Power Splitter Characterisation – EM Day

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James Miall (james.miall@npl.co.uk)
National Physical Laboratory
Teddington, UK

Contents

- Why we need to measure power splitters
- How they get measured
- Actual devices
- Circles!

Why do we need splitter measurements

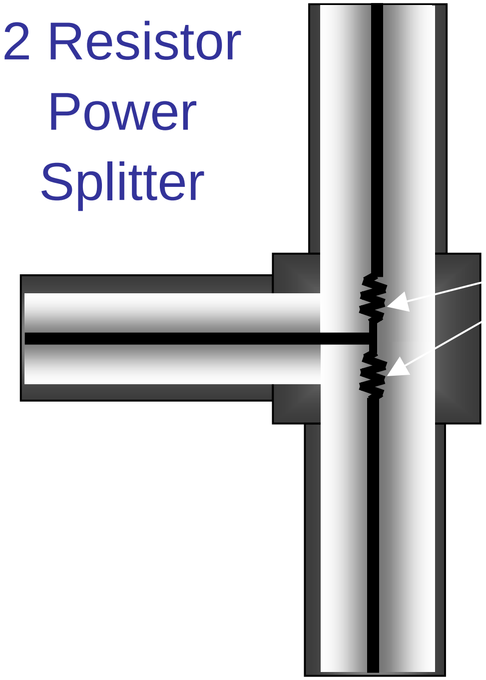
- ***Power Splitters*** and ***Couplers*** are very useful in power sensor calibrations
- A ***splitter*** or coupler ***plus sidearm power sensor*** can form a ***transfer standard*** to calibrate 1 power sensor against another
- In order to do an accurate calibration a ***Mismatch Correction*** should be made
- This requires the reflection coefficient of any power sensors and the ***Equivalent Output Port Match*** of the splitter or coupler

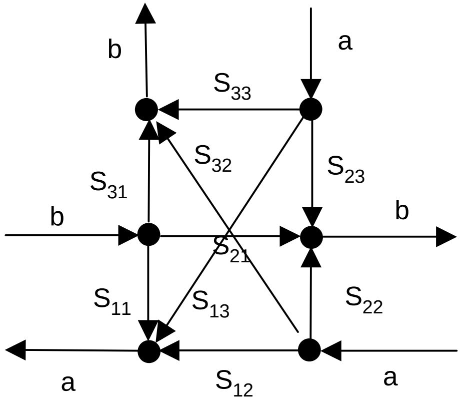
2 Resistor splitters

- If used in a levelling-loop or ratio system a 2 resistor splitter gives a broadband low value for effective source reflection coefficient

- If used as a simple passive device it has

  S_{22} (or S_{33}) ~ 0.25





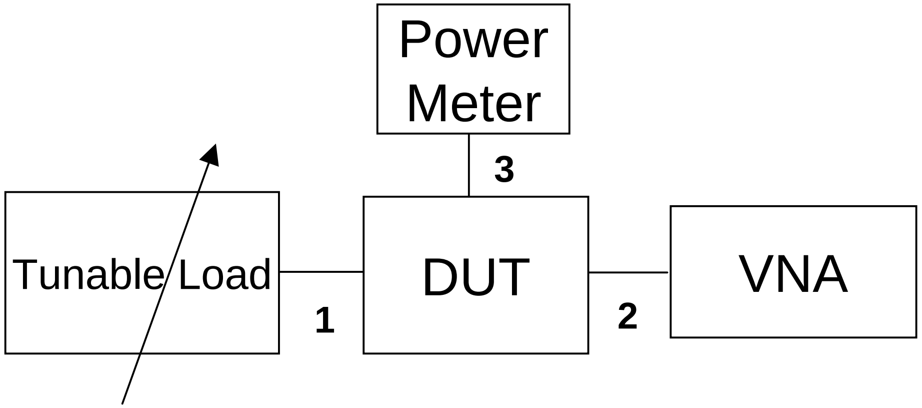
Ways to characterise splitters:

- 2-port S-parameter Measurements - equivalent output mismatch can then be calculated

