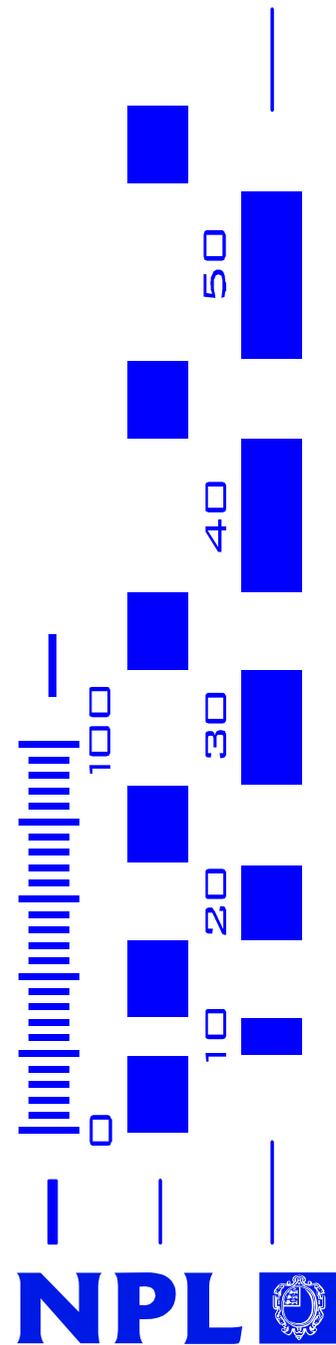


Environmental Radioactivity Comparison  
Workshop, 2003

Limits of Detection

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# What's the problem?

Is it above background?

Can it be quantified?

# Where we're at

**'Currie solution' widely used since 1968**

- **Lower backgrounds**
- **More computing power**
- **Customer pressure for lower and lower detection limits**
- **Low backgrounds (<100 counts are problematic)**

# Definitions

**Critical Limit,  $L_c$**  - the level above which an error is made if it is stated there is no detection;

$$L_c = k_c \cdot \delta b$$

**Detection Limit,  $L_d$**  - the level below which an error is made if it is assumed there is detection;

$$L_d = L_c + k_d \cdot \delta b$$

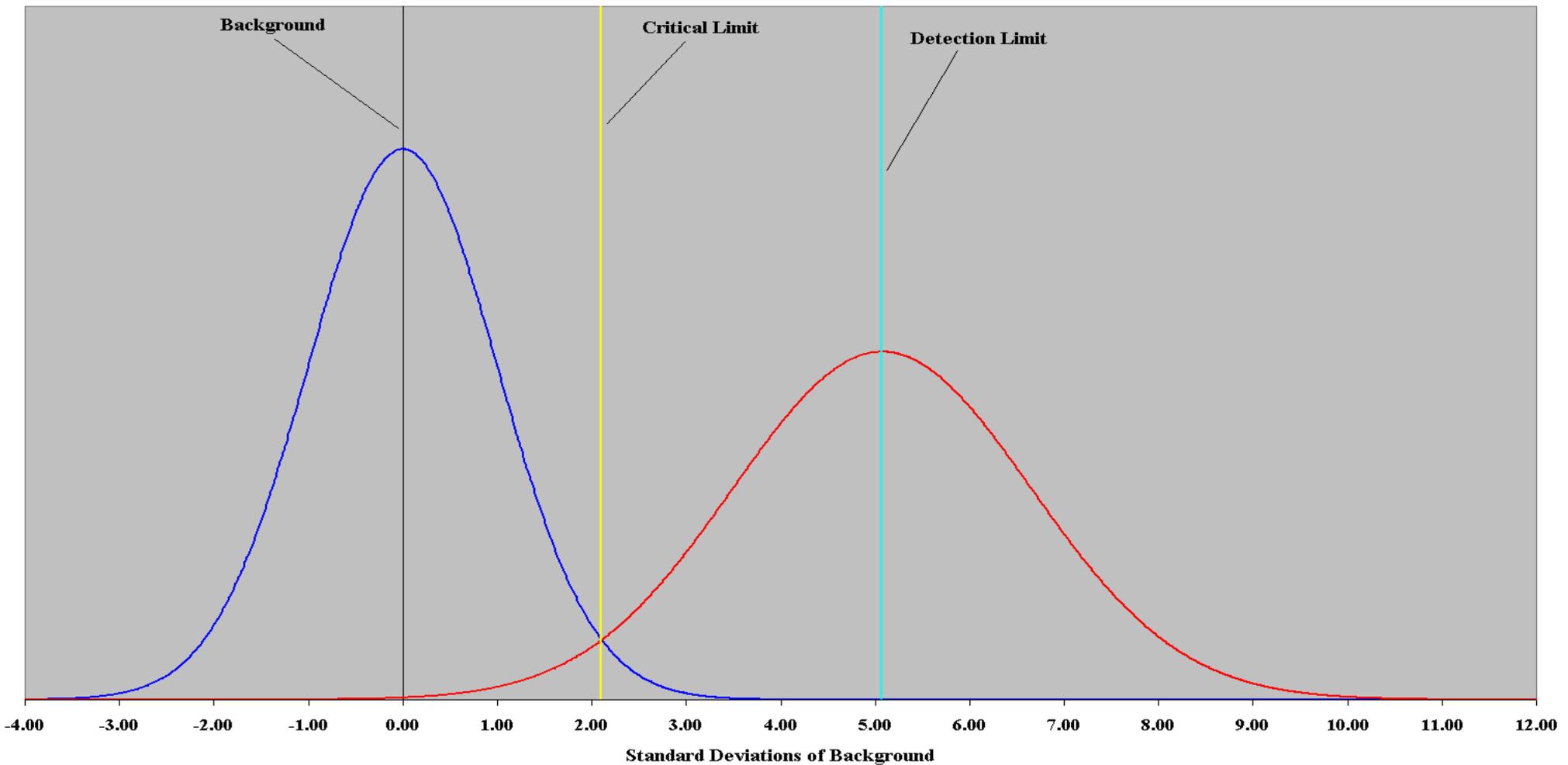
**Quantification Limit,  $L_q$**  - the level above which the signal is quantifiable;

