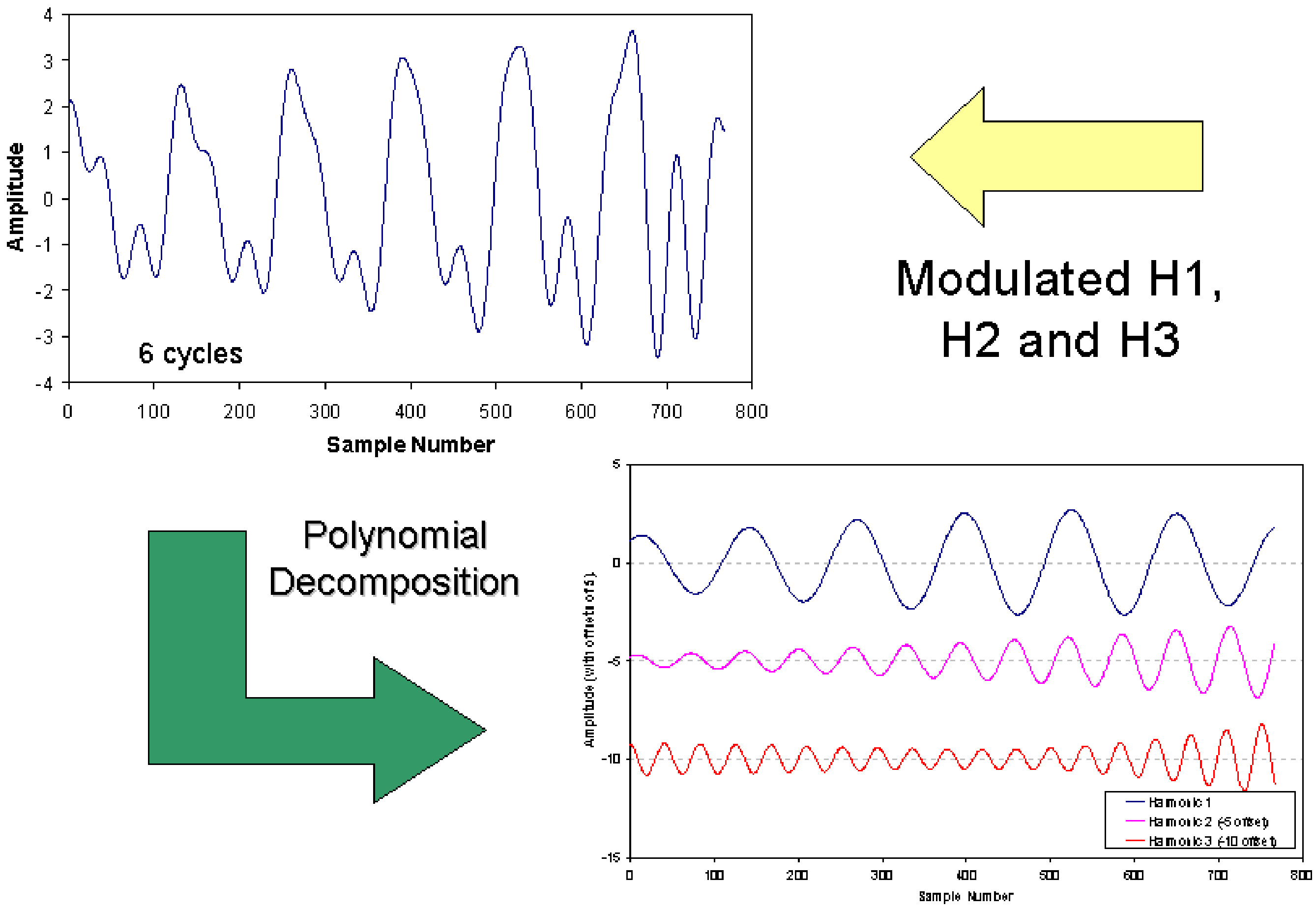
$$q_n(t) = b_{n,0} m^{<0>} + b_{n,1} m^{<1>} + \dots b_{n,K} m^{<K>}$$

