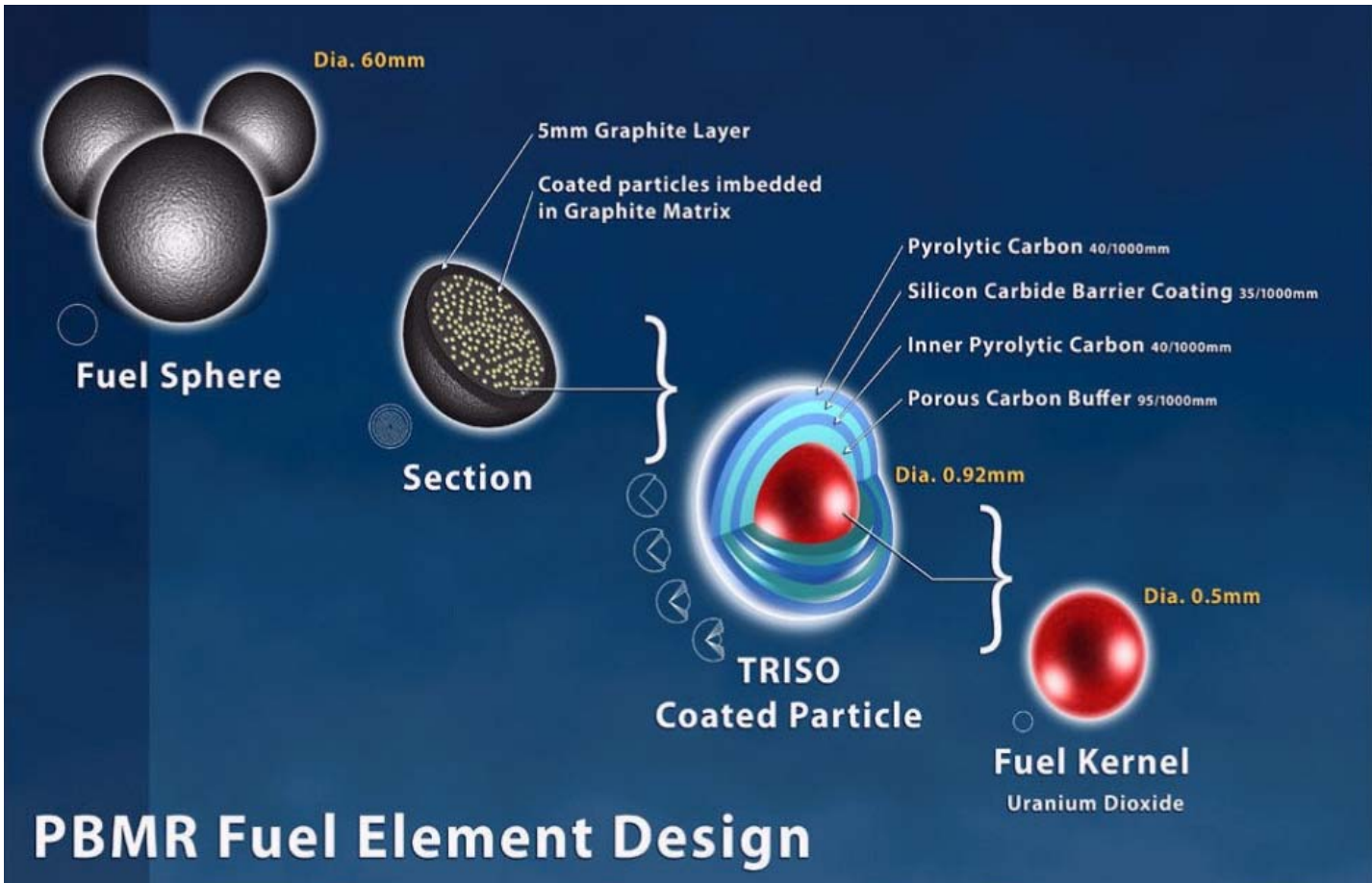
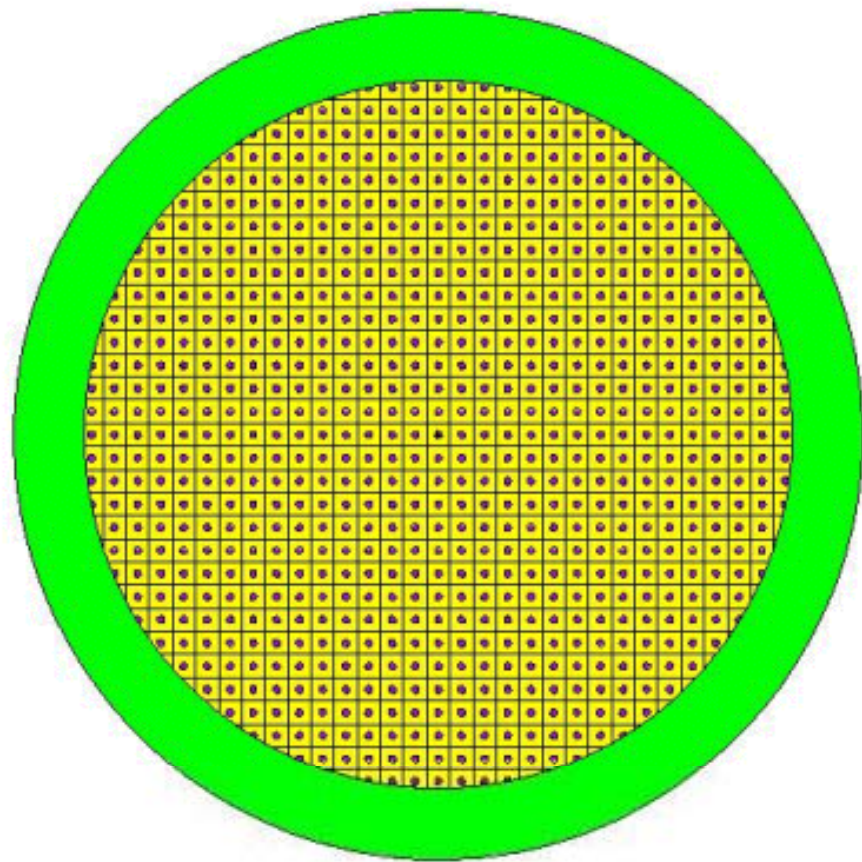
