Pulses and Parameters in the Time and Frequency Domain

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Signal Processing Seminar 30 November 2005
Structure

- Introduction
- Waveforms and parameters
- Instruments
- Waveform metrology and correction
Waveforms and parameters
Parameters for a pulse waveform

- Pulse Center Instant
- 90% Reference Level
- 50% Reference Level
- 10% Reference Level
- Offset
- Pulse Amplitude
- Pulse Duration
- First Transition Occurrence Instant
- First Transition Duration
- Second Transition Occurrence Instant
- Second Transition Duration
- Base State
- Waveform Epoch
- $t_0$
Why use parameters?

• Convenient way to compare information
• Reduce data complexity
• Specification of instruments
• These documents standardise how to characterise a waveform but do not specify units
Measuring and characterising a signal

I. Instrument capabilities
II. Waveform metrology and correction
III. Parameter analysis and extraction

Determine parameters that describe a signal or response
Assumptions

- In practice we want to define a response in terms of single parameters such as width or risetime or bandwidth

- Assume oscilloscope response has smooth bell-shaped
e.g. Gaussian
  Simple analytically
  Can calculate frequency response, convolution simply, reasonably realistic

Only 3 parameters – width/risetime; amplitude and position
Instruments
Time-domain

Real-time oscilloscope
High-speed A/D converter
Memory subsystem e.g.
32 Msamples
• Time-domain

Real-time oscilloscope
Sample rates up to 40Gsamples/s
Multiple high-speed A/D converters
Fast memory subsystem
Memory architecture may limit the length of trace e.g. 1 Msamples
Resolution typically 8 bits
• Time-domain

Real-time oscilloscope
Sampling oscilloscope
Requires a repetitive waveform
High-speed >70 GHz
Short trace length (e.g. 4096 samples)
Instrumentation

• Time-domain
  Real-time oscilloscope
  Sampling oscilloscope

• Frequency domain
  Heterodyne spectrum analyser
  Narrowband IF
  Not well suited to RF pulse measurements
Instrumentation

- Time-domain
  - Real-time oscilloscope
  - Sampling oscilloscope
- Frequency domain
  - Heterodyne spectrum analyser
    - Wideband IF
    - Suitable for RF pulse measurements
    - Lower frequency operation (8 GHz)
Instruments to measure pulse waveform

- Digital oscilloscope – can acquire single-shot data
  Current models up to 15 GHz  40 Gsamples/s.
  Typically 8-bit resolution
- Sampling oscilloscope – requires repetitive waveform –
  Sampling rate <100 ksamp/s.
  Bandwidth up to 100 GHz.
  Typical trace lengths 1024 - 4096 points
  Typical resolution 10-bit
- Heterodyne RF Spectrum Analyser – unsuitable for this task
- Real-time Spectrum Analyser – Down-converts RF signals to low frequency
  Typical frequency range 8 GHz
  IF Bandwidth/digitiser typically 10-80 MHz
Waveform metrology and correction
• Convolution of stimulus and system response

\[ y(t) = \int_{-\infty}^{+t} x(\tau)h(t - \tau)\,d\tau \]

\[ Y(s) = H(s)X(s) \]
Oscilloscope automatically performs a convolution when it records a waveform. Problem is to carry out the inverse operation and that is much more difficult.

Approximate the convolution integral to a quadrature addition. Information required is given by quadrature subtraction.

\[ TR_{CRO} = \sqrt{TR_{meas}^2 - TR_{PG}^2} \]

- Gaussian approximation to both the pulse shapes is valid
- the oscilloscope response is much faster than the pulse duration.

The rule of thumb normally adopted is that
\[ TR_{meas} > 3 \cdot TR_{PG} \]
Deconvolution of measured waveform

- Convolution of stimulus and system response
- Deconvolution – correction for the system response

\[ x(t) \rightarrow h(t) \rightarrow y(t) \]

\[ h^{-1}(t) \text{ is the inverse of the system response } h(t) \]

\[ x'(t) \]
Deconvolution of measured waveform

- Convolution of stimulus and system response
- Deconvolution – correction for the system response

Signal $x(t)$ → System response $h(t)$ → Resultant waveform $y(t)$

Estimate for Signal $x'(t)$ → Filter $r(t)$ → Deconvolve System response $h^{-1}(t)$

$h^{-1}(t)$ is the inverse of the system response $h(t)$
Deconvolution

\[ X(j\omega) = \frac{Y(j\omega) + \text{noise}}{H(j\omega)} \]

- Inverse problem
- Ill-posed
- Noise
- System response errors
Deconvolution with filter

\[ X(j\omega) = \frac{Y(j\omega) + \text{noise}}{H(j\omega)} R(j\omega) \]

- Inverse problem
- Ill-posed
- Noise
- System response errors
- Filter added to limit noise/errors
Deconvolution with filter

\[ X(j \omega) = \frac{Y(j \omega) + \text{noise}}{H(j \omega)} R(j \omega) \]

- Inverse problem
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- System response errors
- Filter added to limit noise/errors

Deconvolution of a 10.2 ps pulse with a Gaussian filter of 7.5 ps using:

a) No filter (blue)
b) A filter of effective width 4 ps (red). Resultant width is 6.2 ps
c) cf quadrature (rds) 6.9 ps.

