SM & T
Standards Measurement & Testing Project No. SMT4-CT97-2165

UNCERT COP 03: 2000

Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 03

The Determination of Uncertainties in Plane Strain Fracture Toughness (K_{IC}) Testing

L.M. Plaza

INASMET
Centro Tecnológico de Materiales
Camino de Portuetxe, 12
ES-20009 San Sebastián
SPAIN

Issue 1

September 2000

CONTENTS

1. SCOPE

This procedure covers the evaluation of uncertainty in the determination *of Plane Strain Fracture Toughness* (K_{IC}) of metallic materials according to the testing Standards

> British Standard, BS 7448. Part 1-1991: Amd 1: August 1999.
> *"Fracture Mechanics Toughness Tests. Part 1. Method for determination of K_{IC}, critical CTOD and critical J values of metallic materials".*
>
> ASTM E399-90 *"Plane-Strain Fracture Toughness of Metallic Materials"*

This Standards give a method for determining plane strain fracture toughness values (K_{IC}), for metallic materials. The method uses fatigue precracked specimens. The tests are carried out in displacement control with monotonic loading, and at a constant rate of increase in stress intensity factor within the range $0.5 - 3\ MPa\sqrt{m}\ s^{-1}$ during the initial elastic deformation. The specimens are loaded to the maximum force associated with plastic collapse. The method is especially appropriate to materials that exhibit a change from ductile to brittle behaviour with decreasing temperature. No other influences of environment are covered.

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

Symbol Definition

Symbol	Definition
a	nominal crack length
B	specimen thickness
c_i	sensitivity coefficient
CoP	Code of Practice
CT	Compact Tension Test Specimen
d_v	divisor associated with the assumed probability distribution, used to calculate the standard uncertainty
f	mathematical function of (a/W)
k	coverage factor used to calculate expanded uncertainty
K	Stress Intensity Factor: the magnitude of the stress field near the crack tip for a particular mode in a homogeneous, ideally linear-elastic body.
K_{IC}	Plane Stress Fracture Toughness: the measure of the resistance of a material to crack extension under conditions of *crack-tip plane strain* (a stress-strain field, near the crack tip that approaches plane strain to the degree required by an empirical criterion).
n	number of repeat measurements

p	confidence level
P	applied force
q	random variable
$\bar{q}$	arithmetic mean of the values of the random variable q
s	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
u	standard uncertainty
u_c	combined standard uncertainty
U	expanded uncertainty
V	value of a measurand
W	effective width of test specimen
x_i	estimate of input quantity
y	test (or measurement) mean result

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty could be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be monitored more closely. This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated. The aim is to produce a series of documents in a common format that is easily understood and accessible to customers, test laboratories and accreditation authorities.

This *CoP* is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into six sections as follows, with all the individual *CoPs* included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials.

This *CoP* can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in

Reference 1. The individual *CoPs* are kept as simple as possible by following the same structure; viz:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)
- A worked example (Appendix B)

This *CoP* guides the user through the various steps to be carried out in order to estimate the uncertainty in the determination of the K_{IC} parameter.

4. A PROCEDURE FOR ESTIMATING THE UNCERTAINTY IN THE DETERMINATION OF PLANE STRAIN FRACTURE TOUGHNESS (K_{IC}), USING A COMPACT TENSION (CT) TEST SPECIMEN

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameter that is usually reported in the test. This measurand is not measured directly, but is determined from other quantities (or measurements).

Table 1. Measurands and Measurements, their units and symbols

Measurand	**Units**	**Symbol**
Plane Strain Fracture Toughness	$MPa\sqrt{m}$	K_{IC}
Measurements		
Thickness of the Specimen	*mm*	B
Width of the Specimen	*mm*	W
Crack Length	*mm*	a
Applied Force	*kN*	P_Q

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2 the user must identify all possible sources of uncertainty that may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (e.g. when a plotter is replaced by a computer). To help the user list all sources, five categories have been defined. The following table (Table 2) lists the five categories and gives some examples of sources of uncertainty in each category for the test method applied.

It is important to note that *Table 2* is not exhaustive and is for guidance only. Relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 2. Sources of Uncertainty, their Type and their likely contribution to Uncertainties on Measurands and Measurements
*(1 = major contribution, 2 = minor contribution, blank = no influence, * = indirectly affected)*

Sources of uncertainty		Measurand and Measurements				
	Type[1]	**K_{IC}**	**B**	**W**	**a**	**P_Q**
1. Apparatus						
Load Cell	B	*1				1
Extensometer	B	*2				2
Plotter X	B	*2				2
Plotter Y	B	*2				2
Caliper	B	*2	1	1	1	
Knife Edges Thickness [2]	B	*2				2
2. Method						
Alignment	B	*2				2
Speed	B	*2				2
Distance between Knife Edges [2]	B	*2				2
3. Environment						
Laboratory ambient temperature and humidity	B	*2				2
4. Operator						
Graph Interpretation	A	*1				1
Specimen Dimensions Measurement	A	*1	1	1		
Crack Length Measurement	A	*1			1	
5. Test Piece						
Specimen Thickness	B	*2	1			2
Specimen Width	B	*2		1		2