- Direct method - measures equivalent output mismatch directly

- Tuned load method

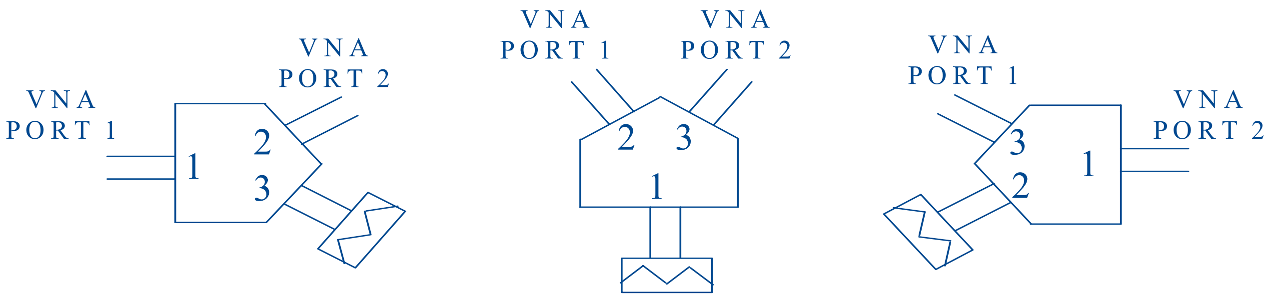
Tuned load method



- Adjust tuned load until zero power appears at port 3
- Reflection coefficient looking into port 2 is effective source match
- Does not work with splitters
  - with size > 0
  - with loss on port 1 (requires tuned source instead)

2-port Measurement method employed (1)

- *S*-parameters of “partial 2-ports” measured using National Standard measurement system (PIMMS)



First partial two-port

Second partial two-port

Third partial two-port

- Also VRC of terminating load measured using PIMMS

2–port Measurement method employed (2)

- Matrix renormalisation employed to obtain *S*-parameters of splitter 3-port following Tippet & Speciale

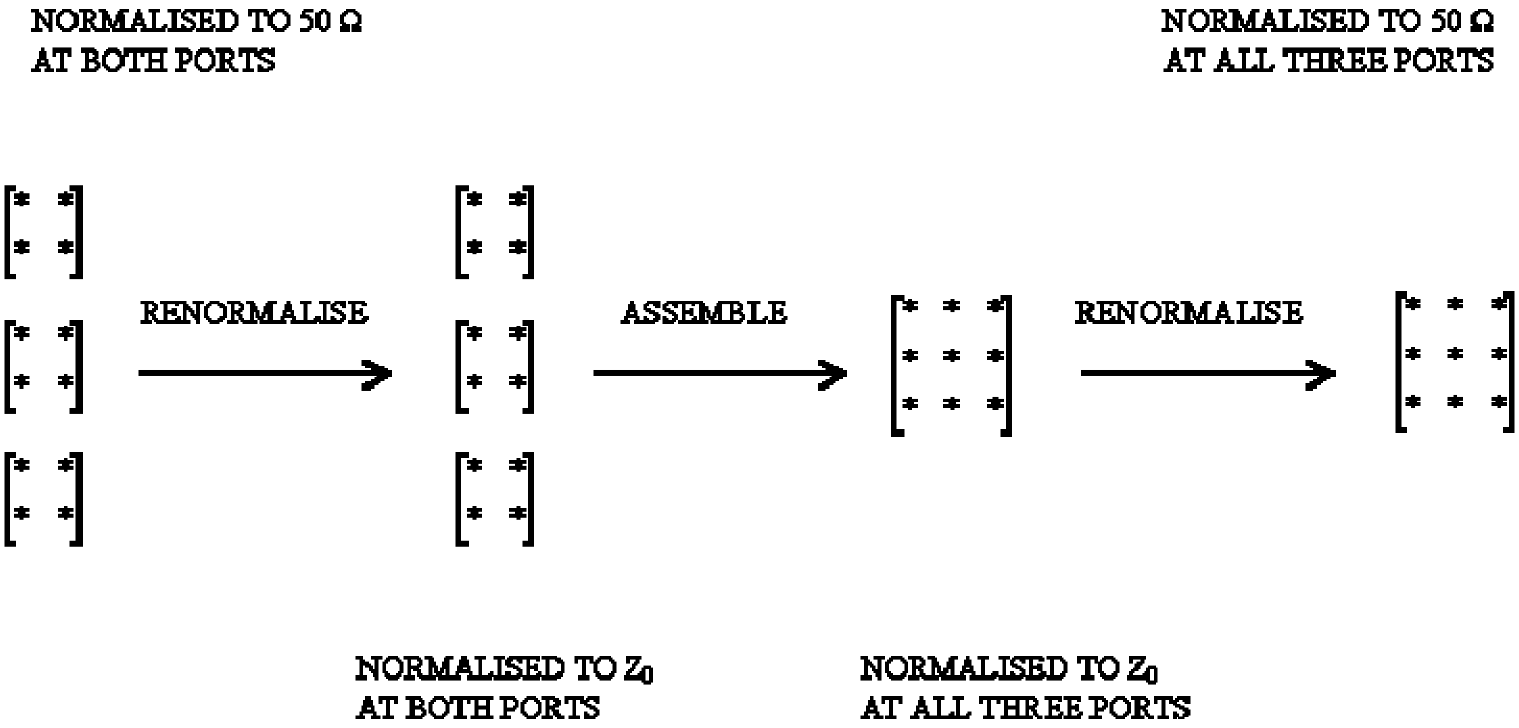
- Measurands calculated from splitter *S*-parameters

