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Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 17

The Determination of Uncertainties of Ramberg-Osgood Parameters (from a Tensile Test)

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CONTENTS

1. SCOPE

This procedure covers the evaluation of uncertainty of Ramberg-Osgood-Parameters from a tensile test. The Code of Practice is restricted to tests with a digital acquisition of load and displacement.

EN 10002 Part 1-1990:*"Tensile Testing - Method of Testing at Ambient Temperature"*

EN 10002 Part 5-1990:*"Tensile Testing - Method of Testing at Elevated Temperature"*

ASTM E8-1998:*"Standard Test Methods for Tension Testing of Metallic Materials"*

ASTM E111-1997:*"Standard Test Method for Young´s Modulus, Tangent Modulus, and*

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

C_{RO}	divisor-parameter
CoP	Code of Practice
d_v	divisor used to calculate the standard uncertainty
E	Young's modulus
k	coverage factor used to calculate expanded uncertainty (normally corresponding to 95% confidence level)
m_{RO}	exponent-Parameter
n	number of repeat measurements
N	number of input parameters x_i on which the measurand depends
p	confidence level
u	standard uncertainty
u_c	combined standard uncertainty
U	expanded uncertainty
V	value of the measurand
x_i	estimate of input quantity
$\bar{x}$	arithmetic mean of the values of the random variable x_i
y	test (or measurement) mean result
σ	stress

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be controlled more closely.

This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated.

The aim is to produce a series of documents in a common format which is easily understood and accessible to customers, test laboratories and accreditation authorities.
This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into 6 sections as follows, with all the individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in Reference 1. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty of Ramberg-Osgoog-Parameters from a tensile testing.

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTY OF POISSON`S RATIO FROM TENSILE TESTING

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are reported.

Table 1 Evaluated quantities, their units and symbols

Measurands	Units	Symbol
Exponent - Parameter	Dimensionless	m_{RO}
Divisor - Parameter	MPa	C_{RO}

Table 2 Measurements, their units and symbols

Measurements	Units	Symbol
Load applied during the test	N	F
Axial displacement	mm	e
Dimension of the specimen	mm	a_0, b_0, or d_0
Gauge length	mm	L_0

Step 2. Identifying all sources of uncertainty in the test

In Step 2, the user must identify all possible sources of uncertainty that may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (eg. when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined. Table 3 lists the four categories and gives some examples of sources of uncertainty in each category. It is important to note that Table 3 is NOT exhaustive and is for GUIDANCE only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 3 Sources of uncertainty and their likely contribution to uncertainty of Ramberg-Osgood-Parameters from tensile testing
[1 = major contribution, 2 = minor contribution, 0 = no contribution (zero effect), ? = unknown]

Source of uncertainty	Type	m_{RO} , C_{RO}
1. Test specimen		
Dimensional compliance	B	2
Surface finish	B	2
Residual stress	B	?
2. Test system		
Original gauge length	B	1
Extensometer angular positioning	B	1
Alignment	B	1
Tooling stiffness	B	2
Uncertainty in force measurement	B	1

Uncertainty in elongation measurement	B	1
3. Environment		
Ambient temperature and humidity	B	2
4. Test Procedure		
Zeroing	B	1
Uncertainty in readings	B	1
Uncertainty in stress rate (strain rate)	B	1
Sampling frequency	B	1
Proportional limits	B	1

To simplify the uncertainty calculations it is advisable to regroup the significant sources affecting Ramberg-Osgood parameters in Table 3 in the following categories:

- Uncertainty due to errors of linear regression
- Further analysis

The worked example in Appendix B uses the above categorisation when assessing uncertainties.

Step 3. Classifying the uncertainty according to Type A or B

In this third step, which is in accordance with Reference 2, *'Guide to the Expression of Uncertainties in Measurement'*, the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified Type A, if it is evaluated by any other means it should be classified as Type B.

The values associated with Type B uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For Type B uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of Reference 1).

It should be noted that, in some cases, an uncertainty can be classified as either Type A or Type B depending on how it is estimated.

