Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 15

The Determination of Uncertainties in Residual Stress Measurement

(Using the hole drilling technique)

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1 SCOPE

This procedure covers the evaluation of uncertainty in residual stress measurement by the hole drilling method, carried out according to the following Standard:

> ASTM: E837-95, "Standard Test Method for Determining Residual Stresses by the Hole-Drilling Strain-Gauge Method"

The hole drilling method is the most established mechanical method of residual stress measurement and can be considered as non-destructive for large structures. A strain gauge rosette is bonded on to the surface and a hole is drilled in the centre. Strains are measured continuously during drilling.

Because the distance between the strain gauges and the hole is small, the drilling has to be performed without significant plastic deformations and heating. Thus, high speed drilling machines of 300,000 revolutions per minute are used or air abrasive particles.

In principle the method is only valid for homogenous and isotropic materials. But, a number of publications show that the influence of the texture of the material can be neglected.

The hole drilling method does not release strains from inherent residual stresses completely. Thus, the stresses cannot be directly calculated from the measured strains. Coefficients for adjustment are necessary, and these are obtained by calculation or experimentation.

2 SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference [1], Section 2. The following symbols and definitions are used in this procedure.

$\bar{A}$, $\bar{B}$	calibration constants
$\bar{a}$	coefficient
$\bar{b}$	coefficient
CoP	Code of Practice
c_i	sensitivity coefficient
d_v	divisor used to calculate the standard uncertainty
D	diameter of the gauge circle
D_o	diameter of the drill hole
E	Young's modulus
k	coverage factor used to calculate expanded uncertainty (normally corresponding to 95 % confidence level)
p	confidence level
u	standard uncertainty
u_c	combined standard uncertainty
U	expanded uncertainty
V	value of the measurand
x_i	estimate of input quantity
y	test (or measurement) mean result
z	depth of the drill hole
α	angle measured counter-clockwise from the direction of σ_{max} to direction of ε_r
β	angle measured clockwise from the location of the reference gauge to the direction of σ_{max}. Gauge 1 is the reference gauge for both CW and CCW rosettes.
ε_r	released strain measured by a radially aligned strain gauge centered at P
ε_1	released strain measured by the gauge 1
ε_2	released strain measured by the gauge 2
ε_3	released strain measured by the gauge 3
σ_{max}	maximum
σ_{min}	minimum principal stresses
μ	Poisson's ratio

Figure 1 shows the definitions of the symbols used in residual stress measurement by the hole drilling method.

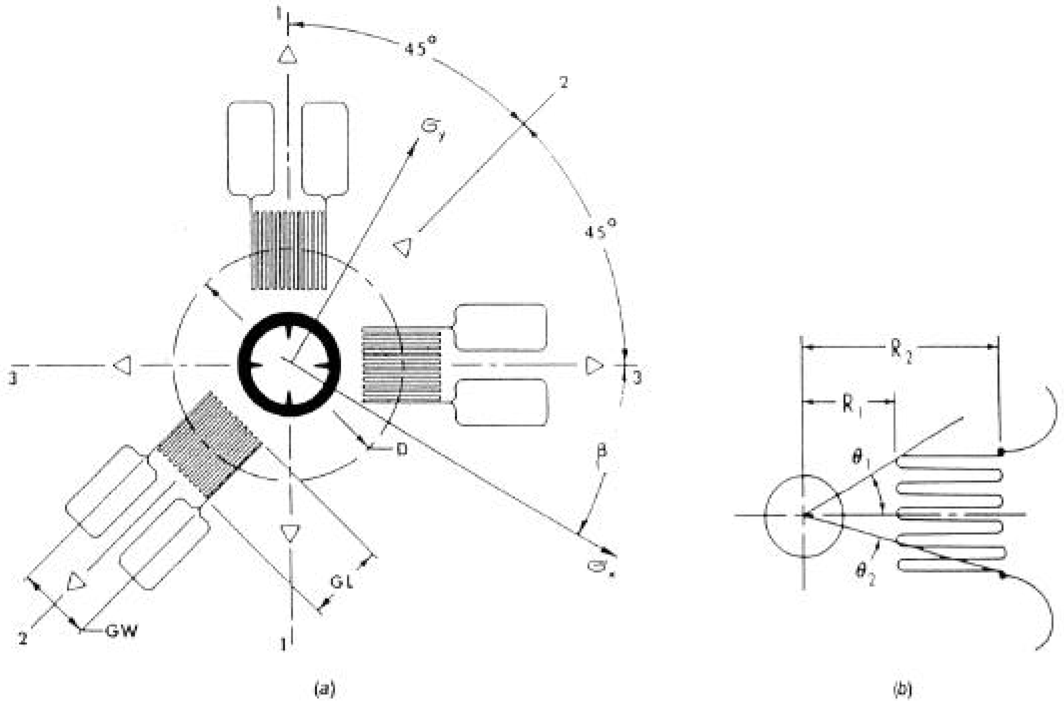

Fig. 1 Typical three-element clockwise (CW) strain gauge rosette for the hole-drilling method.

