NPL
National Physical Laboratory

Digital Sampling and Application of the DFT: Good Practice and Pitfalls

Paul Wright

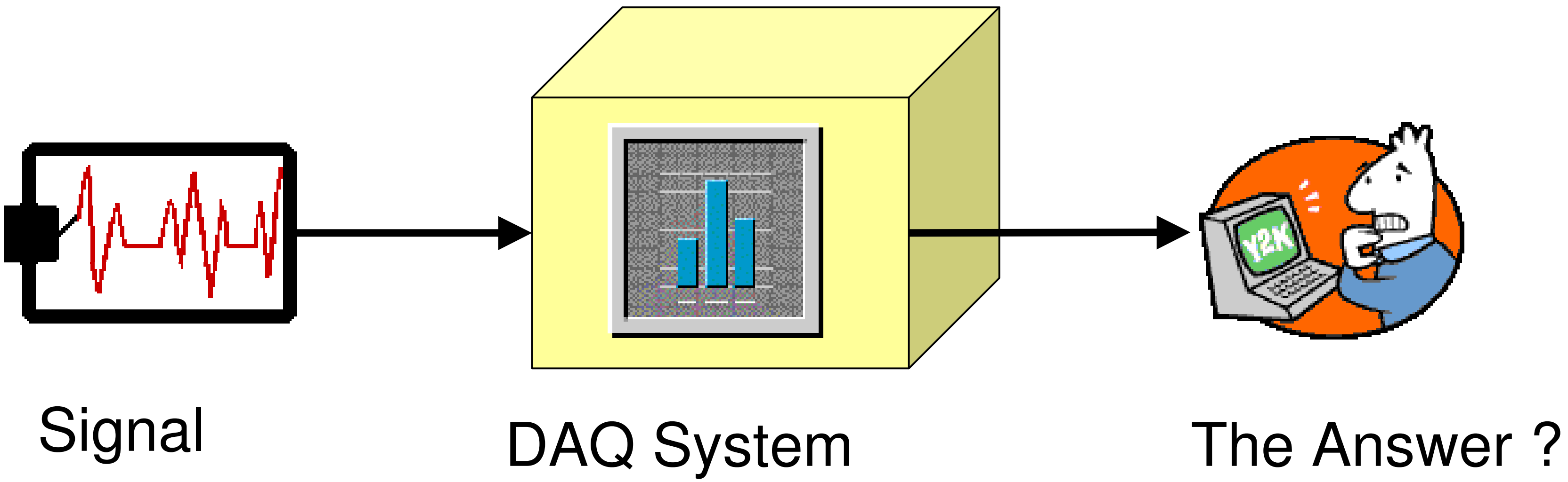
Applied Electrical and Magnetics Group, NPL.

Outline

- Analysis of Waveforms and Transforms
- How many Samples to Take – Aliasing
- Negative Spectrum
- Frequency Resolution
- Synchronizing Sampling
- Non-repetitive Waveforms
- Picket Fencing
- A Sampled Data System

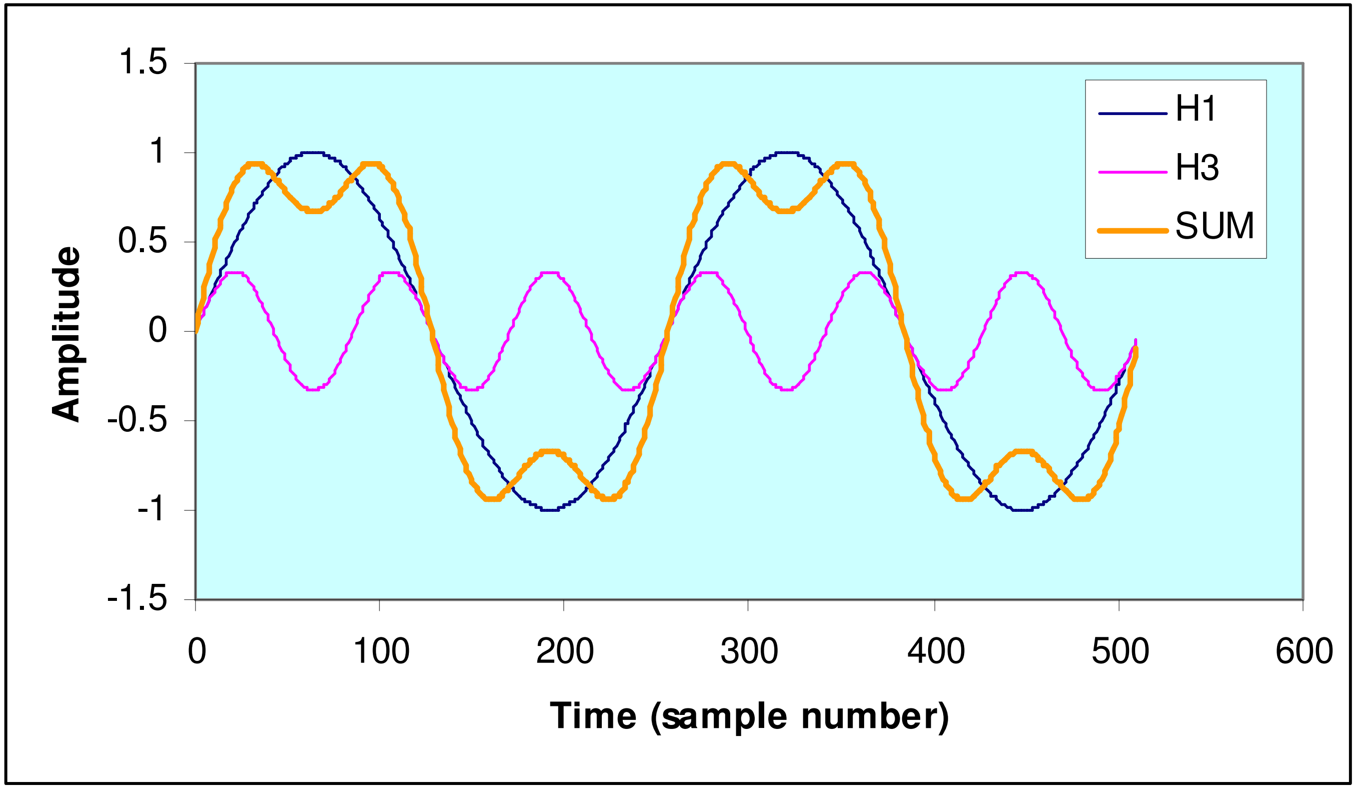
NPL

Data Acquisition Systems



Fourier Series for Repetitive Waves

$$x(t) = p_0 + \sum_{n=1}^{N} p_n \cos(n\omega t) + q_0 + \sum_{n=1}^{N} q_n \sin(n\omega t)$$



P's & Q's determine the resulting wave

NPL

Finding the Frequency Components

How much sinewave at frequency h.f is in signal s(t)

$$S(h) = Corrolation[s(t), \sin(2\pi.f.h)]$$

The correlation is carried out as follows:

$$S(h) = \int_{-\infty}^{\infty} s(t).\sin(2\pi.f.h)\, dt$$

Similarly (to account for phase angles)

$$C(h) = \int_{-\infty}^{\infty} s(t).\cos(2\pi.f.h)\, dt$$

Using complex number operations

$$C(h) + j\,S(h) = \int_{-\infty}^{\infty} s(t).e^{-j.2\pi.f.h.t}\, dt$$

NPL

Fourier Transforms

The component in waveform *s(t)* at frequency *f* is found by a Fourier Transform:

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\,2\pi\, f\, t}\, dt$$

For sampled data, the discrete Fourier transform (DFT) of repetitive waveform:

$$S(f) = \frac{1}{N} \sum_{n=0}^{N-1} s(n)\, e^{-j\,2\pi\, f\, \frac{n}{N}}$$

The Fast Fourier Transform (FFT) is a fast calculation method for the DFT

NPL

Aliasing Issues

How Many Samples Are Needed ?