NPL
Realisation of Deconvolution process

- System impulse response
- Direct time-domain convolution (Digital filtering - FIR)
- Transform approach (e.g. Fourier transform)

\[ R(j\omega) = \frac{|H(j\omega)|^2}{|H(j\omega)|^2 + \alpha|C|^2} \]

1. \( \alpha \) is a user controlled parameter
2. \( C \) may be constant or frequency dependent e.g. \( \omega^2 \) maximises the smoothness of the result
Jitter
Analysis and correction of measurement jitter

- A measured signal will contain both jitter and noise.
- Jitter and noise can be analysed as a Taylor expansion.
- The variance of the signal will be a function of time.
- If the jitter distribution is assumed then the underlying signal can be estimated by deconvolution.

$$x_m(t) = x(t + \xi) + \varepsilon$$

$$\langle x_m(t) \rangle = \int_{-\infty}^{+\infty} x(t - \tau) \phi_{\text{Jitter}}(\tau) \, d\tau$$

$$x_m(t) = x(t) + \varepsilon + \xi \frac{d x(t)}{dt} + \ldots$$

$$\sigma_m^2(t) \approx \sigma_{\text{noise}}^2 + \sigma_{\text{Jitter}}^2 \left(\frac{d x(t)}{dt}\right)^2$$

$$\langle x_m(t) \rangle \approx x(t) + \sigma_{\text{Jitter}}^2 \frac{d^2 x(t)}{dt^2}$$

Removal of measurement jitter using time-correlation between sampling gates

- If two sinusoidal signals in quadrature and with the same amplitude are measured using the same trigger event then the sample time can be determined.
- This information can be used to determine the timing of a waveform measured in another channel.

![Diagram of measurement system with random and deterministic jitter, and signal vs time graphs.](image-url)
Measurement jitter: uncertainty in time between the trigger and measurement events on sampling or real-time digital oscilloscope.

In Datacomm application the total measured jitter is the combination of data-dependent-jitter DDJ and random-jitter (RJ) in addition to any contribution added by the measurement system.

As DDJ is to be accurately quantified to qualify components, it is important to minimise the measurement jitter.

Jitter

- Jitter affects the measured signal by broadening it
- Measure the jitter using oscilloscope software (histogram method)
- Ideally we need to know the jitter distribution
- Usually assumed to be a Normal distribution
- Averaging over too long a period should be avoided since in addition to high frequency jitter, low frequency drift might also occur which produces non-Gaussian shaped jitter
- Removal of jitter - philosophical should it be removed?
- The problem is to identify which jitter is associated with the calibration and which is associated with the measurement
Waveforms and parameters
Parameters for a pulse waveform

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• Definition of terms
• Definitions for calculation of levels (typically 0% and 100%)
  Histogram methods and how to apply them (Main technique)
  Peak magnitude
  Initial (final) instant
• Definitions for a single transition (positive and negative)
• Definitions for a pulse waveforms
• Definitions for compound waveforms containing several transitions
• Classification of aberrations such as overshoot and undershoot before/after the transition
Care must be taken in defining the baseline and topline of the pulse and measuring the risetime, as has been described elsewhere.

Waveforms can be measured in two ways:
- built in acquisition and processing algorithms of the oscilloscope
- transferring the waveforms to a computer and processing them with proprietary software.

Whatever method is used care must be taken to ensure that the processing method is understood and that it gives the "correct" results, in other words the correct operation can be verified.

Levels must be defined for the specified region of interest.
Record the waveform on the longer epoch and find the levels. Switch off the tracking on the oscilloscope to keep the levels fixed and change the timebase to the faster setting. Record the risetime.
For processing the data by computer use an analogous process.
Example of RF transition duration pulse parameters

N.B. histogram is for example only
• Common Units Power or voltage envelope
• Do the important features occur within the specified ranges?

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<tr>
<th>Voltage</th>
<th>Power</th>
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<tr>
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<td>100%</td>
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<tr>
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<td>81%</td>
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<tr>
<td>10%</td>
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<tr>
<td>0%</td>
<td>0%</td>
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Parameters defined as Power

<table>
<thead>
<tr>
<th>Power</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
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<td>100%</td>
</tr>
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<td>10%</td>
<td>31.6%</td>
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<tr>
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</tr>
</tbody>
</table>

Numerical issues

- Number of points defining the parameter
- Interpolation
- Noise
- Jitter

- 15 Measurements of the response
- Transition duration 316 ps
- $1\sigma$ uncertainty due to measurement variations 4.8 ps
The maximum sampling frequency must be higher than the oscilloscope or source bandwidth otherwise aliasing will occur.
Summary

- Why use parameters?
- Instruments
- Waveform metrology and correction
- Deconvolution
- Jitter
- Parameters
Thank you for your attention