Step 4. Estimating the standard uncertainty for each source of uncertainty

In this step the standard uncertainty, u, for each input source is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d_v, associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of Reference 1.

In many cases the input quantity to the measurement may not be in the same units as the output quantity. In such a case, a sensitivity coefficient, c_T (corresponding to the partial derivative), is used to convert from input quantity to output quantity.

Step 5. Computing the combined uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y)=\sqrt{\sum_{i=1}^{N}[c_i.u(x_i)]^2} \qquad (1)$$

where c_i is the sensitivity coefficient associated with x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.27%.

Step 6. Computing the expanded uncertainty U

The expanded uncertainty, U, is defined in Reference 2 as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand".
It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, k, which is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2 which corresponds to a confidence interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty, U, is, therefore, broader than the combined uncertainty, u_c. Where a higher confidence level is demanded by the customer (such as for Aerospace industry, Electronics, ...), a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in Type A analysis is small), the value of k should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference 1, Section 4 for more details).

Table B1 in Appendix B shows the recommended format of the calculation worksheets for estimating the uncertainty of Ramberg-Osgood-Parameters. Appendix A presents the mathematical formulae for calculating uncertainty contributions.

Step 7. Reporting of results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U \qquad (2)$$

Where V is the estimated value of the measurand, y is the test (or measurement) mean result, U is the expanded uncertainty associated with y. An explanatory note, such as that given in the following example should be added (change when appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k = 2, which for a normal distribution corresponds to a coverage probability, p of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 17:2000.

5. REFERENCES

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APPENDIX A

Mathematical Formulae for Calculating Uncertainties of Ramberg-Osgood-Parameters (from a Tensile Test)

The Ramberg-Osgood-Concept is used in most modern strength-concepts to describe the limited plastic deformation as stress-strain-relationship. This procedure uses the mathematical model of linear regression for the determination of Young's Modulus and the Ramberg-Osgood parameters. In both cases the evaluation stops at the minimum variance of the slope (upper limits).

It is recommended to estimate the uncertainty of Young's Modulus and the proof stress before you start with this CoP - see CoP07 "The Determination of Uncertainties in Tensile Testing". The following procedure is based on a test series. At least 7 Specimens should be used to determine the uncertainty of Ramberg-Osgood parameters. 7 specimens and up it is possible to use test methods concerning normal distribution.

A1. The discussed mathematical model

$$e_t = \frac{s}{E} + \left(\frac{s}{C_{RO}}\right)^{m_{RO}} \qquad (3)$$

$$e_p = \left(\frac{s}{C_{RO}}\right)^{m_{RO}} \qquad (4)$$

e_t Total strain

e_p Permanent strain

<u>Transformation to linear model:</u>

$$LN(s) = \frac{1}{m_{RO}} \; LN(e_p) + LN(C_{RO}) \qquad (5)$$

A2. Formulae of linear regression (general):

$$y = m\,x + b \qquad (6)$$

Slope:

$$m = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \qquad (7)$$

Intercept equation:

$$b = \frac{\sum_{i=1}^{n} y_i - m\sum_{i=1}^{n} x_i}{n} \quad \textbf{(8)}$$

Empirical covariance:

$$S_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}\right] \quad \textbf{(9)}$$

Standard deviation of x-values:

$$S_x = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]} \quad \textbf{(10)}$$

Standard deviation of y-values:

$$S_y = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]} \quad \textbf{(11)}$$

Correlation coefficient:

$$r = \frac{S_{xy}}{S_x S_y} \quad \textbf{(12)}$$

Standard deviation of the slope:

$$S_m = \sqrt{\frac{(1-r^2)S_y^2}{(n-2)S_x^2}} \quad \textbf{(13)}$$

Standard deviation of the intercept:

$$S_b = \sqrt{S_m^2 \frac{(n-1)S_x^2 + \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n}} \quad \textbf{(14)}$$

Bound regarding the upper proportional limit for the determination of Young's modulus:

$$S_{m(rel)} = \frac{S_m}{m} \rightarrow \text{ minimum} \tag{15}$$

The data pair at the minimum of $S_{m(rel)}$ (variance) means the upper proportional limit.