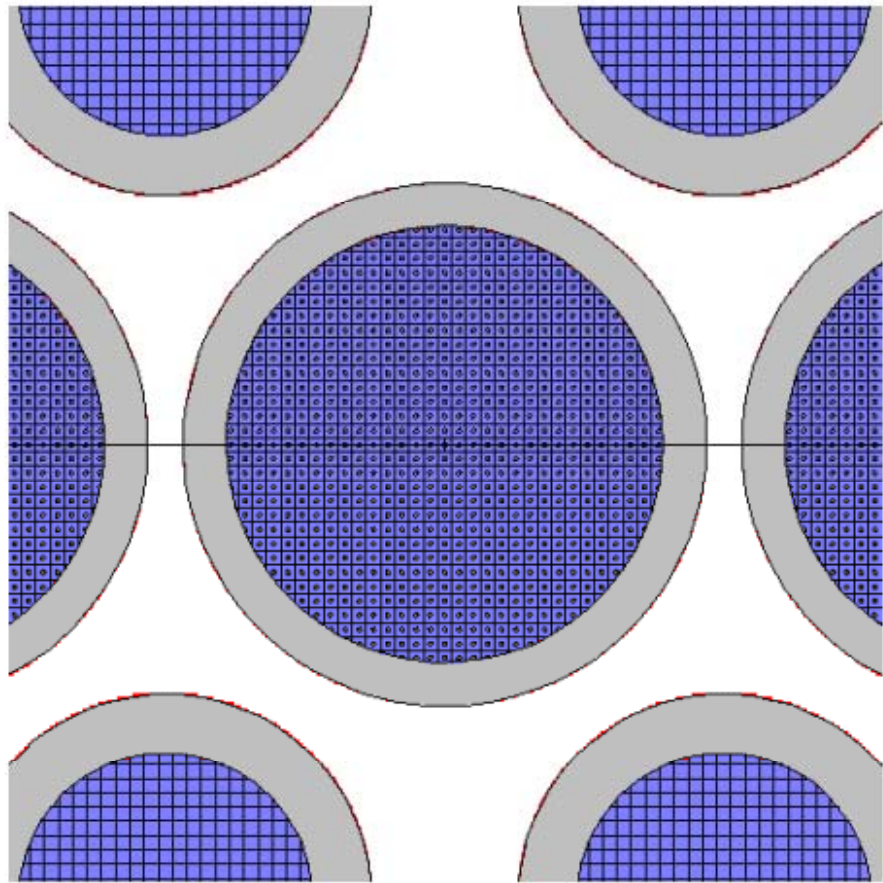