- Monte-Carlo Simulation employed to estimate uncertainties in measurands

References:

• Tippet & Speciale ‘A Rigorous Technique for Measuring the Scattering Matrix of A Multiport Device with a 2-Port Network Analyser’, IEEE Trans. Microwave Theory Tech., May 1982

Matrix renormalisation to obtain *S*-parameters of splitter 3-port



NORMALISED TO 50 Ω
AT BOTH PORTS

RENORMALISE

ASSEMBLE

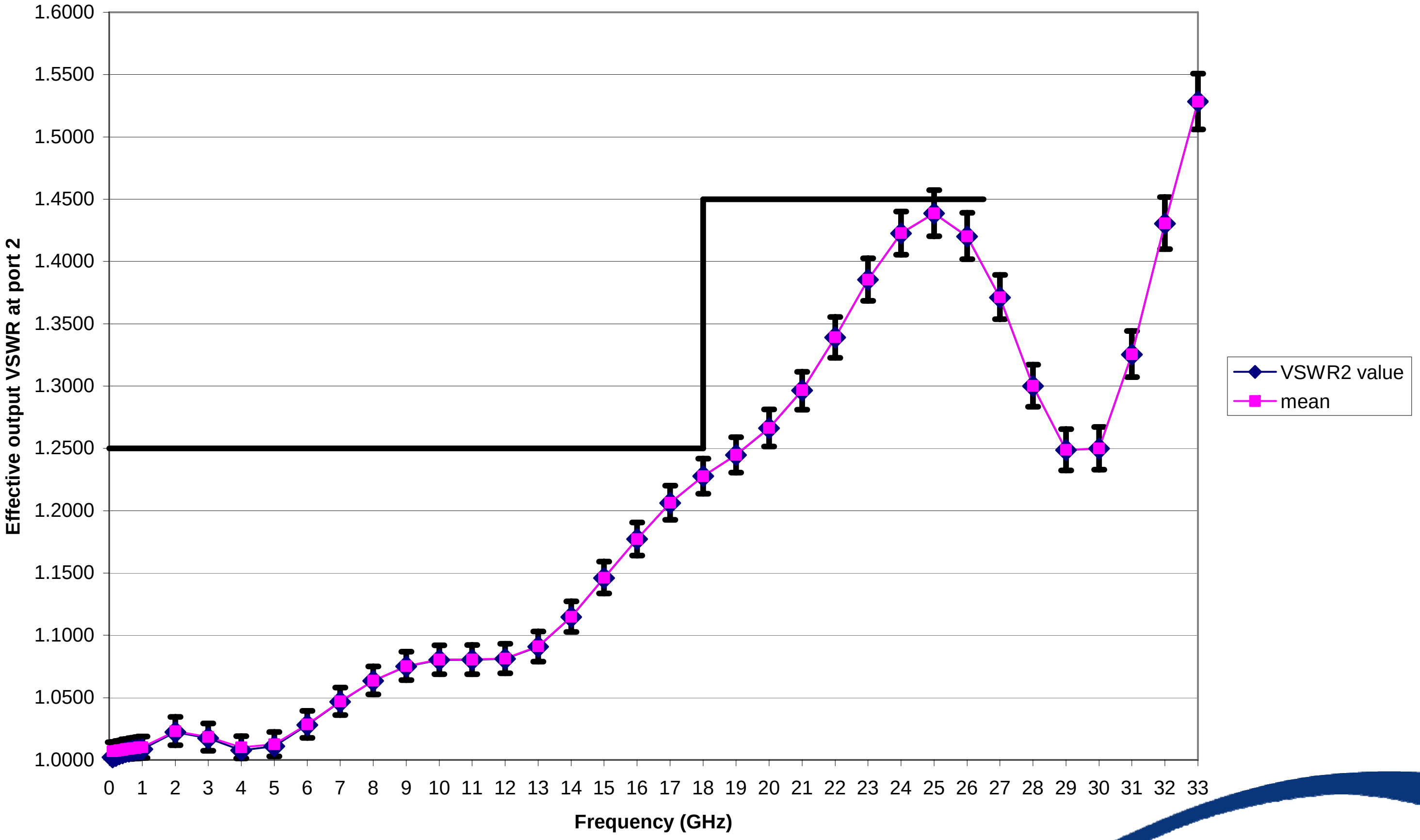
RENORMALISE

NORMALISED TO 50 Ω
AT ALL THREE PORTS

NORMALISED TO Z_0
AT BOTH PORTS

NORMALISED TO Z_0
AT ALL THREE PORTS