Remark: Angles α and β are identical in there amount and differ only in the reference directions. Angle α is used in the theoretical case where it is necessary to define the direction of the strain ε_r relative to a known principal stress direction. Angle β is used in the practical case when it is necessary to define a principal stress direction relative to a known ε_r direction, such as for hole-drilling residual stress calculations.

3 INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be controlled more closely. This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated. The aim is to produce a series of documents in a common format which is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. The Codes of Practice have been collated in a single Manual [1] that has the following sections.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 of the Manual [1]. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure.
- Details of the uncertainty calculations for the particular test type (Appendix A)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in residual stress measurement by the hole drilling method.

4 A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTY IN RESIDUAL STRESS MEASUREMENT

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are usually reported in residual stress measurement by the hole drilling method. None of the measurands are measured directly, but are determined for other quantities (or measurements).

Table 1 Measurands, measurements, their units and symbols

Measurands	**Units**	**Symbol**
Modulus of Elasticity	Mpa	E
Poissons ratio	Dimensionless	μ
Maximum principal stress	MPa	σ_{max}
Minimum principal stress	MPa	σ_{min}
Direction of principal stress	deg (°)	β

Measurements	**Units**	**Symbol**
Strain from strain gauge 1	$\mu m / m$	ε_1
Strain from strain gauge 2	$\mu m / m$	ε_2
Strain from strain gauge 3	$\mu m / m$	ε_3
Drilling hole depth	mm	z
Drilling hole diameter	mm	D_o
Gauge circle diameter	mm	D
Calibration constant	MPa^{-1}	$\bar{A}$
Calibration constant	MPa^{-1}	$\bar{B}$
Coefficient	dimensionless	$\bar{a}$
Coefficient	dimensionless	$\bar{b}$

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2, the user must identify all possible sources of uncertainty that may have an effect (either directly of indirectly) on the test. The list cannot be identified comprehensively beforehand, as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (for example when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined. Table 2 lists the four categories and gives some examples of sources of uncertainty in each category.

It is important to note that Table 2 is NOT exhaustive and is for GUIDANCE only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 2 Typical sources of uncertainty and their likely contribution to uncertainties in residual stress measurement by the hole drilling method
[1 = major contribution, 2 = minor contribution, blank = insignificant (zero effect)]

Source of uncertainty	**Type**+	**Measurands**			**Measurements**					
		μ	E	σ_{max} σ_{min}	β	$\varepsilon_{1,2,3}$	D_o	D	$\bar{A}, \bar{B}$	$\bar{a}, \bar{b}$
1. Test piece										
Surface finish	B			2	2	2	2	2		
Material characteristics	B	1	1	1	1	1			1	
2. Test system										
Alignment *										
Measuring the drilling hole dimensions	A or B			1			1			1
Gauge circle dimensions	B			1				1		1
Uncertainty in strain measurement	B			1	1	1				
Drift in strain measuring system	B			2	2	2				
Stress and temperature initiation from drilling	B			2	2	2	2	2		
3. Environment										
Temperature and humidity	B									
4. Test Procedure										
Calculation of $\bar{A}$,	B	1	1	1					1	1
Calculation of $\bar{B}$	B		1	1					1	1
$\bar{a}$ (draw from/3/, Table 2)	B			1			1	1		1
$\bar{b}$ (draw from /3/, Table 2)	B			1			1	1		1

+ see step 3.
* contain in: measuring the drilling hole dimensions

Step 3. Classifying the Uncertainty According to Type A or B

In this third step, which is in accordance with Reference [2], 'Guide to the Expression of uncertainties in Measurement', the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified Type A, if it is evaluated by any other means it should be classified as Type B.

The values associated with Type B uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For Type B uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of Reference [1]).