1 See Step 3
2 The specimen must be provided with a pair of accurately machined knife edges that support the gauge arms and serve as the displacement reference points

Step 3. Classifying the Uncertainty According to *Type A or B*

In this third step, which is in accordance with Reference 2, *'Guide to the Expression of Uncertainties in Measurement'*, the sources of uncertainty are classified as *Type A or B* depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations) it is classified *Type A*, if it is evaluated by any other means it should be classified *Type B* (see *Table 2*).

The values associated with *Type B* uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For *Type B* uncertainties, it is necessary for the users to estimate the most appropriate probability distribution for each source (further details are given in Section 2 of Reference 1).

It should be noted that, in some cases, an uncertainty could be classified as either *Type A* or *Type B* depending on how it is estimated.

Step 4. Estimating the standard Uncertainty for Each Source of Uncertainty

In this step the standard uncertainty, $u(x_i)$, for each measurement is estimated (see *Appendix A*). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter d_v, associated with the assumed probability distribution. The divisors for the distributions most likely to be encountered are given in Section 2 of Reference 1.

The significant sources of uncertainty and their influence on the evaluated quantity are summarised on *Table 3.*

Table 3. Example Worksheet for Uncertainty Calculations in K_{IC} Tests

Column No. ①	②	③	④	⑤	⑥	⑦	⑧
Sources of Uncertainty		**Measurements**		**Uncertainties**			
Source	**Value** (1) **or** (2)	**Measurement Affected**	**Nominal or Averaged Value (Units)**	**Type**	**Probabl. Distribt.**	**Divisor (d_v)**	**Effect on Uncertainty in Measurement**
Apparatus							
Load Cell		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(load cell)
Extensometer		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(extensom)
Plotter Y		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(plotterX)
Plotter X		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(PlotterY)
Knife Edges Thickness		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(knife edges)
Caliper		W B a	(mm) (mm) (mm)	B	Rectang.	$\sqrt{3}$	u(caliper)
Method							
Alignment		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(alignm)
Speed		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(speed)
Environment							
Room Temperature		P_Q	(kN)	B	Rectang.	$\sqrt{3}$	u(room temp)
Operator							

Graph Interpretation		P_Q	(kN)	A	normal	1	u(graph)
B Measurement		B	(mm)	A	normal	1	u(B msrment.)
W Measurement		W	(mm)	A	normal	1	u(W msrment.)
Crack Length Measurement		a	(mm)	A	normal	1	u(a msrment.)
Test Piece							
Specimen Thickness		B P_Q	(mm) (kN)	B	Rectang.	$\sqrt{3}$	u(thickness)
Specimen Width		W P_Q	(mm) (kN)	B	Rectang.	$\sqrt{3}$	u(width)

(1) permissible range for the measurand according to the test standard
(2) maximum range between measures made by several trained operators, on the same test

This table is structured in the following manner:

- column ①: sources of uncertainty
- column ②: source's value. There are two types:
  - (1) permissible range for the measurement according to the test standard
  - (2) maximum range between measures made by several trained operators on the same test
- column ③: measurements affected by each source
- column ④: measurement values obtained in a real test
- column ⑤: source of uncertainty type
- column ⑥: assumed probably distribution
- column ⑦: correction factor for *Type B* sources (d_v)
- column ⑧: measurand standard uncertainty produced by the input quantity uncertainty.
  This column is obtained by two different ways:
  - if the influence of the source of uncertainty on the measurand is proportionally direct.
    (column ②)· (column ④)/(column ⑦)
  - if the influence is not direct it should be obtained by calculating the measurand for both the maximum and minimum value of the range in column ② without variation in the rest of the sources of uncertainty and applying the appropriate correction factor for the probability (column ⑦).

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty of the measurand, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} [c_i \times u(x_i)]^2} \qquad (1)$$

where c_i is the sensitivity coefficient associated with the measurement x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.27%.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, ***U***, is defined in Reference 2 as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand". It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, ***k***, which is selected on the basis of the level of confidence. For a normal probability distribution, the most generally used coverage factor is 2, which corresponds to a confidence level interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty, ***U***, is, therefore, broader than the combined uncertainty, u_c. Where the customer (such as for Aerospace, Electronics...) demands a higher confidence level, a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in *Type A* analysis is small), the value of ***k*** should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference 1, Section 4 for more details).

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following format:

$$V = y \pm U$$

where:

where:	V	is the estimated value of the measurand
	y	is the test (or measurand) mean result
	U	is the expanded uncertainty associated with ***y***

An explanatory note, such as that given in the following example should be added (change when appropriate):

"The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k=2, which for a normal distribution corresponds to a coverage probability, ***p***, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 03:2000."