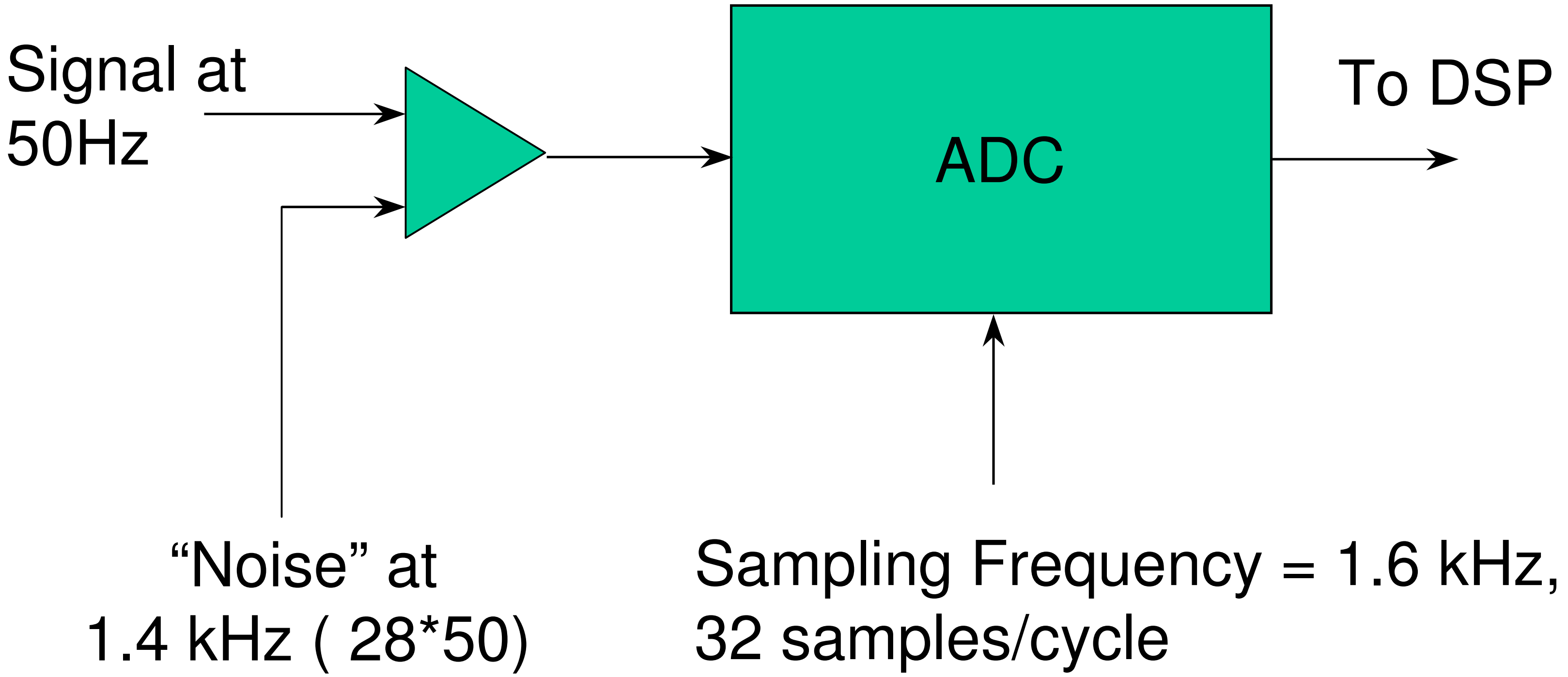
Sampling Theorem:

The Sampling Frequency must be At Least Twice the Bandwidth of the System !

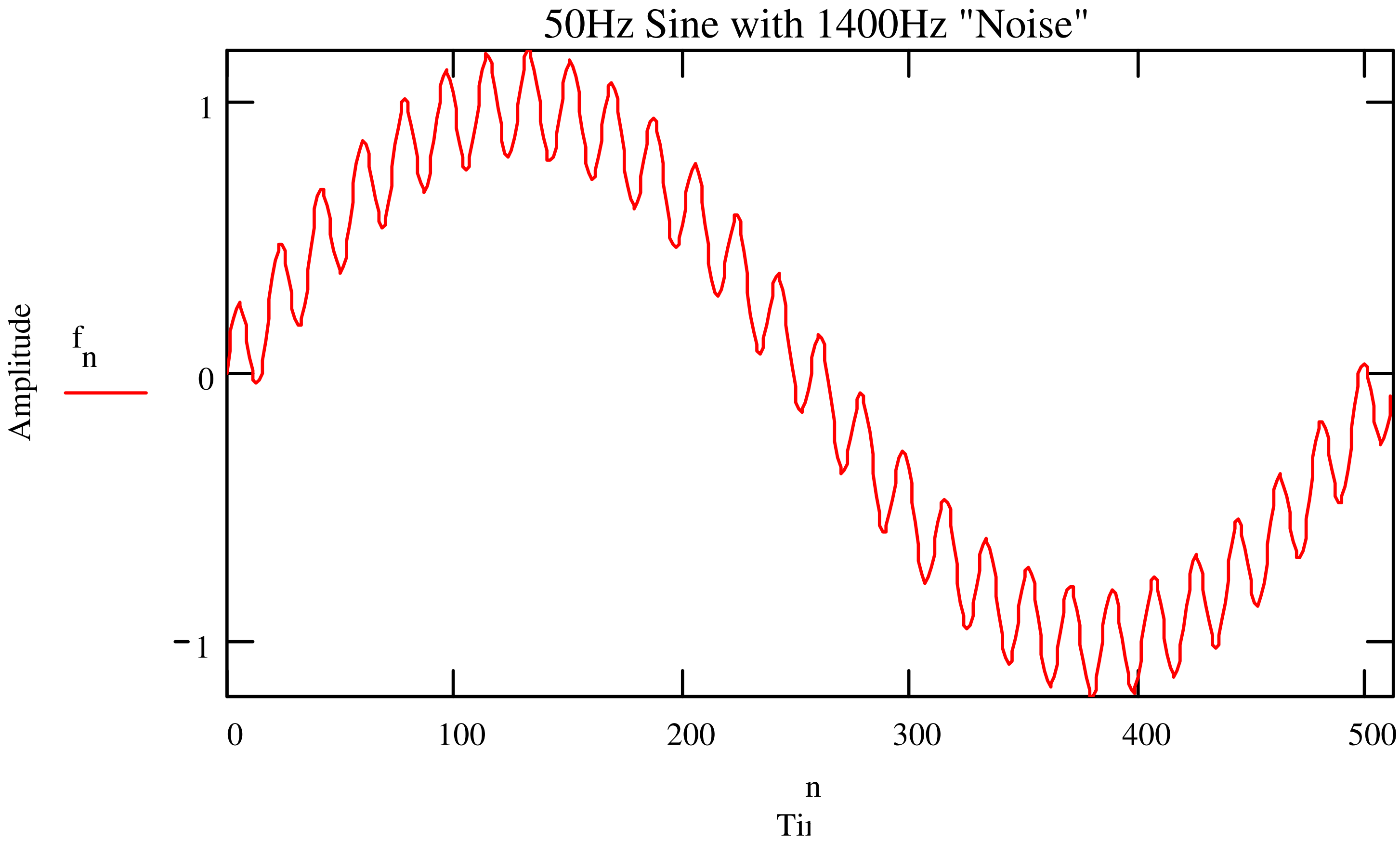
So for a 50 Hz signal will 101 Hz be OK?