Systematic error in MCNP's pebbles modelling

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$$N_K = \sum_{i=-M}^M \sum_{j=-M}^M \sum_{k=-M}^M f_{ijk}$$

with

$$M = \left[\frac{R_P}{P} \right]$$

$$f_{ijk} = \begin{cases} 1 & R_{ijk} < R_P \\ 0 & \textit{Otherwise} \end{cases}$$

with

$$R_{ijk} = P \sqrt{i^2 + j^2 + k^2}$$

where

N_K is the number of kernels inside the pebble,

R_P the radius of the pebble matrix,

P the pitch of the cubic lattice,

and the brackets represents the largest integer number lower than the argument (“integer part”).

Number of kernels (design parameter)	Number of points of the cubic lattice inside the pebble	Number of effective kernels inside the pebble
10000	10395	9969.3
11000	11363	11003.0
12000	12293	11975.9
13000	13397	13054.1
14000	14411	13966.1
15000	15515	15040.0
16000	16519	15980.9
17000	17461	16997.0
18000	18613	17975.6
19000	19549	19049.1
20000	20479	19985.1

Table 1: geometric and mass effects on the numbers of kernels for a pebble matrix of 2.5 cm radius and kernels of 0.025 cm radius.

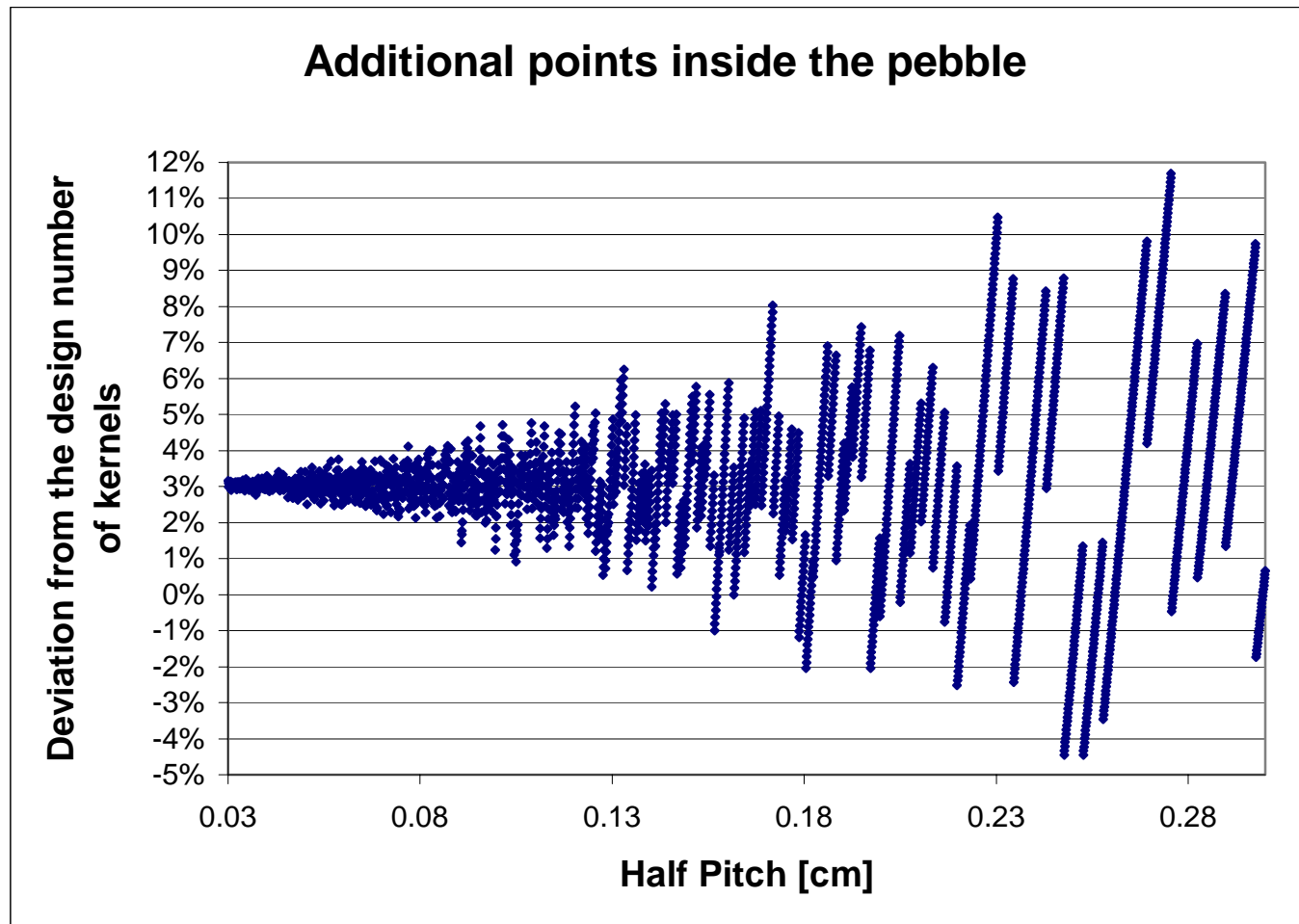