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0 946754 41 1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, *Guide to the expression of Uncertainty in Measurement.* International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. [This Guide is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that drafted it.]

3. British Standard, BS 7448. Part 1-97. “Fracture Mechanics Toughness Tests. Part 1. Method for determination of K_{IC}, critical *CTOD* and critical *J* values of Metallic Materials”.

4. ASTM E399-90 *“Plane-Strain Fracture Toughness of Metallic Materials”*

ACKNOWLEDGEMENTS

This document was written as part of project “Code of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials”. The project was partly funded by the Commission of European Communities through the Standards, Measurement and Testing Programme Contract No. SMT4-CT97-2165.

The author gratefully acknowledges the efforts of Mrs. Yeyes San Martín of INASMET in the development of this Code of Practice.

Many thanks are also due to many colleagues from UNCERT for invaluable helpful comments.

APPENDIX A

Mathematical Formulae for Calculating Uncertainties in Plane Strain Fracture Toughness (K_{IC}) Testing, using CT testpieces

The formula for the calculation of the *Plane Strain Fracture Toughness* parameter K_{IC} for a CT specimen is:

$$K_{IC} = \frac{P_Q}{B\sqrt{W}} f(\frac{a}{W}) \qquad \textbf{(A1)}$$

where f is a mathematical function of $\frac{a}{W}$, which for a CT specimen is:

$$f(\frac{a}{W}) = \frac{(2+\frac{a}{W})\left[0.886+4.64\frac{a}{W}-13.32(\frac{a}{W})^2+14.72(\frac{a}{W})^3-5.6(\frac{a}{W})^4\right]}{(1-\frac{a}{W})^{3/2}} \qquad \textbf{(A2)}$$

and P_Q is a load value obtained from the test load-displacement curve (*see Figure A1*). It is calculated by drawing a secant line through the origin of the test record with a slope of 95% of the slope of the tangent to the initial linear part of the record. This secant crosses the test curve at point P_d. The P_Q load is then defined as follows:

- if the load at every point on the record that precedes P_d is lower than P_d, then P_d is P_Q (see *Figure A1, III*)
- if, however, there is a maximum load preceding P_d which exceeds it, then this maximum load is P_Q (see *Figure A1, I and II*)

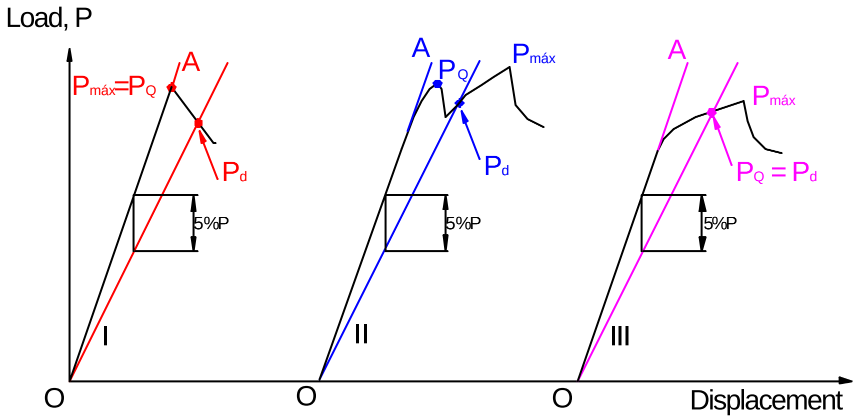

Figure A1. Definition of P_Q

The derived measurand $\boldsymbol{K_{IC}}$ ($\boldsymbol{y}$) is a function of four measurements $\boldsymbol{P_Q}$, $\boldsymbol{f}$, $\boldsymbol{B}$ and $\boldsymbol{W}$ (x_i), and each x_i is subject to uncertainty $\boldsymbol{u(x_i)}$. The general combined standard uncertainty $\boldsymbol{u_c(y)}$ is expressed by equation (1) in main procedure:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}[c_i \times u(x_i)]^2} \qquad \textbf{(A3)}$$

where
$$c_i = \frac{\partial y}{\partial x_i} \qquad \textbf{(A4)}$$

Using these formulae, it is possible to write the combined standard uncertainty of the $\boldsymbol{K_{IC}}$ parameter:

$$[u_c(K_{IC})]^2 = c_{P_Q} \times u(P_Q)^2 + c_f \times u(f)^2 + c_B \times u(B)^2 + c_W \times u(W)^2 \qquad \textbf{(A5)}$$

The partial derivatives for equation **A5** are given by:

$$c_{F_Q} = \frac{\partial K_{IC}}{\partial P_Q} = \frac{f}{B \times W^{1/2}} \qquad \textbf{(A6)}$$

$$c_f = \frac{\partial K_{IC}}{\partial f} = \frac{P_Q}{B \times W^{1/2}} \qquad \textbf{(A7)}$$

$$c_B = \frac{\partial K_{IC}}{\partial B} = \frac{-f \cdot P_Q}{B^2 \times W^{1/2}} \qquad \textbf{(A8)}$$

$$c_W = \frac{\partial K_{IC}}{\partial W} = \frac{-f \cdot P_Q}{2 \times B \times W^{3/2}} \qquad \textbf{(A9)}$$

A5 equation is composed by four terms that will be analysed in next paragraphs.

