Nb-95 and Zr-95
Sr-89 and Sr-90

NPL Environmental Radioactivity Comparison Workshop

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Nb-95 results GL

Deviation Nb-95 GL

Deviation (%)

Lab 10 - Lab 61
Nb-95 results GH

Deviation Nb-95 GH

[Graph showing deviation (%) for various labs, with deviations ranging from -150 to 150.]
Nb-95 and Zr-95 decay

Zr-95 decays to Nb-95 or Nb-95m \((p = 0.0118)\)

[Nb-95m decays to Nb-95 \((q = 0.976)\) or Mo-95]

Nb-95 decays to Mo-95

**Half-lives**

1. Zr-95 \(64.032(6)\) d
2. Nb-95m \(3.61(3)\) d
3. Nb-95 \(34.991(6)\) d

The Nb-95 activity may be a function of the Zr-95 activity
Transient equilibrium

\[
\frac{A_3(t)}{A_1(t)} = \frac{(1-p)\lambda_3}{(\lambda_3 - \lambda_1)} \left(1 - e^{(\lambda_1 - \lambda_3)t}\right) + \frac{q p \lambda_2 \lambda_3}{(\lambda_2 - \lambda_1)} \left[\frac{1 - e^{(\lambda_1 - \lambda_2)t}}{(\lambda_3 - \lambda_1)} - \frac{e^{(\lambda_1 - \lambda_2)t} - e^{(\lambda_1 - \lambda_3)t}}{(\lambda_3 - \lambda_2)}\right]
\]

for \( t \to \infty \), and \( \lambda_1 < \lambda_3 < \lambda_2 \) then

\[
\frac{A_3(\infty)}{A_1(\infty)} = \frac{(1-p)\lambda_3}{(\lambda_3 - \lambda_1)} + \frac{q p \lambda_2 \lambda_3}{(\lambda_2 - \lambda_1)} \left[\frac{1}{(\lambda_3 - \lambda_1)}\right] = \frac{\lambda_3}{(\lambda_3 - \lambda_1)} \left[1 - p + \frac{q p \lambda_2}{(\lambda_2 - \lambda_1)}\right] = 2.206
\]

This equation reduces to a transient equilibrium equation by setting \( p = 0 \) and \( q = 1 \)

\[
\frac{A_3(\infty)}{A_1(\infty)} = \frac{\lambda_3}{(\lambda_3 - \lambda_1)} = 2.205
\]
$^{95}$Zr, $^{95m}$Nb and $^{95}$Nb activity

Zr-95, Nb-95m and Nb-95 decay

<table>
<thead>
<tr>
<th>Time (d)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td>100</td>
<td>0.2</td>
</tr>
<tr>
<td>150</td>
<td>0.3</td>
</tr>
<tr>
<td>200</td>
<td>0.4</td>
</tr>
<tr>
<td>250</td>
<td>0.5</td>
</tr>
<tr>
<td>300</td>
<td>0.6</td>
</tr>
<tr>
<td>350</td>
<td>0.7</td>
</tr>
<tr>
<td>400</td>
<td>0.8</td>
</tr>
<tr>
<td>450</td>
<td>0.9</td>
</tr>
<tr>
<td>500</td>
<td>1.0</td>
</tr>
</tbody>
</table>

- Zr-95: Yellow
- Nb-95: Blue
- Total: Red
- Nb-95/Zr-95 Ratio: Black
- Nb-95m: Cyan
- Reference time: Dashed line

NPL
How to decay-correct for Nb-95?

Three possible cases:

- Nb-95 and Zr-95 are in equilibrium (ratio = 2.21)
  
  Also assume equilibrium after correction and use the half-life value of Zr-95

- Ratio is found to be higher than 2.21
  
  Subtract the equilibrium part and use the half-life values of Nb-95 and Zr-95

- Ratio is found to be lower than 2.21 (this exercise: 2.09)
  
  Use the following formulae;
\[
\frac{A_3(t)}{A_1(t)} = \frac{\lambda_3}{(\lambda_3 - \lambda_1)} \left(1 - e^{(\lambda_1 - \lambda_3)t}\right)
\]

Rearrange to calculate \( t_{stm} \):

\[
t_{stm} = \frac{\ln \left[1 - \frac{A_3(t_{stm}) (\lambda_3 - \lambda_1)}{A_1(t_{stm}) \lambda_3}\right]}{(\lambda_1 - \lambda_3)}
\]