NPL

Sampling a 50 Hz Signal

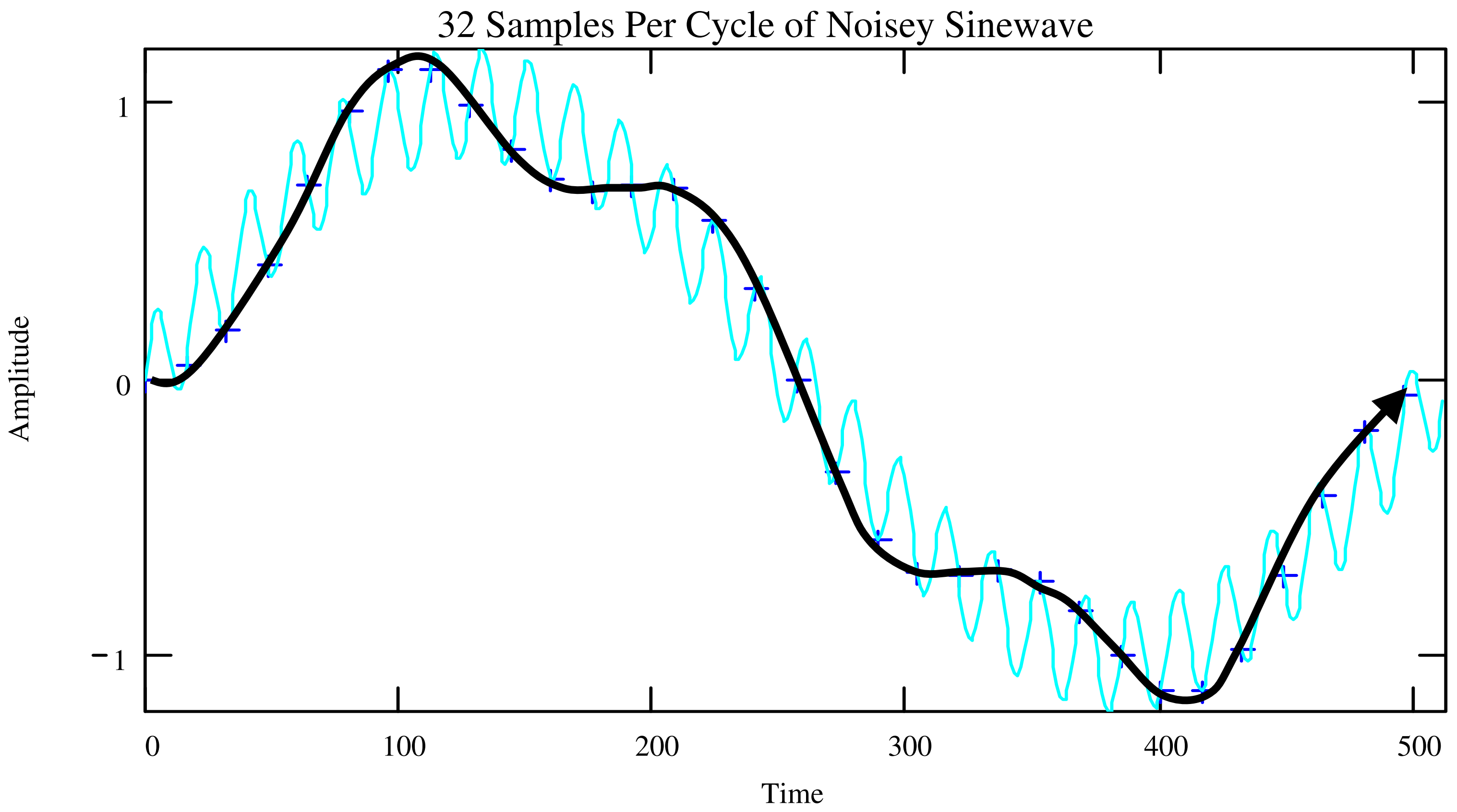


50 Hz Signal with Noise

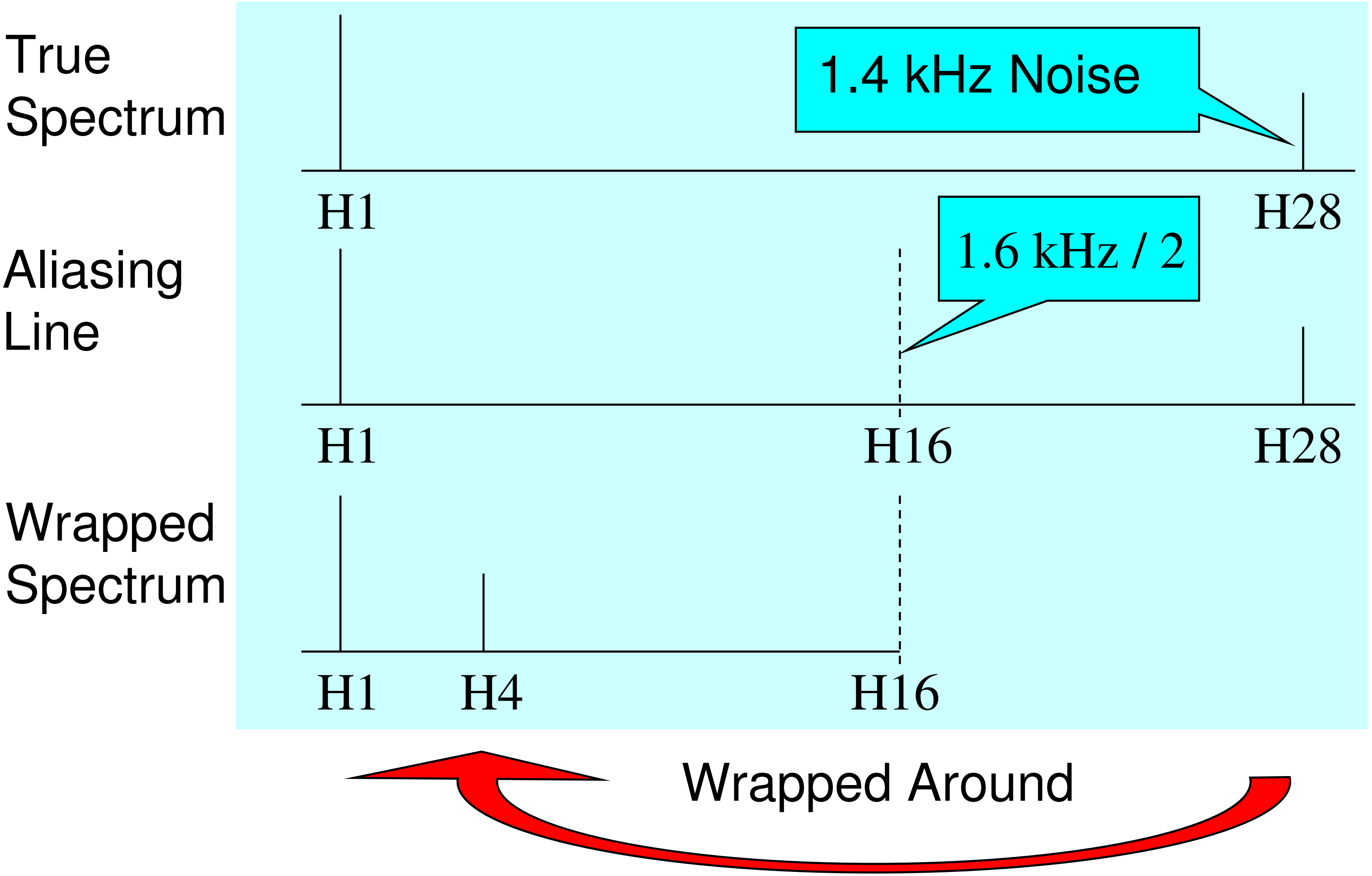


Now Sample at 1.6 kHz, (32 times 50 Hz)….