A.1 UNCERTAINTY IN $\boldsymbol{P_Q}$ ($u(P_Q)$)

$\boldsymbol{P_Q}$ is a measurement affected by different sources of uncertainty (see *Table 2* and *Table 3* in the main procedure). Assuming that the individual uncertainty sources are uncorrelated, the $\boldsymbol{P_Q}$ combined uncertainty can be computed using the root sum squares:

$$u_{PQ} = \sqrt{u(s_i)^2} = \sqrt{u(\textit{load cell})^2 + u(\text{graph interpretation})^2 + u(\text{thickness})^2 + u(\text{width})^2} \qquad \textbf{(A10)}$$

where $\boldsymbol{u(s_i)}$ is the effect of each source on the uncertainty in $\boldsymbol{P_Q}$.

Terms ***u(s$_i$)*** can be estimated using *Table 3* in this procedure.

A.2 UNCERTAINTY IN *f* ($u(f)$)

f is a mathematical function of ***a/W*** (equation **A2**).

Assuming that this source of uncertainty (***f***) is considered as *Type A*, the contribution to the total uncertainty can be calculated from the standard deviation of the arithmetic mean:

$$u(f) = s(\bar{f}) = \frac{s(f_j)}{\sqrt{n}} \quad \textbf{(A11)}$$

Where:

$$s(f_j) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(f_j - \bar{f})^2} \quad \textbf{(A12)}$$

Using the maximum and the minimum values to calculate *s(f$_j$)*, (n=2):

$$u(f) = \frac{\sqrt{\frac{1}{2-1}\left[(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2\right]}}{\sqrt{2}} = \sqrt{\frac{(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2}{2}} \quad \textbf{(A13)}$$

Where:

$$f_{max} = f\left(\frac{a_{max}}{W_{min}}\right) \quad and \quad f_{min} = f\left(\frac{a_{min}}{W_{max}}\right) \quad \textbf{(A14)}$$

and

$$\begin{aligned} a_{max} &= a_0 + 2 \times u(a) \\ a_{min} &= a_0 - 2 \times u(a) \\ W_{max} &= W_0 + 2 \times u(W) \\ W_{min} &= W_0 - 2 \times u(W) \end{aligned} \quad \textbf{(A15)}$$

(assuming that both ***a*** and ***W*** have a normal probability distribution, k=2)

A.2.1 Uncertainty in a ($u(a)$)

a is a measurement affected by two sources of uncertainty (see *Table 2* and *Table 3*). Assuming that the individual uncertainty sources are uncorrelated, the combined uncertainty in ***a*** can be computed using the root sum squares:

$$u(a) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(crack\ length\ measurement)^2} \quad \textbf{(A16)}$$

Terms ***u(s$_i$)*** can then be calculated using Table 3 in this procedure.

A.3 UNCERTAINTY IN *B* ($u(B)$)

B is a measurement affected by three sources of uncertainty (see *Table 2* and *Table 3* in the procedure). Assuming that the individual uncertainty sources are uncorrelated, the combined uncertainty in ***B*** can be computed using the root sum squares:

$$u(B) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(B\ measurement)^2 + u(thickness)^2} \quad \textbf{(A17)}$$

Terms ***u(s$_i$)*** can then be calculated using Table 3 in this procedure.

A.4 UNCERTAINTY IN *W* ($u(W)$)

W is a measurement affected by three sources of uncertainty (see *Table 2* and *Table 3*). Assuming that the individual uncertainty sources are uncorrelated, the combined uncertainty in ***W*** can be computed using the root sum squares:

$$u(W) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(W\ measurement)^2 + u(width)^2} \quad \textbf{(A18)}$$

The combined uncertainty of each measurement is shown in table A1.