Assignment of the symbols:

Symbol	Young's Modulus	Ramberg - Osgood parameters
y	$\boldsymbol{s}$ or F	$LN(\boldsymbol{s})$
x	$\boldsymbol{e}$ or e	$LN(\boldsymbol{e}_p)$
m	E	$1/m_{RO}$
b	b	$LN(C_{RO})$
S_{xy}	S_{es}	$S_{LN(e_p),LN(s)}$
S_x	S_e	$S_{LN(e_p)}$
S_y	S_s	$S_{LN(s)}$
r	r	r
S_m	S_E	$S_{1/m_{RO}}$
S_b	S_b	$S_{LN(C_{RO})}$
$S_{m(rel)}$	$S_{E(rel)}$	$S_{1/m_{RO}(rel)}$
$\sum_{i=1}^{n_{Sm(rel,\min)}} x_i y_i$	$\sum_{i=1}^{n_{SE(rel,\min)}} \boldsymbol{e}_i \boldsymbol{s}_i$	$\sum_{i=1}^{n_{S1/m_{RO}(rel)}} LN(\boldsymbol{e}_p) LN(\boldsymbol{s})$

$$m_{RO} = \frac{1}{1/m_{RO}} \tag{16}$$

$$C_{RO} = \exp\left[LN(C_{RO})\right] \tag{17}$$

A3. Statistical evaluation of the test series (general)

Mean value:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \textbf{(18)}$$

Empirical standard deviation:

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \quad \textbf{(19)}$$

Uncertainty of the mean value:

$$u_{\bar{x}} = \frac{t_{n,P} \; s_x}{\sqrt{n}} \quad \textbf{(20)}$$

A4. Combined and expanded uncertainty for test series

In order to obtain information concerning additional affecting parameters (e.g. strain rate, temperature etc.) there are some procedures like design of experiments - see [7], [8], and [9]. After such procedures continue with Eqn. 1(step 5,6) and Eqn. 2(step 7).

A5. Analysis if only one series has been tested

On principle it is not possible to calculate the uncertainty by partial derivation of Eqn. 3 or Eqn. 4. In fact we have a linear regression with error in the variables of Eqn. 5. There are some models but it leads to extensive calculations. The following formulae shows the problems.

- Error only in "x-axis" - $LN(e_p)$

Slope:

$$m = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n}(x_i)^2 - n(\bar{x})^2 - (n-1)s_x^2} \quad \textbf{(21)}$$

Intercept:

$$b = \bar{y} - m\bar{x} \quad \textbf{(22)}$$

An increasing variance, s_x^2, leads to a decreasing slope. In our case the variance should be calculated for the mean value of the linear region, $\bar{x}$.

- Error only in "y-axis" - $LN(s)$

Slope:

$$m = \frac{\sum_{i=1}^{n}(y_i)^2 - n\bar{y}^2 - (n-1)s_y^2}{\sum_{i=1}^{n} x_i y_i - n\overline{xy}} \qquad \textbf{(23)}$$

Intercept:

$$b = \bar{y} - m\bar{x} \qquad \textbf{(24)}$$

An increasing variance, s_y^2, leads to a decreasing slope. In our case the variance should be calculated for the mean value of the linear region, $\bar{y}$.

- Error in both axis

It is a very extensive formula and therefore we refer to [6] and [10]

In the worked example we didn´t use the mathematical model of error in the variables. We looked only for possible influence of the first linear regression (Young´s Modulus) to the results of the second linear regression (Ramberg-Osgood).

APPENDIX B

A Worked Example for Calculating Uncertainty of Ramberg-Osgood-Parameters (from a Tensile Test)

B1. Introduction

The object of this worked example are sheet type specimens - see CoP07 - of a cold rolled steel.

B2. Testing conditions

The exact testing conditions are described in CoP07.

B3. Example of Uncertainty Calculations and Reporting of Results

All calculations based on the formulae in Appendix A. Every table is produced for a certain measurand or evaluated quantity. The worked example shows the procedure concerning m_{RO} (Exponent-Parameter), and C_{RO} (Divisor-Parameter).