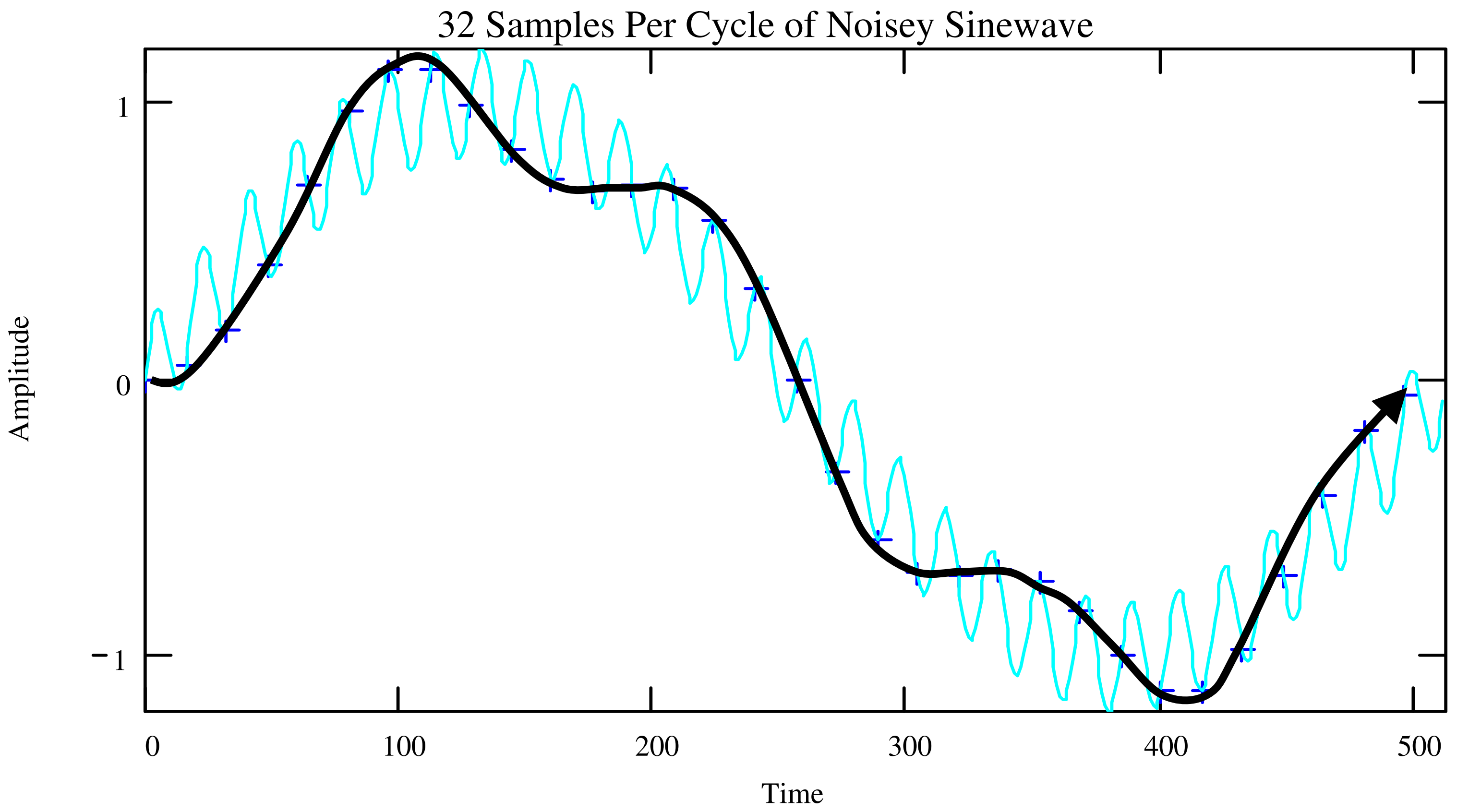
Sampling Too Slowly



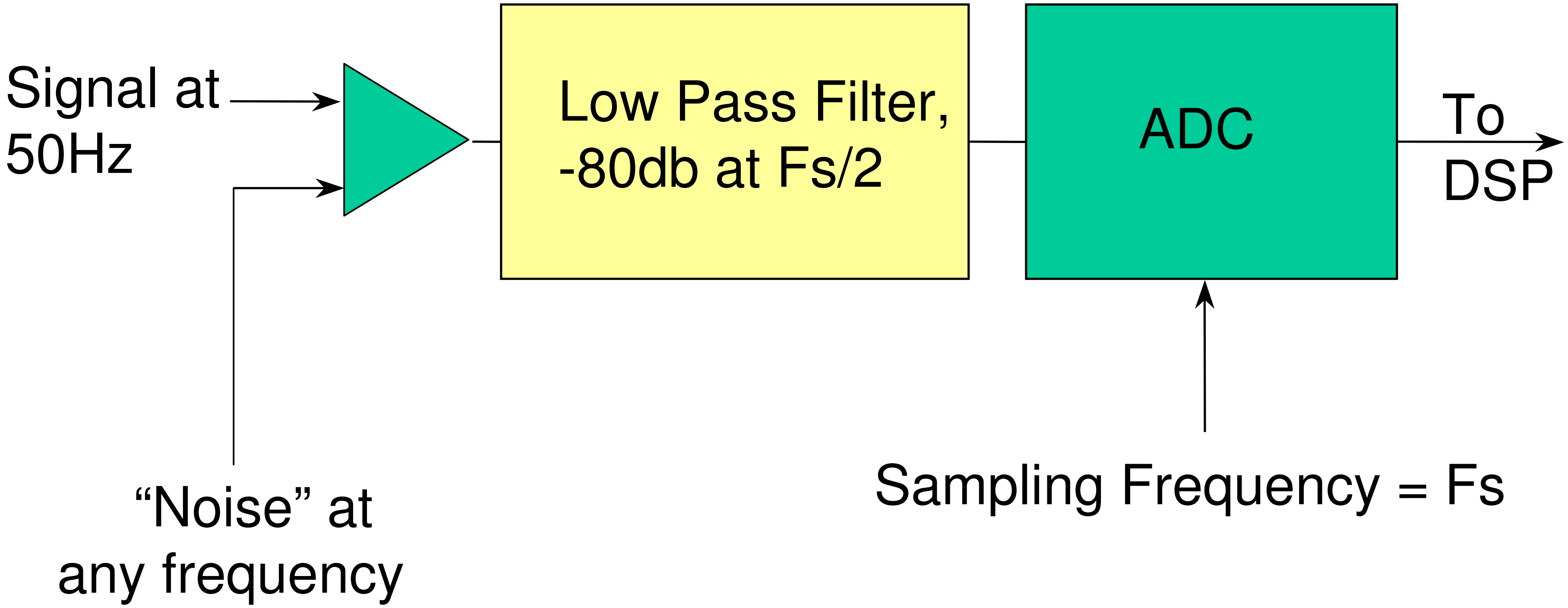
Aliasing of 1.4 kHz Noise in 1.6kHz Sampling System



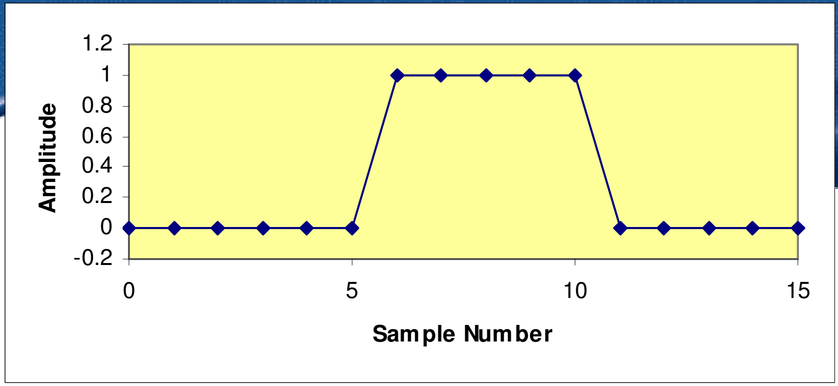
Sampling Too Slowly



Use of an Anti-Aliasing Filter

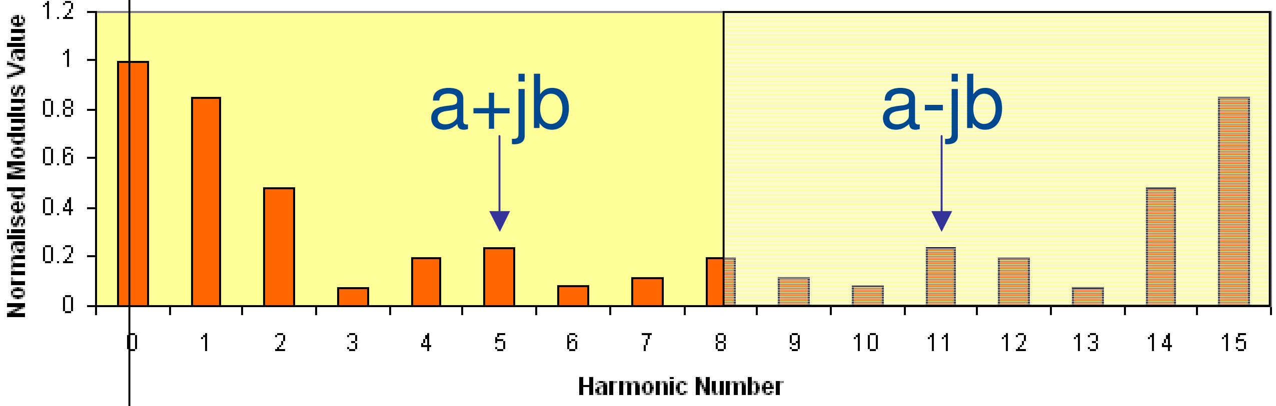


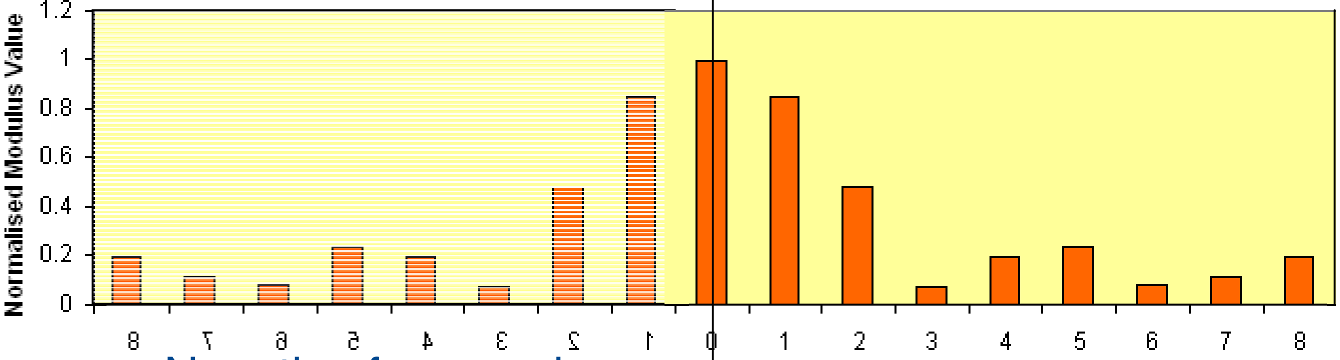
Negative Spectrum



16 Point

FFT





Negative frequencies

Use Hilbert Transform to Cancel –'ve Spectrum

Inverse FFT and Negative Frequencies

Negative Frequencies

Inverse FFT $$x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(jk\omega t)$$

For *k* negative, we get *Negative* Frequencies

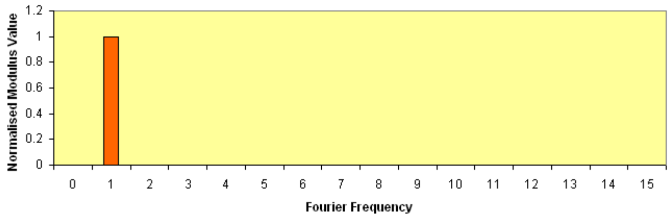
As the negatives are conjugates of the positives,
they cancel the imaginary parts and give a real series *x(t)*

Inverse FFT

0
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0

IFFT

6.25E-002
5.77424707819554E-002+2.39177145228181E-002i
4.41941738241592E-002+4.41941738241592E-002i
2.39177145228181E-002+5.77424707819554E-002i
6.25E-002i
-2.39177145228182E-002+5.77424707819554E-002i
-4.41941738241593E-002+4.41941738241592E-002i
-5.77424707819555E-002+2.3917714522818E-002i
-6.25E-002
-5.77424707819554E-002-2.39177145228181E-002i
-4.41941738241592E-002-4.41941738241592E-002i
-2.39177145228181E-002-5.77424707819554E-002i
-6.25E-002i
2.39177145228182E-002-5.77424707819554E-002i
4.41941738241593E-002-4.41941738241592E-002i
5.77424707819555E-002-2.3917714522818E-002i



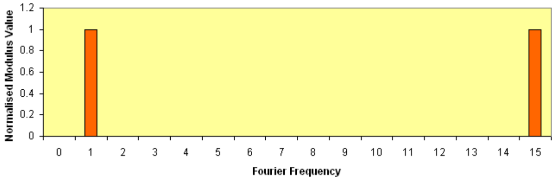
Time Series is

Complex !