Table A1. Formulae for calculating Combined Uncertainties

Measurement	Sources of uncertainty (s_i)	$u(x_i)$ (Units)	Uncertainty of Measurements $u(x_i)$
Applied Force P_Q	Load cell	(kN)	$u(P_Q) = \sqrt{u(load\,cell)^2 + u(graph\,interp)^2 + u(thickness)^2 + u(width)^2}$
	Graph Interpretation		
	Specimen Thickness		
	Specimen Width		
Crack Length *a*	Caliper	(mm)	$u(a) = \sqrt{u(caliper)^2 + u(crack\ length\ measurement)^2}$
	Crack Length Measurement		
Specimen Width *W*	Caliper	(mm)	$u(W) = \sqrt{u(caliper)^2 + u(W\ measurement)^2 + u(width)^2}$
	W measurement		
	Specimen Width		
Specimen Thickness *B*	Caliper	(mm)	$u(B) = \sqrt{u(caliper)^2 + u(B\,measurement)^2 + u(thickness)^2}$
	B measurement		
	Specimen Thickness		

APPENDIX B

A Worked Example for Calculating Uncertainties in Plane Strain Fracture Toughness (K_{IC}) Testing

B1. INTRODUCTION

A customer asked a testing laboratory to carry out a fracture toughness test to determine the plane strain fracture toughness (K_{IC}) at room temperature on compact tension test pieces, according to British Standard, BS 7448. Part 1-97. The laboratory has considered the sources of uncertainty in its test facility, and found that the sources of uncertainty in the test results are identical to those described in Table 2 of the Main Procedure.

B2. ESTIMATION OF INPUT QUANTITIES TO THE UNCERTAINTY ANALYSIS

1 All tests were carried out according to the laboratory's own procedure using an appropriately calibrated tensile test facility. The test facility was located in a temperature-controlled environment, at room temperature.

2 The specimen was a compact tension type, and its dimensions were:
thickness B = 30 mm ± 0.5%
width W = 60 mm ± 0.5%
The dimensions of the specimen were measured using a caliper with an uncertainty of *0.05 mm*, typical for calipers used in the laboratory.

3 The crack length (***a***) was measured with the same caliper and the value obtained was 30.38 mm.

4 The test was carried out on a universal test machine using a strain rate of $0.5\ kNs^{-1}$. (A constant loading rate of $0.34 – 1.7\ kNs^{-1}$ for a standard compact specimen corresponds to a rate of increase in stress intensity factor within the range $0.55 – 2.75\ MPa\sqrt{m}\ s^{-1}$, according to ASTM E 399 Standard).

5 The machine was calibrated to Grade 1 of BS 1610.

6 Strain was measured using a clip gauge extensometer with a nominal gauge length of *10 mm*. The extensometer complied with *Class 0.5*, specification according to EN 10002-4:1994.

7 The thickness of the knife-edges attached to the specimen were 1.5 mm (*BS 7448 Clause 5.1.3* permits a thickness of *1.5 - 2 mm*) and were attached at a distance of *10 mm*.

8 The accuracy of the plotter used to record the load-displacement curve was within ± 0.5% in both axes.

The curve obtained in this test is presented in *Figure B1*. The maximum load value recorded by the machine P_{max} was *85 kN*. The value indicated in *Figure B1* as P_Q was obtained graphically by the operator (P_Q=*72.5* k*N*), according to the procedure described in Step 2 of this CoP.

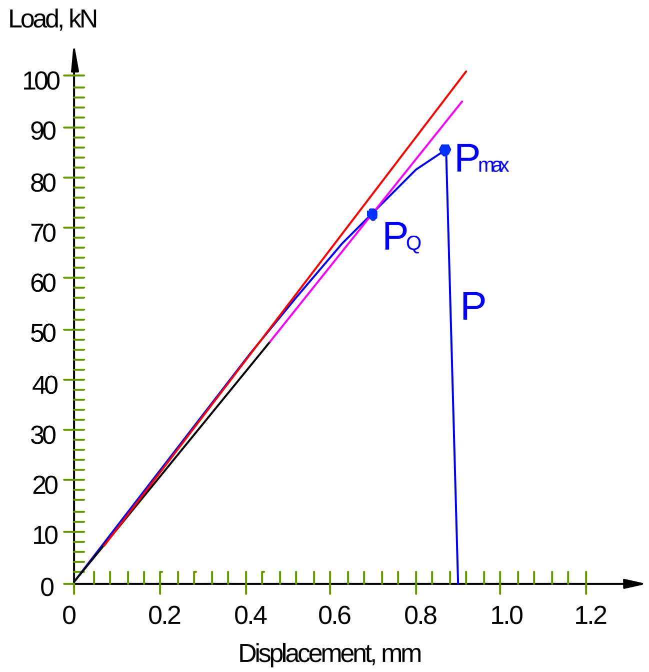

Figure B1. Test Record

9 An indication of the uncertainty associated with interpreting the graph was obtained using another test record (*Figure B2*). Four trained operators analysed the record and obtained the values indicated in Table B1.