It should be noted that, in some cases, an uncertainty could be classified as either Type A or Type B depending on how it is estimated. Table 2 contains an example where, if the diameter of a drilling hole is measured once, that uncertainty is considered Type B. If the mean value of two or more consecutive measurements is taken into account, then the uncertainty is Type A.

Step 4. Estimating the Sensitivity Coefficient and Standard Uncertainty for each Source

In this step the standard uncertainty, u, for each major input source identified in Table 2 is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d_v, associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of Reference [1].

The standard uncertainty requires the determination of the associated sensitivity coefficient, c, which is usually estimated from the partial derivatives of the functional relationship between the output quantity (the measurand) and the input quantities. The calculations required to obtain the sensitivity coefficients by partial differentiation can be a lengthy process, particularly when there are many individual contributions and uncertainty estimates are needed for a range of values. If the functional relationship for a particular measurement is not known, the sensitivity coefficients may be obtained experimentally.

To help with the calculations, it is useful to summarise the uncertainty analysis in a spreadsheet - or 'uncertainty budget'. Appendix A includes the mathematical formulae for calculating the uncertainty contributions and Appendix B gives a worked example.

Step 5. Computing the combined uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[c_i \cdot u(x_i)\right]^2} \qquad (1)$$

where c_i is the sensitivity coefficient associated with x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.27 %.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, U, is defined in Reference [2] as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand". It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, k, which is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2 which corresponds to a confidence interval of 95.4 % (effectively 95 % for most practical purposes). The expanded uncertainty, U, is, therefore, broader that the combined uncertainty, u_c. Where a higher confidence level is demanded by the customer (such as for Aerospace or the Electronics industries), a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73 %.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in Type A analysis is small), the value of k should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference [1], Section 4 for more details).

Remark: For the calculation of the combined and the expanded uncertainty by the hole drilling method we have to consider, that with our equipment only one measurement can be performed for each measuring point. An additional measurement requires another measuring point, that may have different residual stresses. Thus, we cannot repeat the measurements for the same conditions for statistical analyses and have only one reference dimension and so no statistical distributions. For this reason we approach the calculation in another way.

Appendix A gives the mathematical formulae used for calculating the uncertainty contributions and Appendix B gives a worked example.

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, U is the expanded uncertainty associated with y. An explanatory note, such as that given in the following example should be added (change when appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k = 2, which for a normal distribution corresponds to a coverage probability, p, of approximately 95 %. The uncertainty evaluation was carried out in accordance with UNCERT COP 15:2000.

5 REFERENCES

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APPENDIX A

MATHEMATICAL FORMULAE FOR CALCULATING UNCERTAINTIES IN RESIDUAL STRESS MEASUREMENT BY THE HOLE DRILLING METHOD

A1. Mathematical Formulae for Calculation of Residual Stresses by the Hole Drilling Method

The following section of the Standard ASTM E837-95 describes the method for calculating the maximum and minimum principal stresses and their direction (for the case of a blind hole).

Computation of Stresses

(authentic excerpt by from ASTM E837-95) [3]

9.1 To obtain the stresses from the measured strains $\varepsilon_1 \varepsilon_2$ and ε_3, use the following procedure:

9.1.1 Assign to the three gauges numbers (1), (2) and (3) in a clockwise order as shown in Figure 1. The directions (1) and (3) are mutually perpendicular and (2) coincides with one of the bisectors.

9.1.2 The principal stress σ_{max} is located at an angle β measured clockwise from the direction of gauge in Figure 1. Similarly, the principal stress σ_{min} is located at an angle β measured clockwise from the direction of gauge 3.

Compute the angle β from $$\beta = \tfrac{1}{2} \arctan \left[\frac{\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2}{\varepsilon_3 - \varepsilon_1} \right] \qquad (3)$$

Direct calculation of the angle β using the common one argument arctan function such as is found on an ordinary calculator, can give an error of $\pm$ 90°. The correct angle can be found by using the two-argument arctan function (function ATAN2 in some computer languages), where the signs of the numerator and denominator are each taken into account. Alternatively, the result from the one-argument calculation can be adjusted by $\pm$ 90° as necessary to place β within the appropriate range defined in the following table:

	$\varepsilon_3 - \varepsilon_1 < 0$	$\varepsilon_3 - \varepsilon_1 = 0$	$\varepsilon_3 - \varepsilon_1 > 0$
$\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2 > 0$	$45° < \beta < 90°$	45°	$0° < \beta < 45°$
$\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2 = 0$	90°		
$\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2 < 0$	$-90° < \beta < -45°$	-45°	$-45° < \beta < -0°$

A positive value of β, say $\beta = 30°$, indicates that σ_{max} lies 30° clockwise of the direction of gauge 1. A negative value of β, say β = -30°, indicates that σ_{max} lies 30° counter-clockwise of the direction of gauge 1.