NPL

Inverse FFT

0
1
0
0
0
0
0
0
0
0
0
0
0
0
0
1



IFFT

Complete the –ve

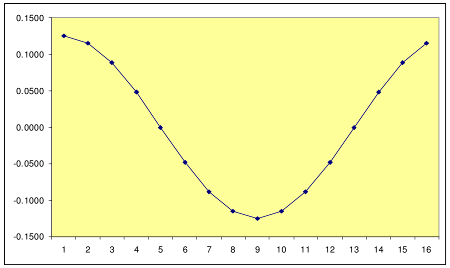
half of the spectrum

(Use complex conjugate)

Time Series is

Real

0.1250
0.1155
0.0884
0.0478
0.0000
-0.0478
-0.0884
-0.1155
-0.1250
-0.1155
-0.0884
-0.0478
0.0000
0.0478
0.0884
0.1155



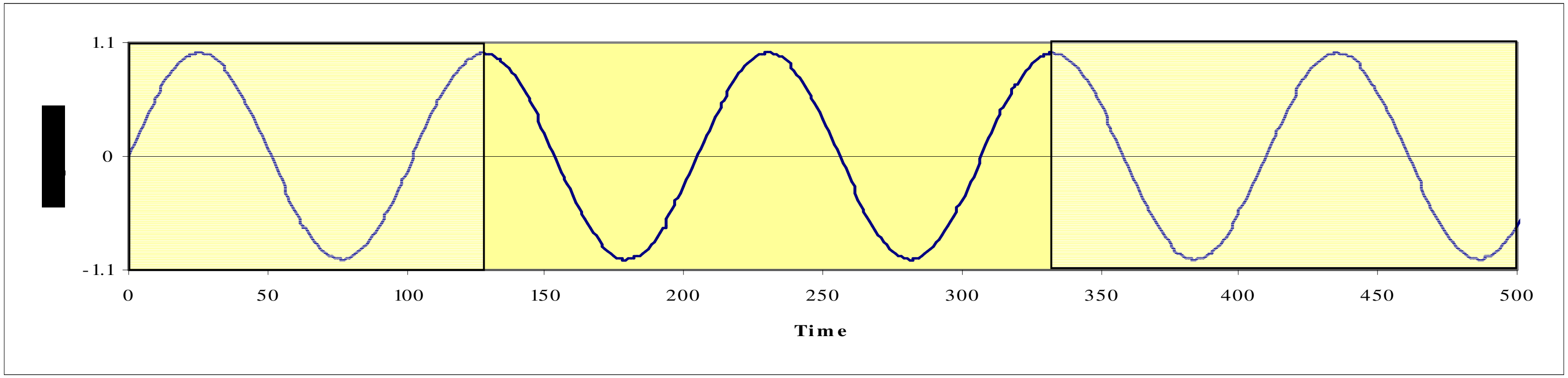
NPL

National Physical Laboratory

Windowing and Related Issues

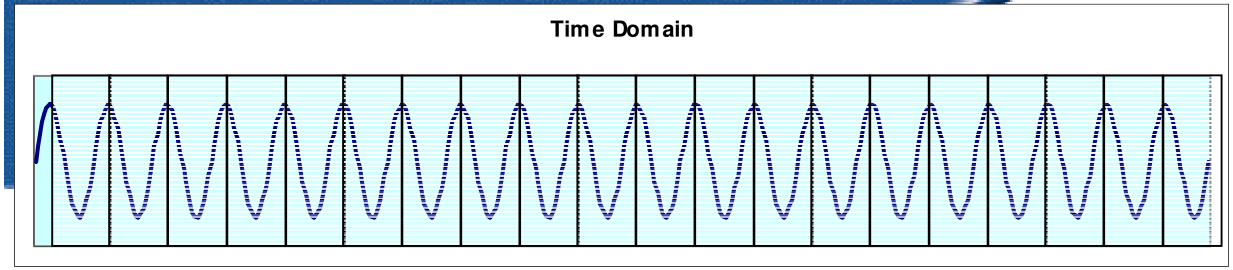
Repetitive Signals and Windowing

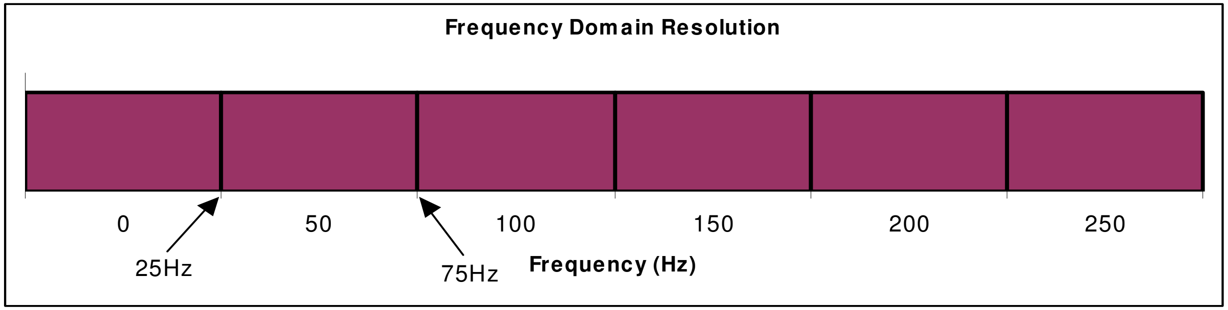
- The Fourier Transform is Defined over Infinite Time.
- In practice only a finite duration signal is available. (we can't wait forever!)
- Repetitive Signals -Take a 'snap shot' of the signal … Use an Integer Number of Cycles