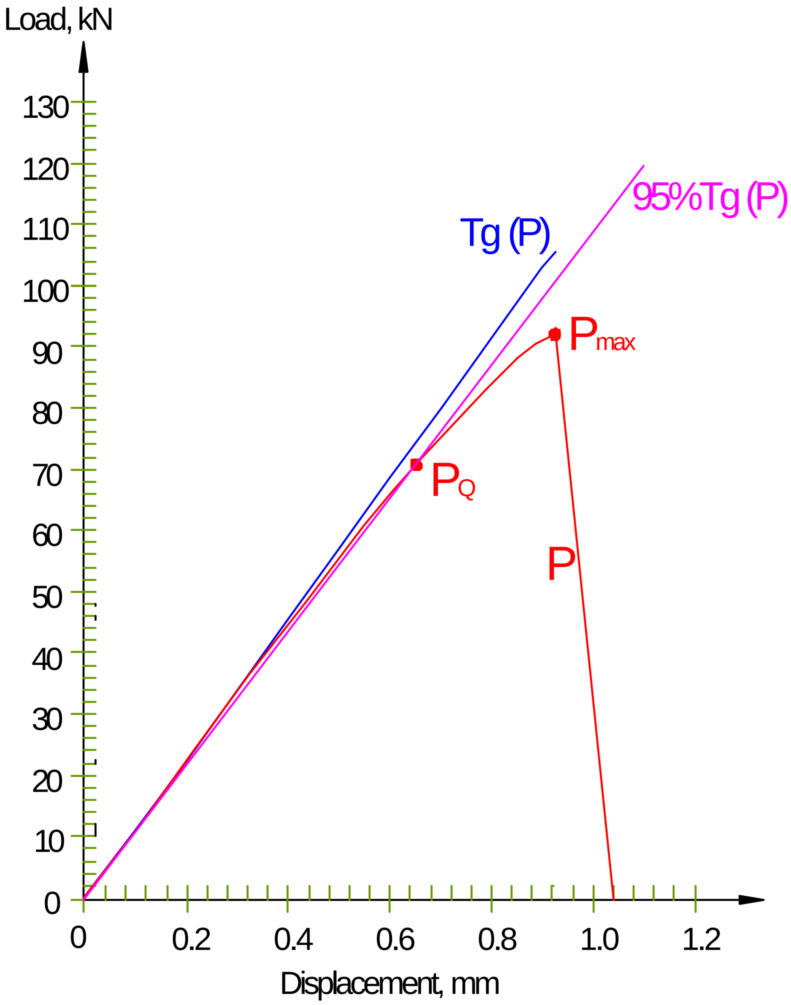

Figure B2. Uncertainty in Interpreting the Graph

Table B1 Uncertainty in Interpreting the Graph

	P_Q (kN)
Operator 1	66.5
Operator 2	70.25
Operator 3	70
Operator 4	69
Mean	68.94

The uncertainty for this input value is its standard deviation:

$$u = \sqrt{\frac{1}{n-1}\sum_{j=1}^{4}(P_j - \bar{P})^2} =$$

$$= \sqrt{\frac{1}{3}[(66.5 - 68.94)^2 + (70.25 - 68.94)^2 + (70 - 68.94)^2 + (69 - 68.94)^2]} =$$

$$= 1.712\ kN$$

10 The uncertainties associated with the error in measurements (***B***, ***W*** and ***a***) were obtained in the same manner as above (using a compact tension test specimen with the same nominal dimensions as the one used in this example, ***B*** = *30 mm* and **W** = 60 mm)

Table B2. Uncertainty in Dimensions Measurement

	B (mm)	**W (mm)**	**a (mm)**
Operator 1	29.95	60.00	30.12
Operator 2	30.10	59.90	30.18
Operator 3	30.05	59.95	30.20
Operator 4	29.97	60.15	30.22
Mean	30.018	60.000	30.180
Standard Deviation	0.070	0.108	0.043
Uncertainty	0.233 %	0.180 %	0.143 %

(*) $\text{uncertainty}(\%) = \frac{s(q_i) \cdot 100}{mean}$

The highest uncertainty associated with all these measurements was in the thickness, so it has been selected for both ***B***, ***W*** and ***a*** an uncertainty of ***0.23%***.

11 The influence of the knife-edge thickness was studied using a *Computer Assistant Design Program*, it proved to be negligible.

B3 EXAMPLE OF UNCERTAINTY CALCULATIONS AND REPORTING OF RESULTS

B3.1 Calculations

Firstly, *Table 3* in main procedure should be completed.