Figure 1: Additional points inside the pebble as a fraction of the design numbers of kernels (for a pebble matrix of 2.5 cm radius and kernels of 0.025 cm radius).

- In the range of PBMR pebbles design the error in the number of points is between 2% and 4%.
- There exist “magical” pitches where the error is 0%.
- Negative (or 0%) errors can be achieved for pitches/2 greater than 0.14 cm ($\sim d/36$).

$$V_T = \sum_{i=-M}^M \sum_{j=-M}^M \sum_{k=-M}^M V_{ijk} \quad \text{with} \quad M = \left[\frac{R_P + R_K}{P} \right]$$

$$V_{ijk} = \begin{cases} \frac{4}{3} \pi R_K^3 & R_{ijk} + R_K \leq R_P \\ 0 & R_{ijk} + R_K \geq R_P \\ \frac{\pi}{24 R_{ijk}^3} \left\{ \begin{aligned} & \left[R_K^2 - (R_{ijk} - R_P)^2 \right]^2 \left[6 R_{ijk} R_P + (R_{ijk} - R_P)^2 - R_K^2 \right] + \\ & \left[R_P^2 - (R_{ijk} - R_K)^2 \right]^2 \left[6 R_{ijk} R_K + (R_{ijk} - R_K)^2 - R_P^2 \right] \end{aligned} \right\} & R_P - R_K < R_{ijk} < R_P + R_K \end{cases}$$

with $R_{ijk} = P \sqrt{i^2 + j^2 + k^2}$

where

V_T is the total volume of kernels inside the model of the pebble,

R_K is the radius of a kernel, and the rest of the parameters were already defined.