NPL

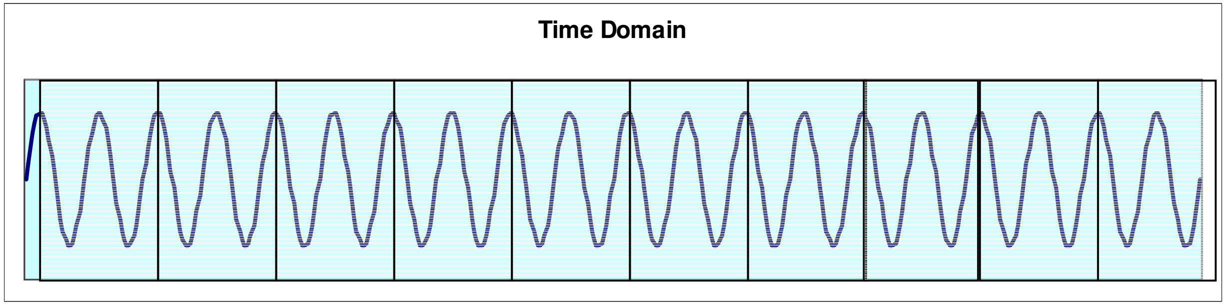
FFT Frequency Resolution





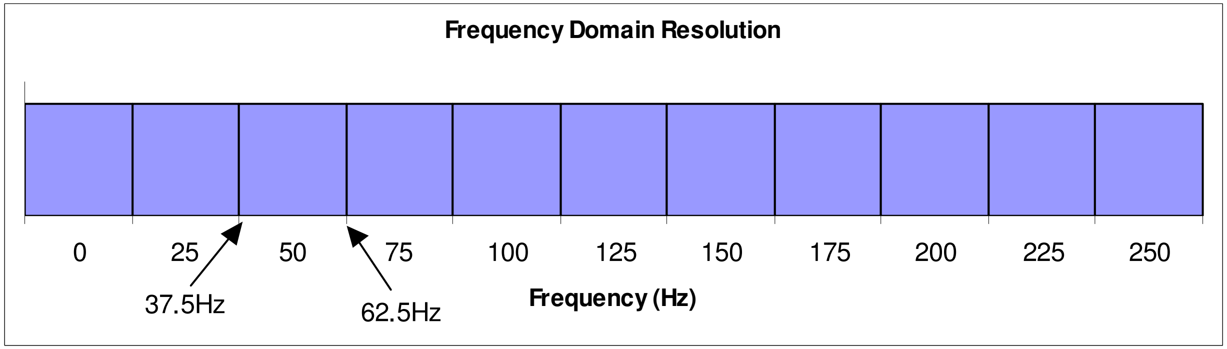
1 Cycle Windows

Resolution = 1/T

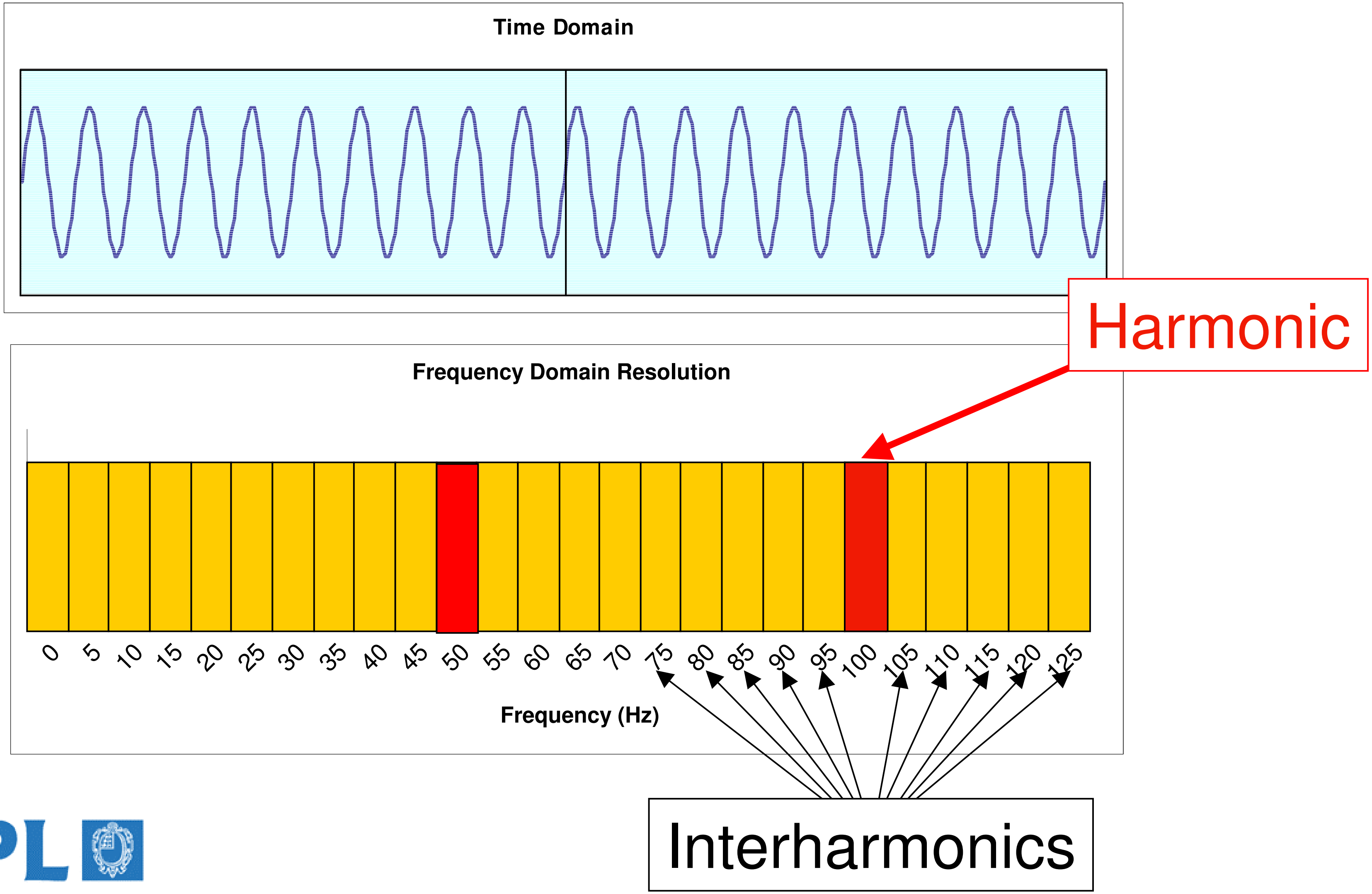


2 Cycle Windows

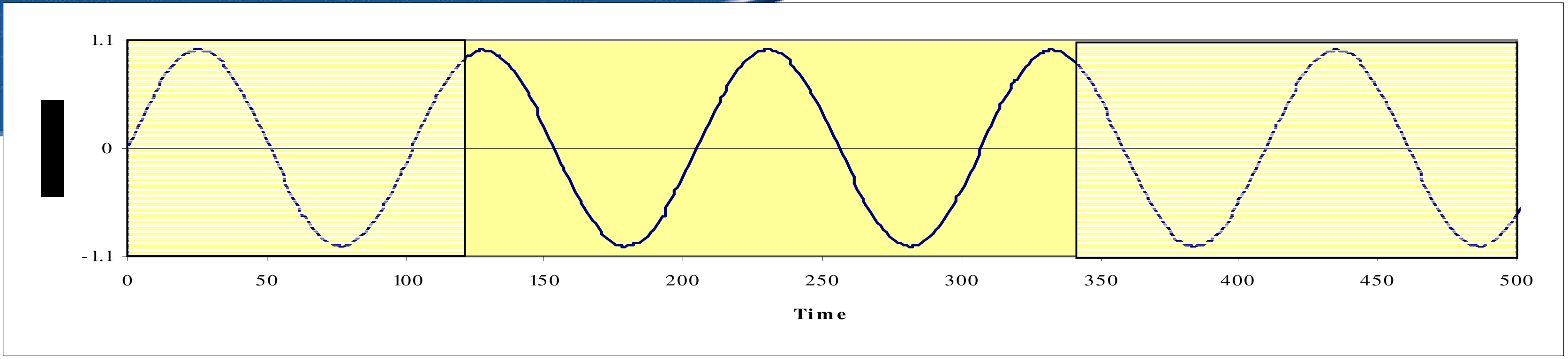
Resolution = 1/2T

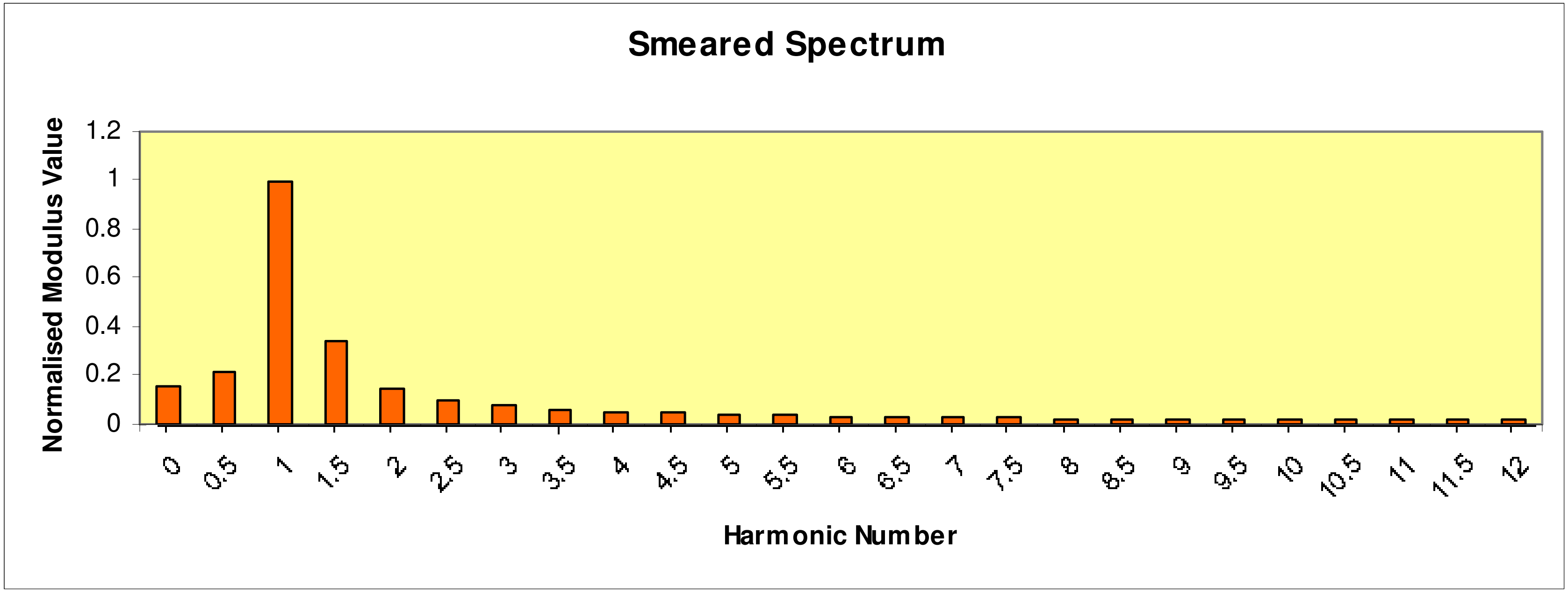


10 Cycle Windows and Interharmonics