$$L_q = k \cdot \delta V$$

{Where  $\delta b$  is the background uncertainty ( $= \sqrt{b}$  for 'paired' observations),  $\delta V$  is the overall measurement uncertainty,  $k_c$ ,  $k_d$  are set to 'reasonable' risk levels (1.645 if 5% and 1.690 if 4.6%) and  $k$  ( $=2$ ) is the measurement uncertainty coverage factor}

# Graphically.....

Critical and Detection Limits



# Small Numbers of Counts

With small (<100) numbers of counts, assuming a normal distribution of data may not be valid, so uncertainty on  $x$  counts is can be evaluated as:

$$\sqrt{(x+1)}$$

(rather than  $\sqrt{(x)}$ )

However, as  $x \rightarrow 0$ ,  $\delta x \rightarrow$  confusion! Need to know background well –  
ie repeat measurements and is it consistent over time?

# Small Numbers of Counts

## Comment from Lloyd Currie:

‘The extreme case (few counts) is important, difficult and controversial. One of the difficulties is that the distribution of the difference between two Poisson variables is no longer Poisson - unlike the difference between two Normal variables (i.e., large numbers of counts)...’

(See also *IEEE Transactions on Nuclear Science*, v. NS-19, (Feb. 1972) pp 119-126)

## One approach....

Probability of not exceeding a given value, M is given by;

$$P = \frac{[n(nM)^{(m_1+m_2+m_3+\dots+m_n)}e^{(-nM)}]}{(m_1+m_2+m_3\dots m_n)}$$

For a single background observation resulting in zero count (ie  $m=0$ ,  $n=1$ ), then we can say we are 99% certain that  $M < 4.6$

If this is repeated 10 times (ie  $m=0$ ,  $n=10$ ), then we can say we are 99% certain that  $M < 0.46$

## And Another...

Evaluate the background by  $n$  repeated measurements, in which case;

$$\delta b = t \cdot \sigma_b$$

(where  $t$  is the appropriate value for the number of observations)

Probably the easier approach.

# Quantification Limit

Given by solving this expression:

$$S_{\text{net}} \text{ (ie } S-B) > k\sqrt{(S+B+2)}$$

(where **S** is the gross sample count, **B** is the gross background count and **k** is the coverage factor)