Column Nb. ①	②	③	④	⑤	⑥	⑦	⑧
Sources of Uncertainty		**Measurements**		**Uncertainties**			
Source	**Value** (1) **or** (2)	**Measurement Affected**	**Nominal or Averaged Value (Units)**	**Type**	**Probabl. Distribt.**	**Divisor (d_v)**	**Effect on Uncertainty in Measurement**
Apparatus							
Load Cell	1% (1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	0.419 kN 1)
Extensometer	0.5%(1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Plotter Y	0.5%(1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Plotter X	0.5%(1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Knife Edges Thickness	1.5 to 2 mm(1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Caliper	0.05 mm(1)	W B a	60 (mm) 30 (mm) 30.38 (mm)	B	Rectang.	$\sqrt{3}$	0.029 mm 2)
Method							
Alignment	-	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Speed	0.3-1.5kN/s	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Environment							
Room Temperature	2 ºC (1)	P_Q	72.5 (kN)	B	Rectang.	$\sqrt{3}$	nglg.
Operator							
Graph Interpretation	2.49 % (2)	P_Q	72.5 (kN)	A	normal	1	1.805 kN 3)
Thickness Measurement	0.23 % (2)	B	30 (mm)	A	normal	1	0.069 mm 4)
Width Measurement	0.23 % (2)	W	60 (mm)	A	normal	1	0.138 mm 5)
Crack Length Measurement	0.23 % (2)	a	30.38 (mm)	A	normal	1	0.070 mm 6)
Test Piece							
Specimen Thickness	0.5 % (1)	B P_Q	30 (mm) 72.5 (kN)	B	Rectang.	$\sqrt{3}$	0.087 mm 7) 0.209 kN 8)
Specimen Width	0.5 % (1)	W P_Q	30 (mm) 72.5 (kN)	B	Rectang.	$\sqrt{3}$	0.173mm 9) 0.209 kN 10)

(1) permissible range for the measurand according to the test standard
(2) maximum range between measures made by several trained operators, on the same test

Column ②:

- ❖ Load Cell: according to the BS 7448, the force sensing device shall comply with grade 1 of BS 1610 (accuracy within $\pm 1\%$)
- ❖ Extensometer and Plotter: our extensometers and plotter comply with Grade 0.5 (accuracy within 0.5%), so the error due to the operator in the calculation of $\boldsymbol{P_Q}$ from the test record minimises the possible uncertainty caused by plotter and extensometer devices.
- ❖ Caliper: generally uncertainty of 0.05 mm is typical for calipers used to measure both test piece dimensions and crack length
- ❖ Knife Edges Thickness: according to the test standard, knife edges thickness must be between *1.5 and 2 mm*, but the error due to the operator in the calculation of $\boldsymbol{P_Q}$ from the test record minimises the possible uncertainty caused by knife edges thickness.
- ❖ Alignment: assuming that test specimen geometry is according to test standard, alignment uncertainty is negligible
- ❖ Room Temperature: this influence is considered negligible for metallic materials because only small variations $\pm 2\,°C$ are allowed by the test standard and its effect can be ignored
- ❖ Graph Interpretation: this is the greatest contribution to total uncertainty because of the manual method used to obtain the $\boldsymbol{P_Q}$ value from the test record
- ❖ Specimen Thickness: according to the test standard, the tolerance for the specimen thickness is ± 0.5%
- ❖ Specimen Width: according to the test standard, the tolerance for the specimen width is ± 0.5%

Column ④: are the values obtained in the real test

Column ⑧: 1), 3), 4), 5), 6), 7) and 9)

(column ⑧) = (column ②)· (column④)/(column⑦)

$$0.419 = 0.01 \times 72.5/\sqrt{3}$$
$$1.805 = 0.0249 \times 72.5/1$$
$$0.069 = 0.0023 \times 30/1$$
$$0.138 = 0.0023 \times 60/1$$
$$0.087 = 0.005 \times 30/\sqrt{3}$$
$$0.173 = 0.005 \times 60/\sqrt{3}$$

2) (column ⑧) = (column ②)/(column⑦)

$$0.029 = 0.05/\sqrt{3}$$

8) and 10):

assuming that the tensile strength of the specimen is a material characteristic, a variation in one dimension of the test specimen would produce a proportional variation on the applied force

$$P_Q(B = 30) = 72.5\ kN$$

$$Tensile\ Strength = \frac{72500}{30 \times 60} = 40.28\ MPa \rightarrow material\ characteristic$$

$$so \qquad P_Q(B = 30 \pm 0.5\%) = 72.5 \pm 0.5\%\ kN$$

It means that:

$$u(specimen\ thickness) = u(specimen\ width) =$$
$$= 0.005 \times 72.5/\sqrt{3} = 0.209 \text{ kN}$$

Filling in *Table A1* in *Annex A*:

Measurement	**Sources of uncertainty**	**u_i**	**Uncertainty of Measurements $u(x_i)$**
P_Q	Load cell	0.419 kN	$u_c(P_Q) = \sqrt{(0.419)^2 + (1.805)^2 + (0.209)^2 + (0.209)^2} = 1.877\ kN$ (A11)
	Graph Interpretation	1.805 kN	
	Specimen Thickness	0.209 kN	
	Specimen Width	0.209 kN	
a	Caliper	0.029 mm	$u(a) = \sqrt{(0.029)^2 + (0.069)^2} = 0.076\ mm$ (A16)
	Crack Length Measurement	0.069 mm	
W	Caliper	0.029 mm	$u(W) = \sqrt{(0.029)^2 + (0.138)^2 + (0.173)^2} = 0.223\ mm$ (A17)
	W Measurement	0.138 mm	
	Specimen Width	0.173 mm	
B	Caliper	0.029 mm	$u(B) = \sqrt{(0.029)^2 + (0.069)^2 + (0.087)^2} = 0.114\ mm$ (A18)
	B Measurement	0.069 mm	
	Specimen Thickness	0.087 mm	