Smearing or Gibb's Phenomenon

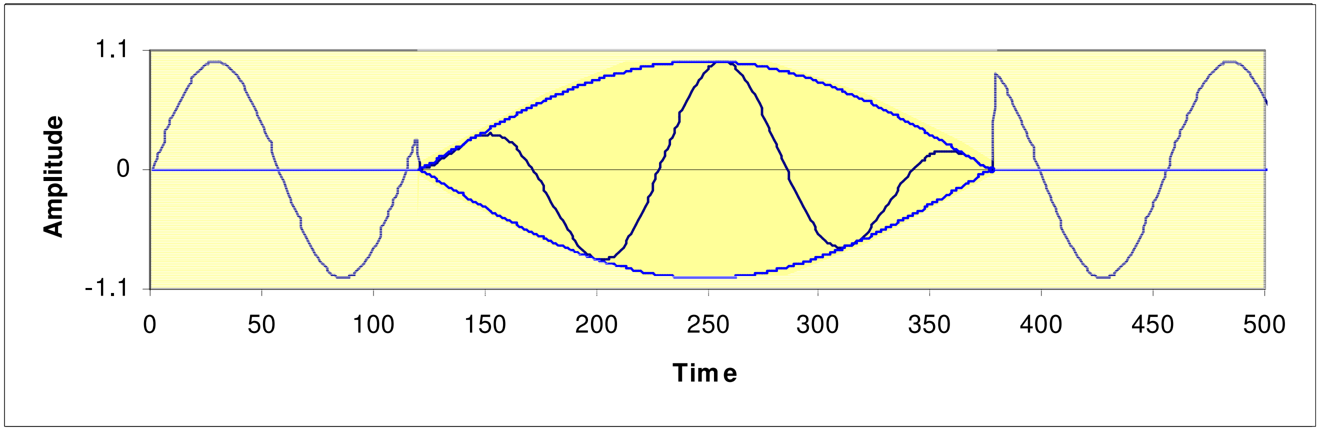


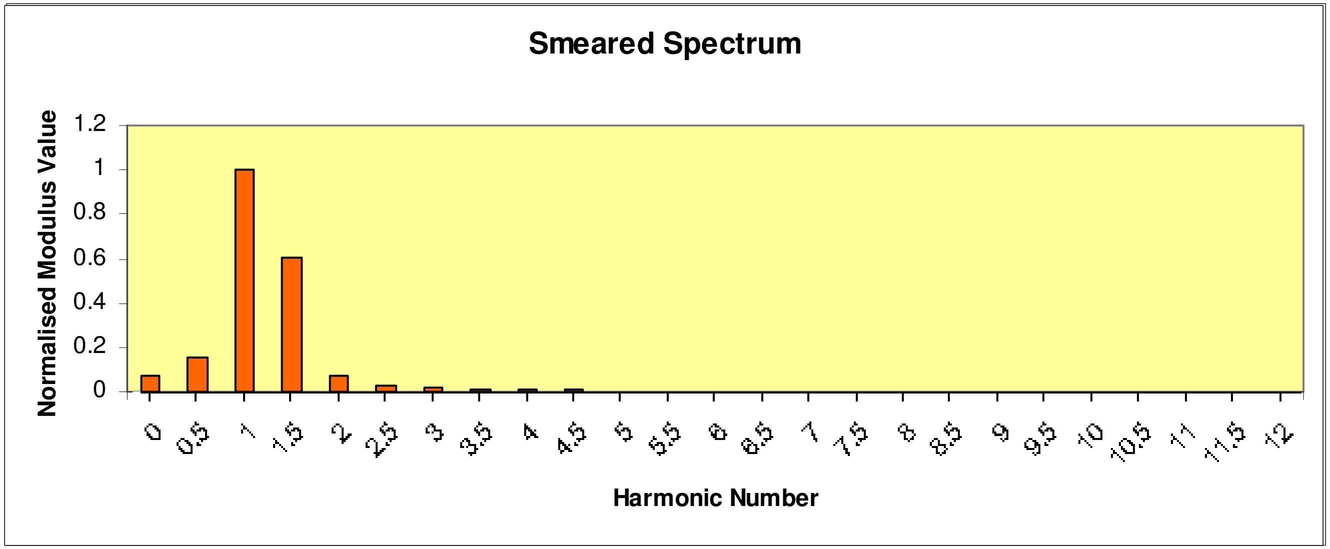


Synchronise the Samples to the Window!

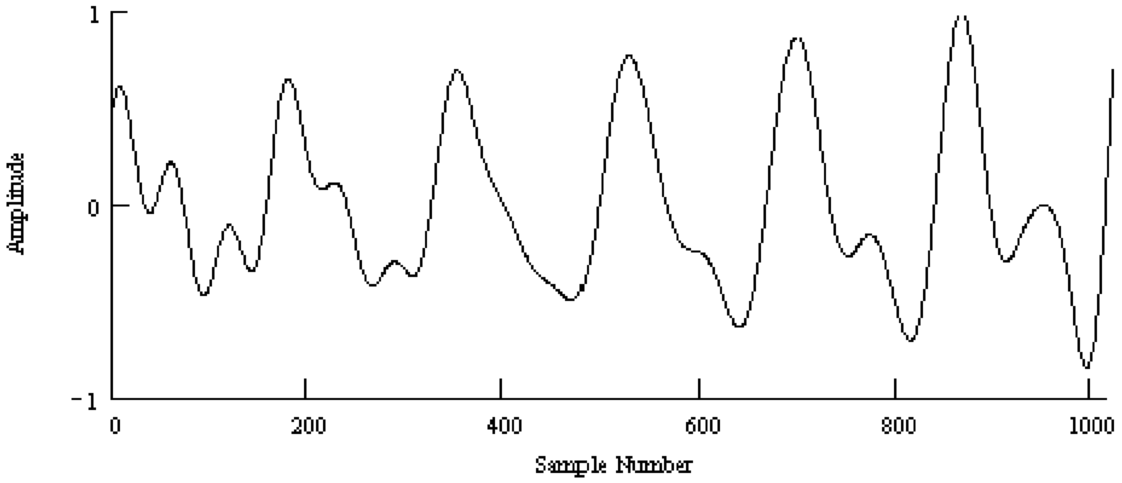
NPL

Shaped Windows



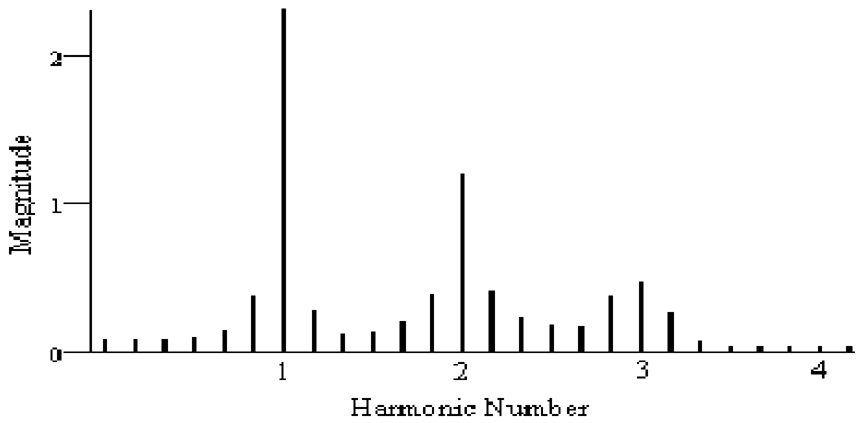


Non-Repetitive Signals



Made of H1, H2 and H3

But…



My Next Talk…..