In general, the direction of σ_{max} will closely coincide with the direction of the numerically most negative (compressive) relieved strain. The case where both $\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2 = 0$ and $\varepsilon_3 - \varepsilon_1 = 0$ corresponds to an equal biaxial stress field, for which the angle β has no meaning.

NOTE 1 - The clockwise measurement direction for angle β defined in 9.1.2 applies only to a strain gauge rosette with CW gauge numbering, such as that illustrated in Figure 1. The opposite measurement direction for β applies to a CCW strain gauge rosette. In such a rosette the geometrical locations of gauges 1 and 3 are interchanged relative to the CW case. The new gauge 1 becomes the reference gauge. For a CCW rosette, a positive value of β, say β = 30°, indicates that σ_{max} lies 30° counter-clockwise of the direction of gauge 1. A negative value of β, say β = -30°, indicates that σ_{max} lies 30° clockwise of the direction of gauge 1. All other aspects of the residual stress calculation are identical for both CW and CCW rosettes.

Table A1. Numerical values of coefficients $\bar{a}$ and $\bar{b}$

	Through-the-thickness hole		Blind hole, depth = 0,4 D	
D_o/D	$\bar{a}$	$\bar{b}$	$\bar{a}$	$\bar{b}$
0.30	0.089	0,278	0.111	0.288
0.31	0.095	0.295	0.118	0.305
0.32	0.101	0.312	0.126	0.322
0.33	0.108	0.329	0.134	0.340
0.34	0.114	0.347	0.142	0.358
0.35	0.121	0.364	0.150	0.376
0.36	0.128	0.382	0.158	0.394
0.37	0.135	0.400	0.166	0.412
0.38	0.143	0.418	0.174	0.430
0.39	0.150	0.436	0.182	0.448
0.40	0.158	0.454	0.190	0.466
0.41	0.166	0.472	0.199	0.484
0.42	0.174	0.490	0.208	0.503
0.43	0.183	0.508	0.217	0.521
0.44	0.191	0.526	0.226	0.540
0.45	0.200	0.544	0.236	0.558
0.46	0.209	0.562	0.246	0.576
0.47	0.218	0.579	0.255	0.594
0.48	0.228	0.596	0.265	0.612
0.49	0.237	0.613	0.275	0.630
0.50	0.247	0.629	0.285	0.648

9.1.3 Compute the stresses σ_{max} and σ_{min} from

$$\sigma_{max}, \sigma_{min} = \frac{\varepsilon_3 + \varepsilon_1}{4\bar{A}} \pm \frac{\sqrt{(\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4\bar{B}} \tag{4}$$

The negative square root in this equation is associated with σ_{max} because the calibration constants $\bar{A}$ and $\bar{B}$ have negative numerical values. A tensile (+) residual stress will produce a compressive (-) relieved strain.

NOTE 2. If the calculated stress σ_{max} or σ_{min}, or both, exceed one half of the yield stress of the material, the stresses on the edge of the drilled hole might exceed the elastic limit of the material. Depending on the material, the inelastic behaviour could affect the accuracy of the results.

9.1.3.1 The following equations may be used to evaluate the constants $\bar{A}$ and $\bar{B}$, including the integrating effect of a finite size strain gauge, the given material properties, and for the possibility of a blind hole situation

$$\bar{A} = -((1+\mu)/2E)\bar{a} \tag{5}$$

$$\bar{B} = -(1/2E)\bar{b} \tag{6}$$

There $\bar{a}$ and $\bar{b}$ are dimensionless, material-independent coefficients given by the equations in 9.1.3.2 and by Table A1. See Note 2.

NOTE 3. The dimensionless coefficients $\bar{a}$ and $\bar{b}$ are both nearly material-independent. They do not depend on Young's modulus, E, and they are correct to within 1 % for Poisson's ratios in the range 0.28 to 0.33. For a through-hole in a thin plate, $\bar{a}$ is independent of Poisson's ratio.