Calculate \( t_{ref} \) and \( A_1(t_{ref}) \) and use:

\[
A_3(t_{ref}) = \frac{\lambda_3}{(\lambda_3 - \lambda_1)} \left(1 - e^{(\lambda_1 - \lambda_3)t_{ref}}\right) A_1(t_{ref})
\]
Example

NPL value Nb-95 / Zr-95 ratio at 1 October 2005: 2.09(2)

Measurement on 1 December 2005: 2.14(2)

Decay correction for Nb-95: $1.935 \times \left(\frac{2.09}{2.14}\right) = 1.89$

Nb-95 overestimation:

- Transient Nb-95 / Zr-95 equilibrium (i.e., Zr-95 decay correction) 2.4%
- Secular equilibrium (1:1) with Nb-95 decay correction 42%
- Simple Nb-95 decay correction 77%
Simplified case, because it:

- Ignores Nb-95m

- Ignores ingrowth and decay during measurement time
Ingrowth / decay during measurement (Desmond MacMahon)

\[ A_3(t_{\text{ref}}) = \frac{\lambda_3 C_3 (t_{em} - t_{stm})}{(e^{-\lambda_3 t_{stm}} - e^{-\lambda_3 t_{em}})} - \frac{\lambda_3 R_1}{\lambda_1} \left( e^{-\lambda_1 t_{stm}} - e^{-\lambda_1 t_{em}} \right) - \frac{\lambda_3 R_2}{\lambda_2} \left( e^{-\lambda_2 t_{stm}} - e^{-\lambda_2 t_{em}} \right) \\
\]

\[ + (R_1 + R_2) \]

Where:

\[ R_1 = \frac{\lambda_3 A_1(t_{\text{ref}})}{\lambda_3 - \lambda_1} \left[ 1 - p + \frac{q \lambda_2}{\lambda_2 - \lambda_1} \right] \]

and

\[ R_2 = \frac{q \lambda_3}{\lambda_3 - \lambda_2} \left[ A_2(t_{\text{ref}}) - \frac{p \lambda_2 A_1(t_{\text{ref}})}{\lambda_2 - \lambda_1} \right] \]
Sr-90 results ABL

Deviation Sr-90 ABL

Deviation (%)

Lab 56  Lab 35  Lab 62  Lab 25  Lab 22-3M  Lab 22-Ech  Lab 8  Lab 28  Lab 29  Lab 21  Lab 10  Lab 13  Lab 7
Sr-90 results ABH

Deviation Sr-90 ABH

Deviation (%)

Lab 48  Lab 22 Ech  Lab 22 3M  Lab 55  Lab 32  Lab 80  Lab 28  Lab 36  Lab 14  Lab 8  Lab 59  Lab 7
Sr-90 and Sr-89

Both high-yield beta emitting fission products

Practically impossible to separate chemically

<table>
<thead>
<tr>
<th></th>
<th>half-live (d)</th>
<th>$E_{\text{max}}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr-89</td>
<td>50.57(3)</td>
<td>1495.1(22)</td>
</tr>
<tr>
<td>Sr-90</td>
<td>10551(14)</td>
<td>545.9(14)</td>
</tr>
<tr>
<td>Y-90</td>
<td>2.6684(13)</td>
<td>2279.8(17)</td>
</tr>
</tbody>
</table>
Sr-89, Sr-90 and Y-90 as a function of time

Sr-89 ($A_0=1$), Sr-90 ($A_0=1$) and Y-90 ($A_0=0$) activity

![Graph showing Sr-89, Sr-90, and Y-90 activity over time](image)

- **Sr-89** (Total) activity increases over time, reaching a peak and then stabilizing.
- **Sr-90** activity is similar toSr-89 but at a lower level.
- **Y-90** activity decreases rapidly over time and then stabilizes at a lower level than Sr-89 and Sr-90.

**Time (d):** 0, 5, 10, 15, 20, 25, 30

**Activity:** 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0

**Legend:**
- Total
- Sr-89
- Sr-90
- Y-90
What to do?


(ii) Separate Y-90 from Sr and note time. Count several times within the first 15 d.

(iii) Combination of Cerenkov counting and LSC. Separate Y-90 from Sr. Determine Sr-89 (Cerenkov). Determine Sr-89 and Sr-90 (LSC). Subtract Sr-89 from Sr-90.

(iv) Spectrum deconvolution.
Thank you.