A.M. and Sidebands

Carrier

$$v_c=V_c\sin(\omega_c)$$

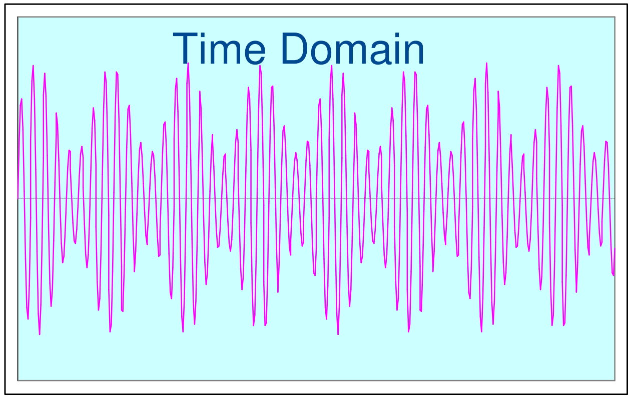
Modulator

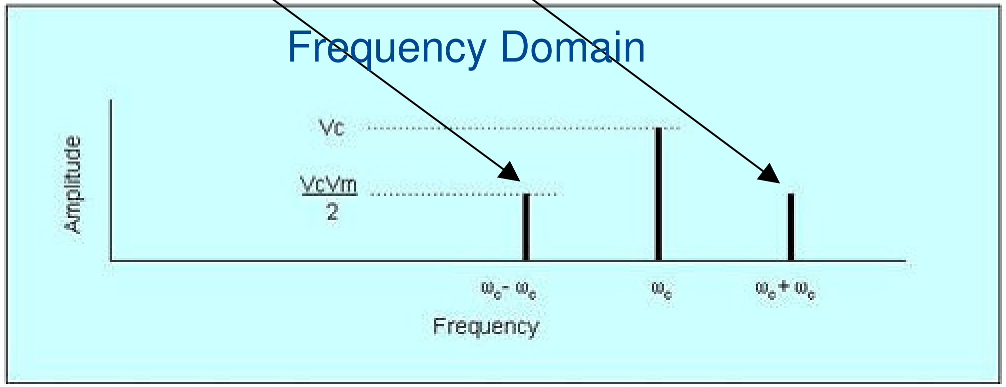
$$v_m=V_m\sin(\omega_m)$$

$$v_{am}= v_c.(1+v_m) = V_c\sin(\omega_c) + V_mV_c\sin(\omega_m)\sin(\omega_c)$$

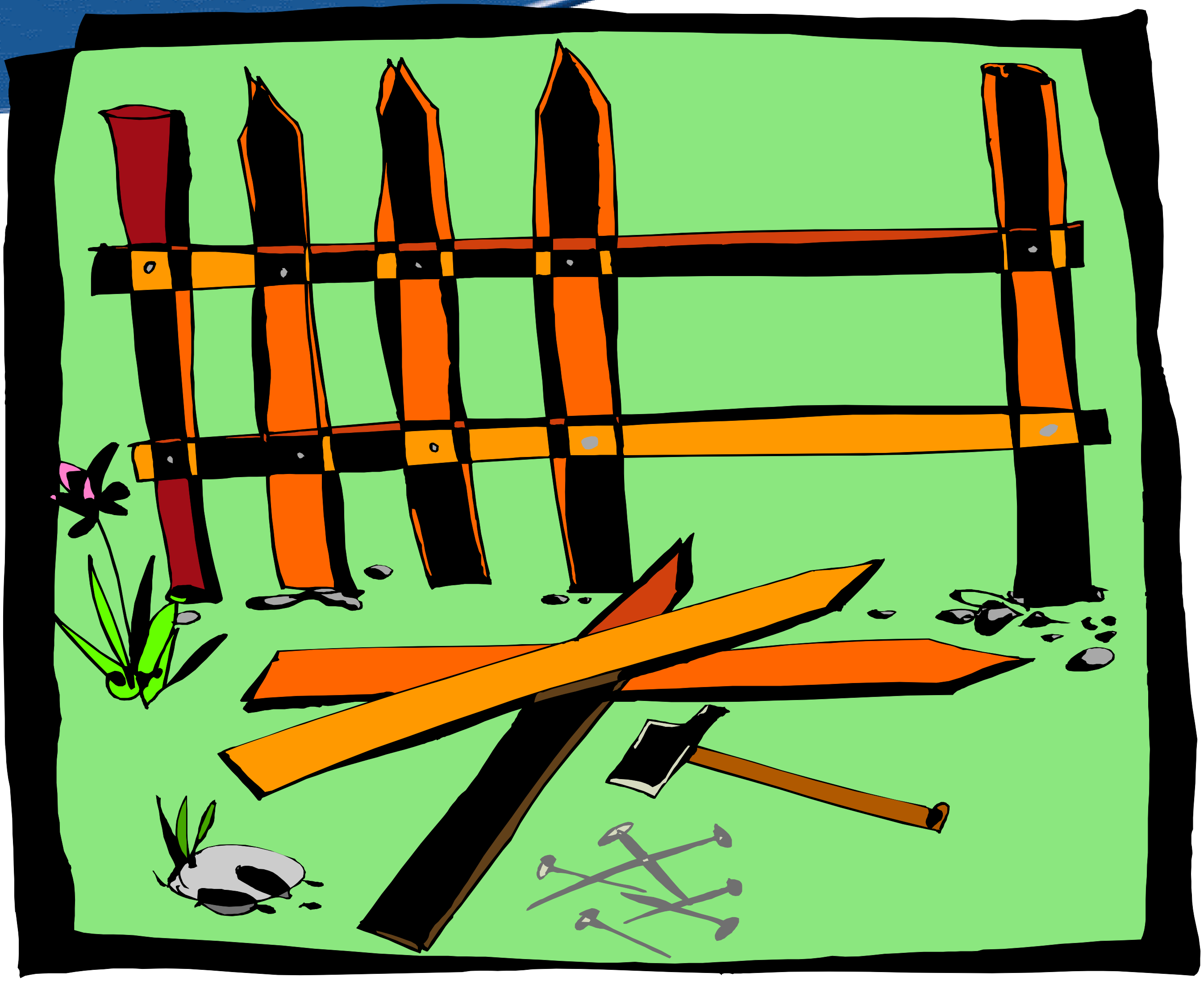
using the trigonometric function for the product of two sine functions,

$$v_{am}= V_c\sin(\omega_c) + ½\, V_mV_c[\cos(\omega_c - \omega_m) + \cos(\omega_c + \omega_m)]$$

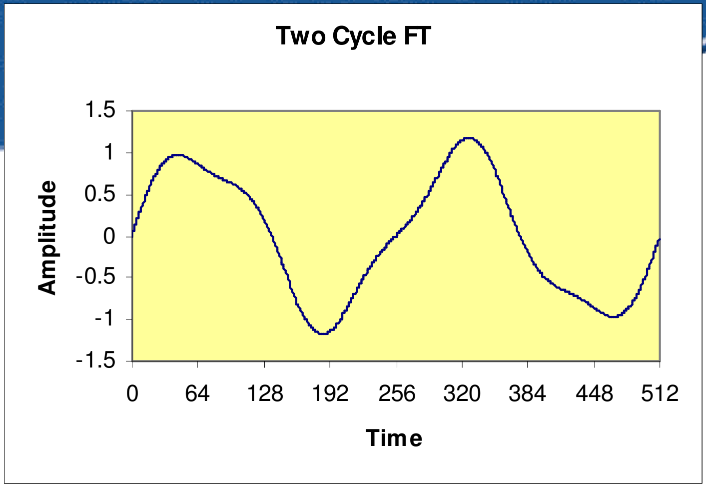


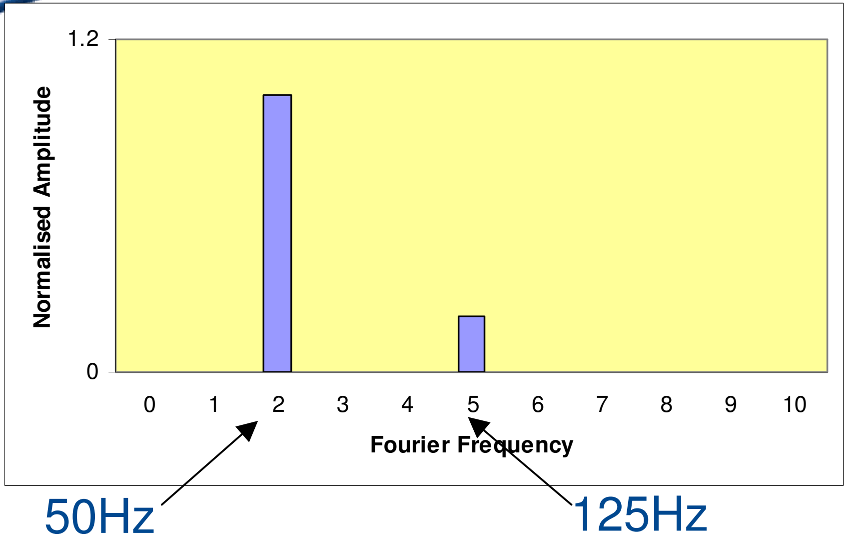


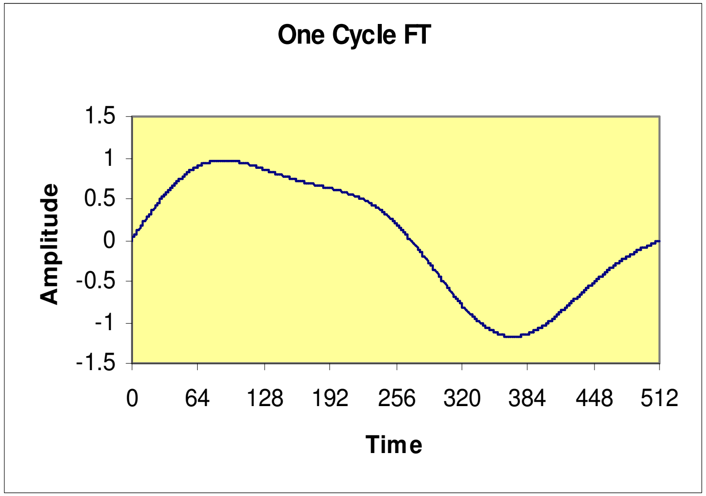
Only Certain Frequencies Can Be seen

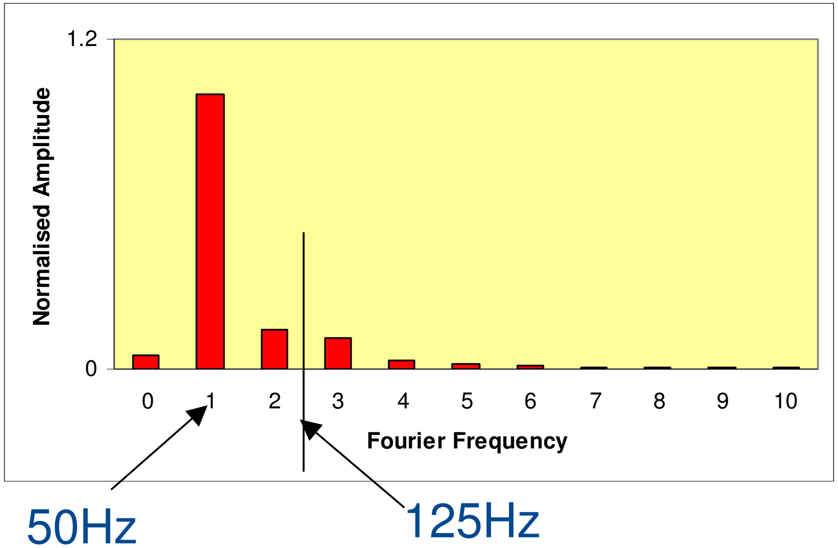


Picket Fencing: 50 Hz + 125 Hz









NPL

Summary
A Sampled Data System