A2. Influence of Factors on the Measuring Uncertainty of the Hole Drilling Method and their Quantification

For the evaluation of the measuring uncertainties of the hole drilling method we have to consider that only one measurement can be performed for each measuring point. An additional measurement requires another measuring point that may have a completely different residual stress state and distribution. Thus, we cannot repeat the measurements for the same conditions for statistical analyses. These quantities were gained from comparison tests with residual stress specimens those were investigated by different residual stress measuring methods. In addition other specimens with defined load/stress ranges, e.g. 4-point bending specimens, were used.

The quantification of the sources of uncertainty listed in the Table A2 is based on a literature study and. The data from the literature are derived from practical and arithmetical investigations.

Table A2. Input Quantities

Influences from the	Uncertainty	Remarks
1. Test piece		
surface	negligible	
heavy mismatch from plane surface	unknown	
Modulus of Elasticity	± 1 %	measure using standard specimens of the same material
Poisson's ratio	± 3 %	see above
stress distribution ♦ 2-axial / biaxial ♦ 3-axial / triaxial	 negligible ± 15 %	
level of residual stresses ♦ <50 % R_p ♦ 50 - 70 % R_p ♦ >70 % R_p	 negligible ± 10 % unknown	
distance between measuring points ♦ 5 times drilling hole diameter ♦ 10 times drilling hole diameter	 ± 8 % ± 2 %	
2. Test system		
measuring the hole drilling dimensions ♦ diameter	 negligible	 measurement by light microscope
♦ irregularities in the drilling hole shape	negligible	use a new drill after 2 hole drillings
♦ drilling hole depth	negligible	the uncertainty of the depth measurement has to be considered for >0,01 mm. The commercial systems have higher accuracies, e.g. 0,001 mm
♦ eccentricity of the hole to the center of the rosette	negligible	for e ≤ 0,05 mm (0,1 mm) for e ≥ 0,05 mm (0,1 mm) the measurement has to be considered invalid
♦ perpendicularity of the hole axis relevant to the surface	for plane surfaces negligible for bent surfaces unknown	
gauge circle dimensions	negligible	producer-data
Uncertainty in strain measurement technique	± 2 till 5 %	experience from the traditional experimental stress analysis
Drift in strain measuring system	negligible	zero adjustment before starting the measurement, short measuring cycle
stress and temperature initiation from drilling	negligible	use of high-speed-drilling equipment and new drill after 2 hole drillings
3. Environment		
Temperature and humidity	negligible	measuring by different conditions (not labor), but zero adjustment before starting the measurement, short measuring cycle
4. Test Procedure		
Calculation of $\bar{A}$ and $\bar{B}$	dependent on the un-	

	certainty of E, μ, $\bar{a}$ and $\bar{b}$	
$\bar{a}$, $\bar{b}$ (from [3], Table 2)	dependent on the uncertainty of D_o, D and z	

A3 Estimating of uncertainty

A 3.1 $\bar{a}$ and $\bar{b}$

The uncertainty from $\bar{a}$ and $\bar{b}$ depends on D_o/D and on the depth of the drilling hole ([1], Table 2). If the calculation of D_o/D is carried out with the maximum of the uncertainties of D_o and D, than the result will show the influence

(and the uncertainties) on $\bar{a}$ and $\bar{b}$.

A 3.2 $\bar{A}$ and $\bar{B}$

The uncertainties are dependent on $U_{\bar{A}}$ (E, μ, $\bar{a}$) ([3], equation 5)

$U_{\bar{B}}$ (E, $\bar{b}$) ([3], equation 6)

Both equations are calculated with all of the combinations from the individual uncertainties. The maximum and minimum each of $\bar{A}$ and $\bar{B}$ represent the uncertainties.

A 3.3 σ_{max}, σ_{min} and β

The reference dimensions in accordance with [3], Equations (3) and (4) of $\varepsilon_1 \varepsilon_2$ and ε_3, are to extend corresponding to their estimated ranges of uncertainty for the specific work example:

ε_1: ε_{1max}, ε_{1min}

ε_2: ε_{2max}, ε_{2min}

ε_3: ε_{3max}, ε_{3min}

In addition the maximum and minimum of $\bar{A}$ and $\bar{B}$ in accordance with the calculated maximum uncertainties have also to be taken. The calculation of the principal stresses σ_{max}, and σ_{min} and the angle β should be carried out for all combinations from $\varepsilon_1 \varepsilon_2$ and ε_3 and $\bar{A}$ and $\bar{B}$. The uncertainties in σ_{max}, σ_{min} and β are represented by the maximum and minimum value of each. It is useful to write software for this calculation, as the number of the combinations is large.