Results of Young's Modulus					
Number	E [GPa]	$\sum_{i=1}^{n_{S_{E(rel,\min)}}} e_i s_i$	Minimum Variance $S_{E(rel)}$ [%]	Upper limit e_{UL} [mm/mm] (total strain)	Upper limit s_{UL} [MPa]
01	206.4	3.34	2.09	7.41E-04	161.5
02	207.9	3.21	1.96	7.52E-04	162.3
03	208.2	2.21	1.53	6.58E-04	145.4
04	207.5	2.25	1.60	6.72E-04	146.7
05	207.5	1.91	1.71	6.18E-04	136.6
06	207.7	2.73	1.63	7.28E-04	156.9
07	208.9	3.12	1.99	7.32E-04	159.8

The upper limits (UL) have been used as lower limits (LL) for the next step of determination of Ramberg-Osgood parameters by linear regression.

Table B1

Results of Ramberg - Osgood parameters							
Number	e_{LL} [mm/mm] (permanent strain)	s_{LL} [MPa]	e_{UL} [mm/mm] (permanent strain)	s_{UL} [MPa]	Minimum Variance [%] $S_{1/m_{RO}}(rel)$	m_{RO} Eqn. 16	C_{RO} Eqn. 17
01	1.66E-06	161.5	2.72E-03	242.3	1.67	14.14	377.1
02	4.09E-06	162.3	1.77E-03	241.0	1.12	13.31	394.2
03	2.44E-06	145.4	1.50E-03	239.7	1.23	11.49	432.5
04	2.71E-06	146.7	1.42E-03	239.2	1.17	12.05	420.3
05	8.51E-07	136.6	2.27E-03	241.2	1.69	11.29	430.0
06	3.56E-06	156.9	1.60E-03	240.1	0.89	13.39	393.7
07	3.86E-06	159.8	1.68E-03	240.7	1.13	12.89	401.6
Mean value (Eqn. 18)						**12.65**	**407.1**
Empirical standard deviation (Eqn. 19)						**1.07**	**20.9**
Uncertainty of the mean value for combined uncertainty; t = 1.09; P = 68.27%						**0.44**	**8.6**
Uncertainty of the mean value for test series; t = 2.45; P = 95 %						**0.99**	**19.4**

Further analysis (figures):