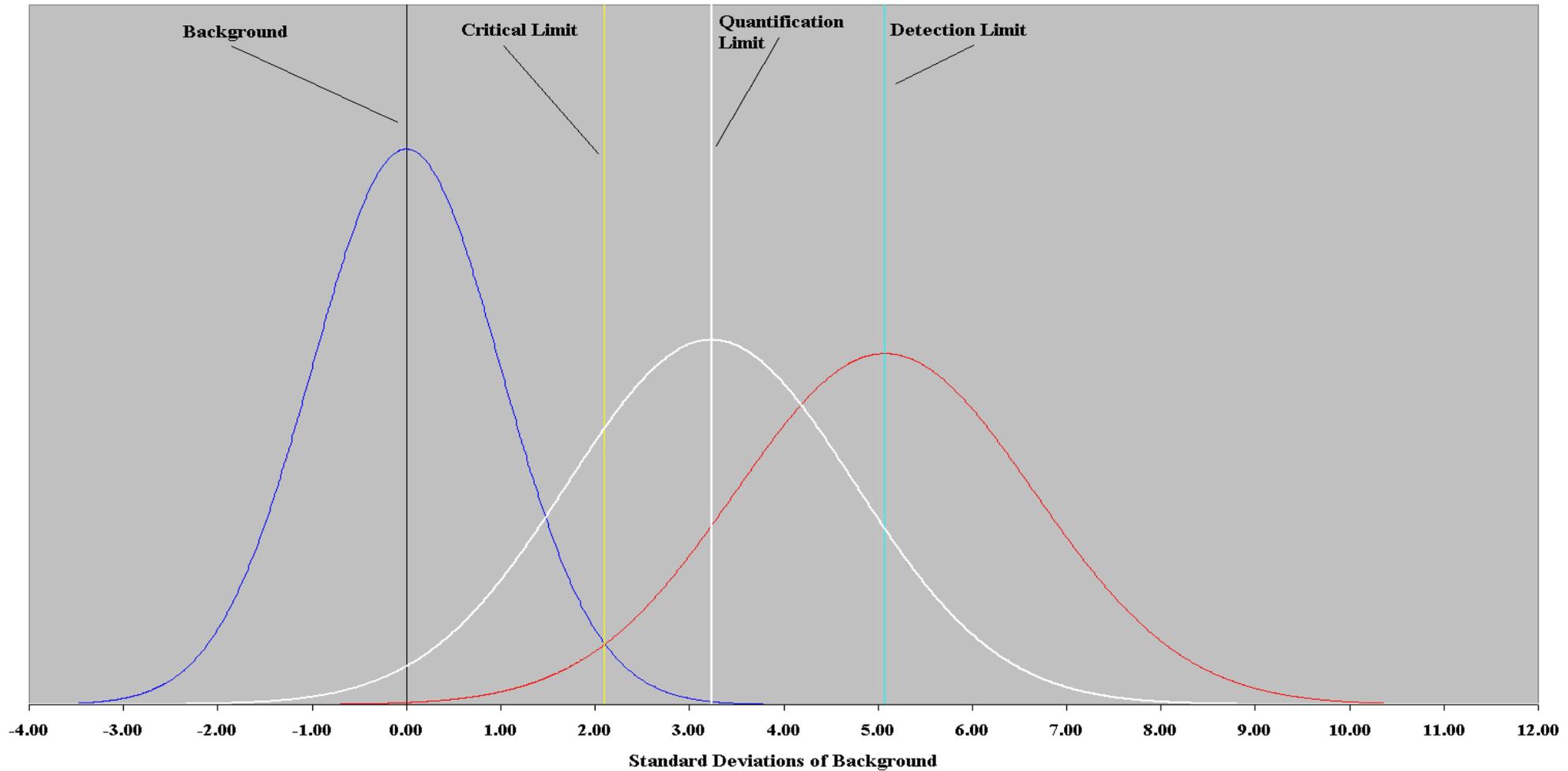
Therefore;

$$S-B = k^2/2 + k\sqrt{[2(B+1)+k^2/4]}$$

But if  $k=2$ , then  $L_d < L_q$  if  $B > 18$  ( $L_d/L_q \rightarrow 1.69$  when  $B \rightarrow \infty$ )

# Graphically again

Critical and Detection Limits



# Reporting

Not a very useful outcome!

So,

$S_{\text{net}} < 0$  report as  $<L_d$  and 'not detected'

$0 < S_{\text{net}} < L_c$  report as  $<L_d$  and 'not detected'

$L_c < S_{\text{net}} < L_d$  report as  $<L_d$  and 'detected, but not quantifiable'

(note that if  $B < 18$ , then reporting as  $<(S_{\text{net}} + 2\delta S_{\text{net}})$  may be better

$L_d < S_{\text{net}}$  report as  $S_{\text{net}} \pm 2\delta S_{\text{net}}$

