Power Splitter Characterisation – EM Day

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Contents

• Why we need to measure power splitters

• How they get measured

• Actual devices

• Circles!
Why do we need splitter measurements

- **Power Splitters** and **Couplers** are very useful in power sensor calibrations

- A **splitter** or coupler **plus sidearm power sensor** can form a **transfer standard** to calibrate 1 power sensor against another

- In order to do an accurate calibration a **Mismatch Correction** should be made

- This requires the reflection coefficient of any power sensors and the **Equivalent Output Port Match** of the splitter or coupler
2 Resistor splitters

- If used in a levelling-loop or ratio system a 2 resistor splitter gives a broadband low value for effective source reflection coefficient

- If used as a simple passive device it has 
  \( S_{22} \) (or \( S_{33} \)) \( \sim 0.25 \)
Ways to characterise splitters:

- 2-port S-parameter Measurements - equivalent output mismatch can then be calculated

- Direct method - measures equivalent output mismatch directly

- Tuned load method
Tuned load method

- Adjust tuned load until zero power appears at port 3
- Reflection coefficient looking into port 2 is effective source match
- Does not work with splitters
  - with size > 0
  - with loss on port 1 (requires tuned source instead)
2-port Measurement method employed (1)

- S-parameters of “partial 2-ports” measured using National Standard measurement system (PIMMS)

- Also VRC of terminating load measured using PIMMS
2–port Measurement method employed (2)

- Matrix renormalisation employed to obtain S-parameters of splitter 3-port following Tippet & Speciale
- Measurands calculated from splitter S-parameters
- Monte-Carlo Simulation employed to estimate uncertainties in measurands

References:
Matrix renormalisation to obtain S-parameters of splitter 3-port

\[
\begin{bmatrix}
* & * \\
* & * \\
\end{bmatrix}
\xrightarrow{\text{RENORMALISE}}
\begin{bmatrix}
* & * \\
* & * \\
\end{bmatrix}
\xrightarrow{\text{ASSEMBLE}}
\begin{bmatrix}
* & * & * \\
* & * & * \\
\end{bmatrix}
\xrightarrow{\text{RENORMALISE}}
\begin{bmatrix}
* & * & * \\
* & * & * \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
* & * \\
* & * \\
\end{bmatrix}
\xrightarrow{\text{NORMALISED TO Z₀}}
\begin{bmatrix}
* & * \\
* & * \\
\end{bmatrix}
\xrightarrow{\text{NORMALISED TO Z₀}}
\begin{bmatrix}
* & * & * \\
* & * & * \\
\end{bmatrix}
\]

NORMALISED TO 50 Ω AT BOTH PORTS
NORMALISED TO 50 Ω AT ALL THREE PORTS
NORMALISED TO Z₀ AT BOTH PORTS
NORMALISED TO Z₀ AT ALL THREE PORTS
Some results for a 3.5 mm splitter

$u_{VSWR} \approx 0.03 \quad u_S \approx 0.015 \quad u_{S11} \approx 0.006$
Direct method - Description

• How it works:
  – Connect unused ports of splitter to VNA
  – Attach 3 known impedances to 3rd port
  – Take 2 of the uncalibrated S-parameters from network analyser measurements for each impedance
  – Solve equations

• Equivalent to a ‘normal’ 1-port calibration

References:
Direct method - Mathematics

\[
\begin{pmatrix}
1 & L \cdot \Gamma_L & \Gamma_L \\
1 & O \cdot \Gamma_O & \Gamma_O \\
1 & S \cdot \Gamma_S & \Gamma_S \\
\end{pmatrix}
\begin{pmatrix}
E_{DF} \\
E_{SF} \\
E \\
\end{pmatrix}
= 
\begin{pmatrix}
L \\
O \\
S \\
\end{pmatrix}
\]

where

\[
E = E_{RF} - E_{DF} \cdot E_{SF}
\]

and

\[
\frac{L}{O} / S = \frac{S_{11,raw}}{S_{21,raw}} \] with \(L/O/S\)-attached
Direct method - Results

Example measurement of the equivalent output port mismatch of a Weinschel 1870A 2-resistor power splitter with type-N connectors.
Direct method – Uncertainties

For the measurement of a well matched 2 resistor splitter with a Short, Open and Load as the known impedances the uncertainty is:

\[ u_{Esf} \approx \sqrt{2u_L^2 + u_O^2 + u_S^2 + \text{random} + VNA} \]

i.e. the Load is an important contribution (although the uncertainty on this should be smaller than on either the Short or Open)
Problems

• Need access to **all 3 ports** of device

• This is not possible in many situations such as *transfer standards* or *Tegam / Weinschel-style* sensors

• How should a calibration laboratory characterise these devices?
Define Equations:

\[ P_{DUT} = P_{TS} \cdot \frac{|1 - |\Gamma_{DUT}|^2| \cdot |1 - |S|^2|}{|1 - S \cdot \Gamma_{DUT}|^2} \]

Where \( S \) is the source match of the output port that we are trying to find.

If you expand out the terms into their real and imaginary parts and use:

\[ |o + j \cdot p|^2 = o^2 + p^2 \]
Mathematics 2

Then you can rearrange the equation into the form:

\[ S_r^2 + S_i^2 + a \cdot S_r + b \cdot S_i + c = 0 \]

with:

\[
\begin{align*}
a &= \frac{-2 \cdot \Gamma_r}{\Gamma_r^2 + \Gamma_i^2 + k} \\
b &= \frac{-2 \cdot \Gamma_i}{\Gamma_r^2 + \Gamma_i^2 + k} \\
c &= \frac{1-k}{\Gamma_r^2 + \Gamma_i^2 + k} \\
k &= \frac{1-\Gamma_r^2 - \Gamma_i^2}{R} \\
R &= \frac{P_{DUT}}{P_{TS}}
\end{align*}
\]

Using just the real parts of \( a \) and \( b \) this is the equation for a \textbf{circle offset from the origin} (actually equation in general is for a conic section but neither a hyperbola or parabola is possible)
A more recognisable form might be:

\[(S_r + d)^2 + (S_i + e)^2 = f^2\]

with:

\[d = \frac{a}{2}, \quad e = \frac{b}{2}, \quad f^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c\]

So from one measurement of power ratio with a sensor of known VRC we define a circle of possible source match values (this does not correspond to knowing the magnitude and not knowing the phase though!)
Circles

Once we have done a second measurement the circles should cross at (1 or) 2 points. If they don't cross at all then there has probably been a mistake in the measurements.

Once we have done 3 measurements then all 3 circles should cross at 1 point which we then need to find.
The problem

• Finding the intersection of 3 circles is not tricky if they do all actually cross at a single point
• As there will be some error associated with the circle centres and radii then they may instead meet each other at 0, 1 or 2 points
• Giving 0-6 potential crossing points
• How do we decide which are the best set?
Finding A Robust Solution

- Often the correct solution will be obvious *to the eye* such as a set of 3 points forming a small triangle
- Sometimes it will be less obvious, for example the situation to the right
- What we really have here is a crossing *area*, however it is useful to define a *single point*
- Several methods were tried and 1 that was fairly simple and worked in most cases tried
- It finds the *set of 3 points* from the 6 that give the *minimum perimeter triangle* (i.e. the closest together) and takes the average of the coordinates of these 3 to define a nominal "meeting point"
Conclusions

• Power splitters can be measured in a variety of ways
• Measuring power splitters can be tricky without access to all ports!