To calculate the uncertainty of ***f*** factor, equations **A13** to **A15** are applied:

$$\begin{aligned} a_{max} &= a_0 + 2\times u(a) = 30.38 + 2\times 0.076 = 30.531 \\ a_{min} &= a_0 - 2\times u(a) = 30.38 - 2\times 0.076 = 30.229 \\ W_{max} &= W_0 + 2\times u(W) = 60 + 2\times 0.223 = 60.447 \\ W_{min} &= W_0 - 2\times u(W) = 60 - 2\times 0.223 = 59.553 \end{aligned} \qquad \textbf{(A15)}$$

then

$$\begin{aligned} \bar{f} &= f\left(\frac{a}{W}\right) = f\left(\frac{30.38}{60}\right) = 9.850 \\ f_{max} &= f\left(\frac{a_{max}}{W_{min}}\right) = f\left(\frac{30.531}{59.553}\right) = 10.048 \\ f_{min} &= f\left(\frac{a_{min}}{W_{max}}\right) = f\left(\frac{30.229}{60.447}\right) = 9.662 \end{aligned} \qquad \textbf{(A14)}$$

$$u(f) = \sqrt{\frac{(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2}{2}} = \sqrt{\frac{(10.048 - 9.850)^2 + (9.662 - 9.850)^2}{2}} = 0.193 \qquad \textbf{(A13)}$$

Now the partial derivatives (equations **A6** to **A9**) are obtained:

$$c_{P_Q} = \frac{\partial K_{IC}}{\partial P_Q} = \frac{f}{B\times W^{1/2}} = \frac{9.850}{30\sqrt{60}} = 0.042\ mm^{-3/2} \qquad \textbf{(A6)}$$

$$c_f = \frac{\partial K_{IC}}{\partial f} = \frac{P_Q}{B\cdot W^{1/2}} = \frac{72.5}{30\sqrt{60}} = 0.312\ kN\,mm^{-3/2} \qquad \textbf{(A7)}$$

$$c_B = \frac{\partial K_{IC}}{\partial B} = \frac{-f\times P_Q}{B^2\times W^{1/2}} = \frac{-9.850\times 72.5}{30^2\sqrt{60}} = -0.102\ kN\ mm^{-5/2} \qquad \textbf{(A8)}$$

$$c_W = \frac{\partial K_{IC}}{\partial W} = \frac{-f\times P_Q}{2\times B\times W^{3/2}} = \frac{-9.850\times 72.5}{2\times 30\times 60^{3/2}} = -0.026\ kN\,mm^{-5/2} \qquad \textbf{(A9)}$$

and the combined standard uncertainty for K_{IC} is (equation **A5**):

$$\begin{aligned} u_c(K_{IC})^2 &= c_{P_Q}{}^2\times u(P_Q)^2 + c_f{}^2\times u(f)^2 + c_B{}^2\times u(B)^2 + c_W{}^2\times u(W)^2 = \\ &= 0.042^2\times 1.877^2 + 0.312^2\times 0.193^2 + (-0.102)^2\times 0.114^2 + (-0.026)^2\times 0.223^2 \end{aligned}$$

$$u_c(K_{IC})^2 = 0.010\ kN^2\ mm^{-3}$$

$$u_c(K_{IC}) = \sqrt{0.010}\ kN\ mm^{-3/2} = 0.101 \frac{10^3 N}{mm\sqrt{mm}} \frac{\sqrt{m}}{\sqrt{10^3 mm}} = 0.101\sqrt{10^3}\ \frac{N}{mm^2}\sqrt{m} =$$

$$= 3.184\, MPa\sqrt{m}$$

The expanded uncertainty with a confidence interval of 95.4% is obtained multiplying the combined standard uncertainty by a coverage factor of 2:

$$U(K_{IC}) = 3.184 \times 2 = 6.368\ \ MPa\sqrt{m}$$

The value of $\boldsymbol{K_{IC}}$ is calculated with equation A1:

$$K_{IC} = \frac{P_Q}{B\sqrt{W}}\, f(\frac{a}{W}) = \frac{0.0725}{0.030\sqrt{0.060}} \times 9.850 = 97.18\, MPa\sqrt{m}$$

B3.2 Reported Results

The Plane Strain Fracture Toughness parameter $\boldsymbol{K_{IC}}$ is $97.18 \pm 6.368\ \ MPa\sqrt{m}$

The above reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k=2, which for a normal distribution corresponds to a coverage probability, p, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 03:2000.