APPENDIX B

A WORKED EXAMPLE FOR CALCULATING UNCERTAINTIES IN RESIDUAL STRESS MEASUREMENT BY THE HOLE DRILLING METHOD

B1 Introduction

For evaluation of the measuring uncertainties of the hole drilling method we have to consider that only one measurement can be performed for each measuring point. An additional measurement requires another measuring point that may have more or less different residual stress state and distribution. Thus, we cannot repeat the measurements for the same conditions for statistical analyses.

The worked example is valid for the following assumptions:

- plane stress condition, with uniform stress distribution throughout the depth
- residual stresses 0.50 R_p
- distance between neighbouring measuring points >10 times drilling hole diameter
- no measuring points close to significant geometry changes.

B2 Worked example

Material: ST 52.3 N

E	=	206.0 GPa	ε_1	=	$18.98 \cdot 10^{-6}$
μ	=	0.3	ε_2	=	$18.75 \cdot 10^{-6}$
D	=	5.14 mm	ε_3	=	$24.22 \cdot 10^{-6}$
D_o	=	1.8 mm	(for the depth = 2 mm)		

Step 1

Calculation of the uncertainty of $\bar{a}$ and $\bar{b}$

The uncertainty of $\bar{a}$ and $\bar{b}$ depends on D_o/D and on the depth of the drilled hole.
([3], Table 2).

The depth of the drilled hole is continuously measured during drilling. Its influence on the evaluated quantities has only to be considered for uncertainty >0.01 mm [5]. In principle, commercial systems have higher accuracies, e.g. 0.001 mm. The drilling hole diameter D_o is measured in the final drilling depth by a dial gauge and light microscope. The uncertainty of the dial gauge is $< \pm$ 0.01 mm. The uncertainty of the gauge circle diameter D is $\leq \pm 0.01$ mm. If the calculation is carried out with the maximum values of the uncertainties, than we obtain the following results:

For the example, D_o = 1.8 mm and D = 5.14 mm. The uncertainties for both are ± 0.01 mm.

Set value: $$\frac{D_O}{D} = \frac{1.8}{5.14} = 0.3502$$

Range of values: $$\frac{D_O}{D} = \frac{1.81}{5.13} = 0.3528$$

$$\frac{D_O}{D} = \frac{1.79}{5.15} = 0.3476$$

From the standard [3], Table 2, the set value results in:

$$\frac{D_O}{D} = 0.3502 \ : \ \bar{a} = 0.150 \qquad \bar{b} = 0.376$$

The maximum and minimum values give:

$$\frac{D_O}{D} = 0.3476 \ : \ \bar{a} = 0.1481 \qquad \bar{b} = 0.3717$$

$$\frac{D_O}{D} = 0.3528 \ : \ \bar{a} = 0.1522 \qquad \bar{b} = 0.3810$$

In order to are:

$\bar{a}$ = 0.150 ± 0.0022 (± 1.5 %)

$\bar{b}$ = 0.376 ± 0.0043 (± 1.2 %)

Step 2

Calculation of the uncertainty from $\bar{A}$ and $\bar{B}$

$\bar{A} = -(1+\mu)/2E \cdot \bar{a}$

$\bar{B} = -(1/2E) \cdot \bar{b}$

with the maximum individual uncertainties

E: ±1 %

μ: ± 3 %

$\bar{a}$: $\pm$1.5 %

$\bar{b}$: $\pm$1.2 %

Both equations can be calculated with all combinations of the individual uncertainties. The maximum and minimum value of $\bar{A}$ and $\bar{B}$ represent the uncertainties.

Step 3

Calculation of the uncertainty from $\sigma_{min}, \sigma_{max}$ and β

$$\sigma_{min}, \sigma_{max} = \frac{\varepsilon_3 + \varepsilon_1}{4\bar{A}} \pm \frac{\sqrt{(\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4\bar{B}}$$

$$\beta = \frac{1}{2}\arctan\left[\frac{\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2}{\varepsilon_3 - \varepsilon_1}\right]$$

with the maximum single uncertainties:

$\bar{A}$ and $\bar{B}$ from Step 2
$\varepsilon_{1/2/3} = \pm 3\%$

Both equations are used to calculate the principal residual stresses and their direction with all combinations of the individual uncertainties.

The maximum and minimum values for $\sigma_{min}, \sigma_{max}$ and β represents the highest possible uncertainties.