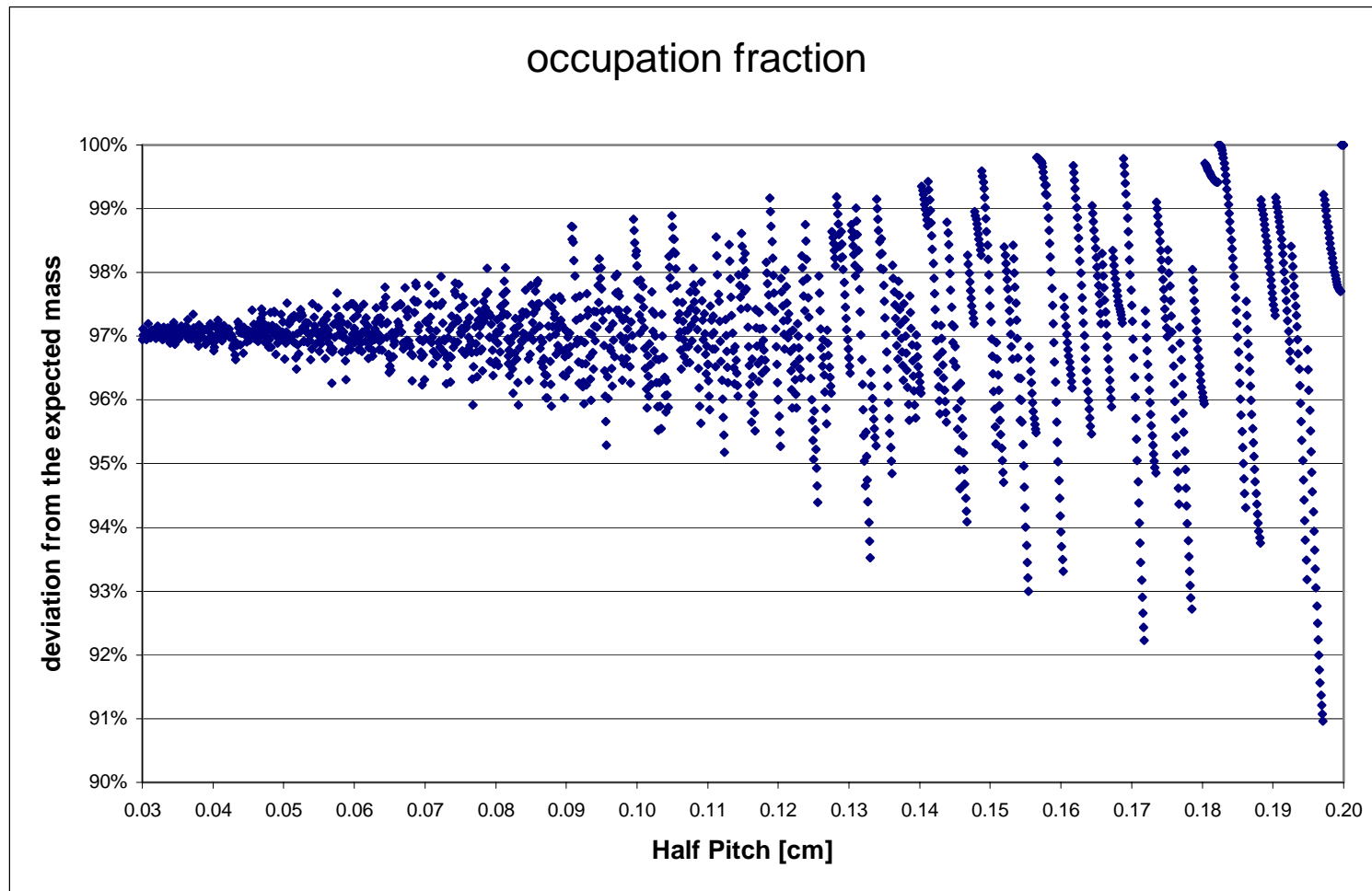


Figure 2: fraction of the expected mass of kernels introduced in the model of a pebble (for a pebble matrix of 2.5 cm radius and kernels of 0.025 cm radius).

- In the range of PBMR pebbles design the error in the mass introduced in the model (with respect to the expected value) is between -2% and -4%.
- There exist “magical” pitches where the error is 0%.