(if $m^{<k>} = t^k$ then the scheme is a polynomial modulation model)

De-modulate using Method of Least Squares

Polynomial modulation functions

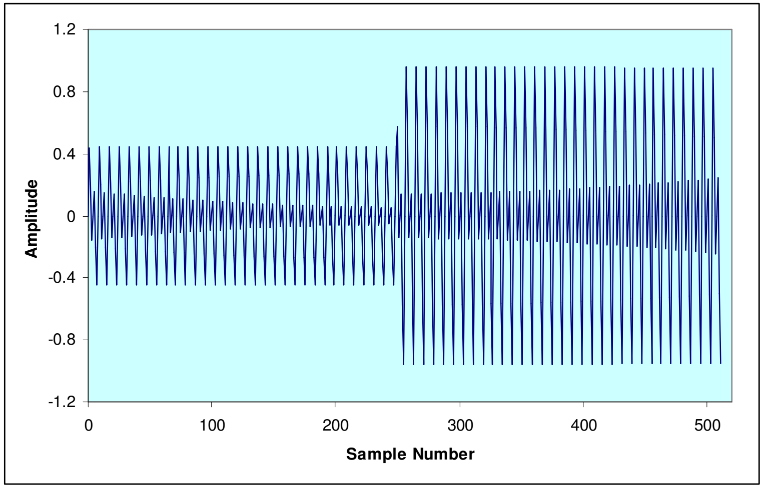
When *m(t) = t*, the modulators are polynomials,

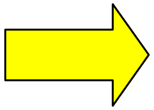


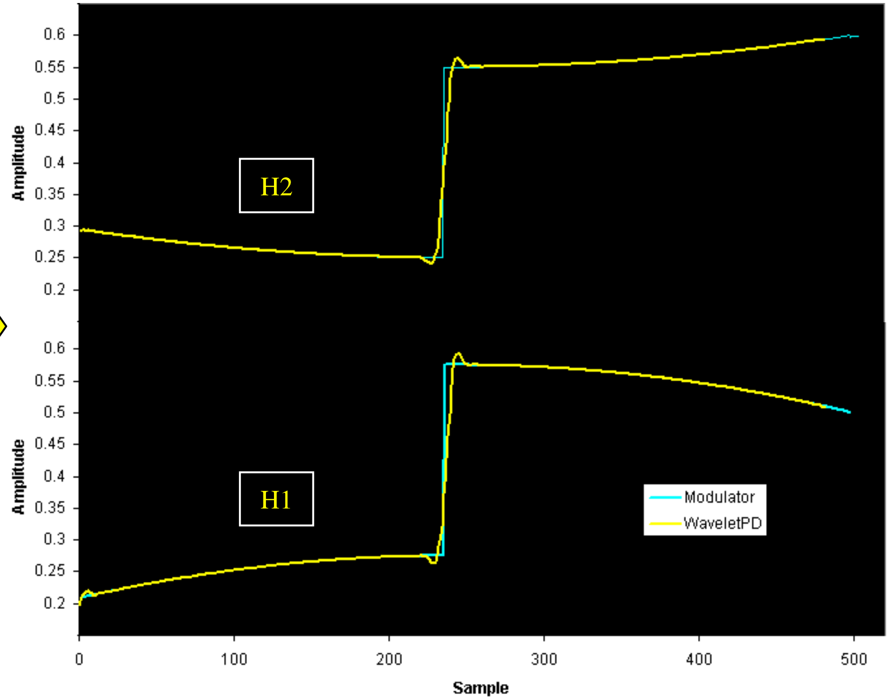
Highly accurate demodulation for smooth modulation functions.

Using Wavelet Basis Functions

For signals with discontinuities, the *m(t)* functions can be wavelets…







Test Signal: Modulated H1 and H2

L.S. D4 Wavelet demodulation

High Accuracy Demodulation – Handles the discontinuity!

Summary

- Short Time Fourier Transforms
- Time Resolution – Frequency Resolution Trade Off
- Wigner Distributions
- Ambiguity Domain Filtering
- Wavelets
- Time Series Modulation Methods

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