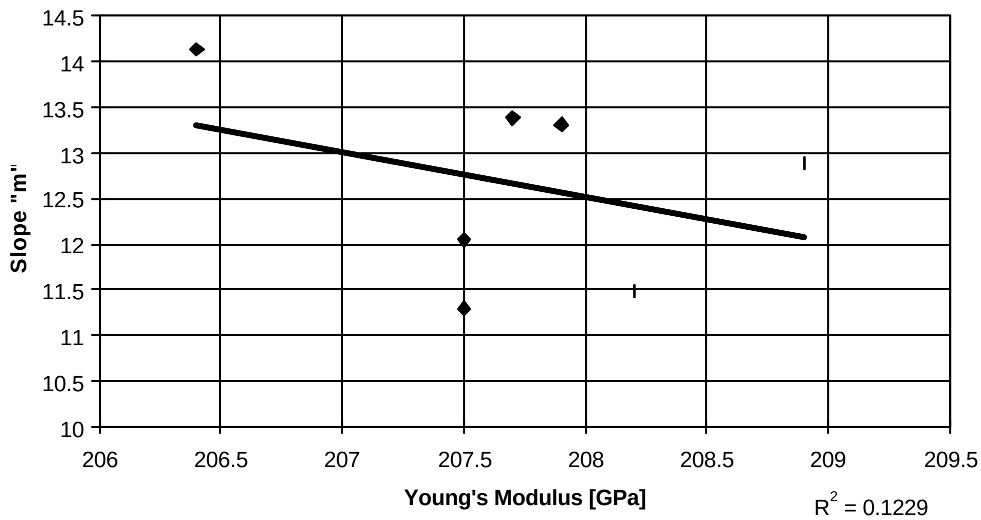

Fig. 1 Slope "m" versus Young's Modulus

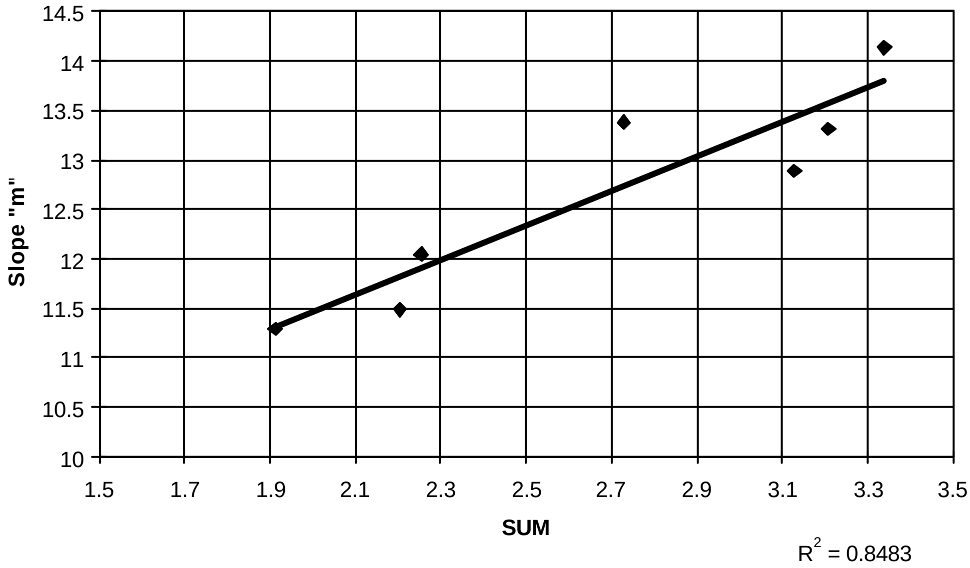

Fig. 2 Slope "m" versus "SUM"

The term "SUM" = $\sum_{i=1}^{n_{S_E(rel,\min)}} \varepsilon_i \sigma_i$

Fig. 2 shows the strong relationship of the ,"SUM", to the slope,"m". The uncertainty of Young´s Modulus itself is very low and is not affecting Ramberg-Osgood parameters. The term ,"SUM", describes the length of the linear elastic region and is affecting the starting point (zero) of permanent elongation. This effect leads to a higher slope,"m".

Fig. 3 shows the goodness of the approximation by Ramberg-Osgood for specimen No.01.

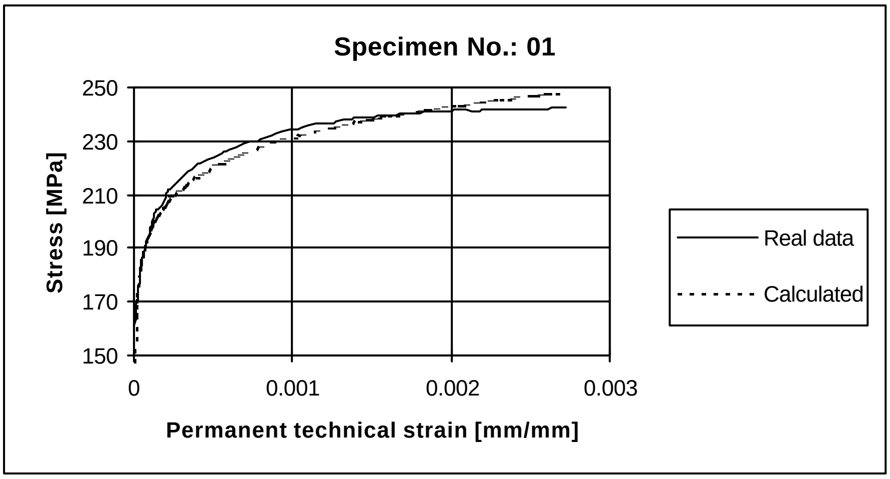

Fig. 3

B4. Reported Results

m_{RO}	=	12.65	± 0.99	(± 7.8 %)
C_{RO}	=	407.1 MPa	± 20.9	(± 5.1 %)

The above reported expanded uncertainties are based on standard uncertainties, providing a level of confidence of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT recommendations.