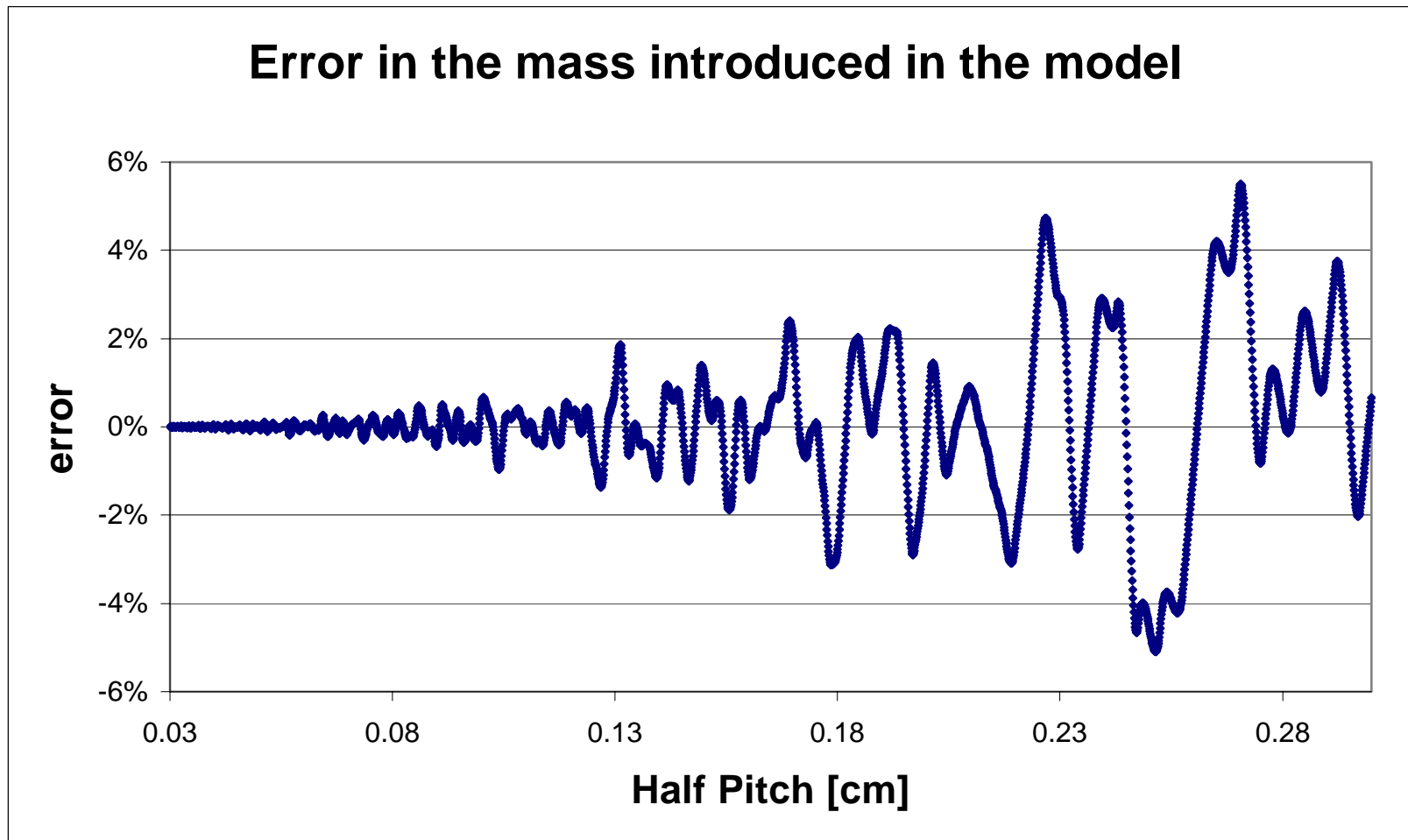


Figure 3: error in the mass of uranium introduced in the model of a pebble (for a pebble matrix of 2.5 cm radius and kernels of 0.025 cm radius).

It is remarkable:

- the cancellation effect.
- the continuum behaviour of the error as a function of the pitch.
- the high number of “magical pitches” where the error introduced in the model is null, even for relative big pitches.

The most relevant issue is that

for today’s pebble design

($R_P=2.5$ cm, $R_K=250$ μm , $U_{\text{mass}}=9$ g, $\text{Enrich}=9.6\%$)

the error in the mass introduced in the model is only 0.24% (negligible)

just because an *error cancellation*,

but simple changes in the design parameters

(bigger kernels, less uranium mass, bigger pebbles, more enrichment)

would lead to errors in the MCNP model of 3% or more.

Conclusion

- Be paranoid,
- and check everything twice.