Some results for a 3.5 mm splitter



u_{VSWR}~0.03 u_S~0.015 u_{S11}~0.006

Direct method - Description

- How it works:
  - Connect unused ports of splitter to VNA
  - Attach 3 known impedances to 3rd port
  - Take 2 of the uncalibrated S-parameters from network analyser measurements for each impedance
  - Solve equations
- Equivalent to a ‘normal’ 1-port calibration

References:

• J. Juroshek ‘A Direct Calibration Method for Measuring Equivalent Source Mismatch’, Microwave Journal, Oct 1997, pp 106-118

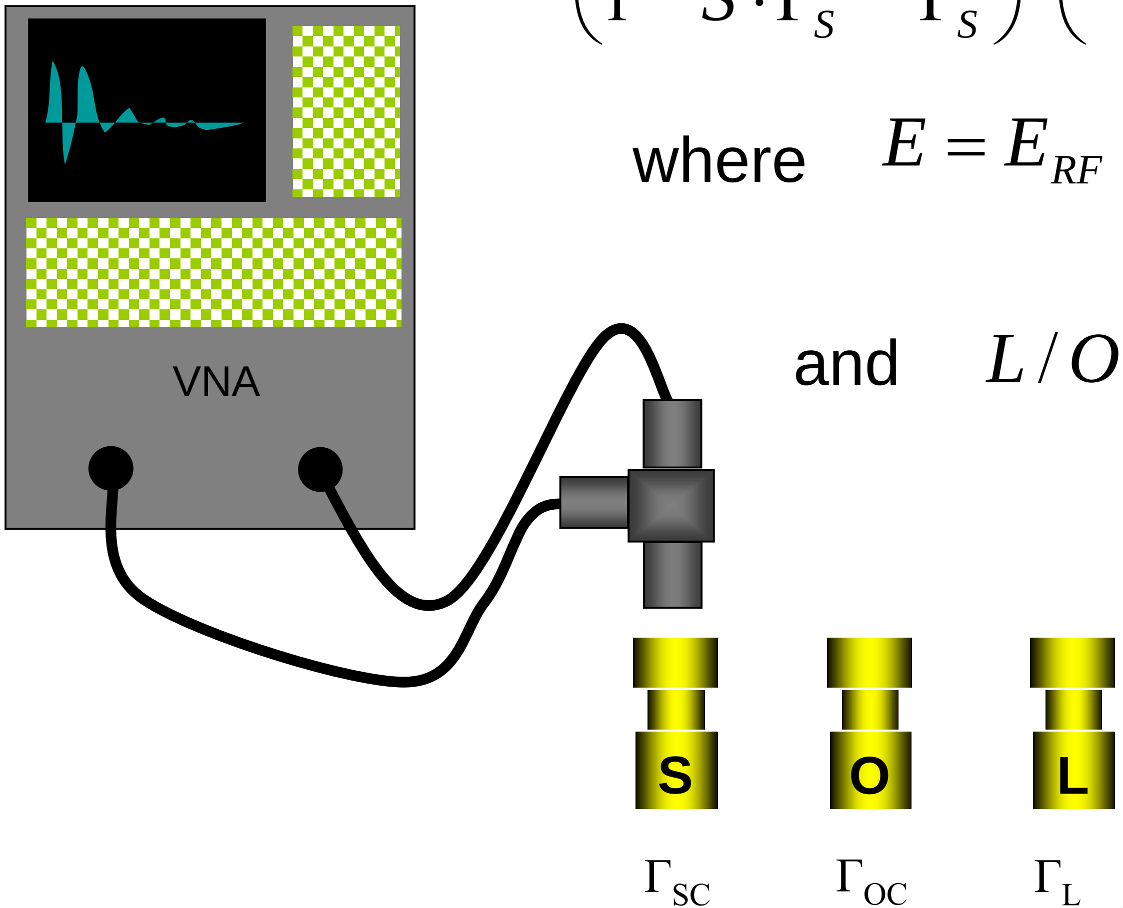
• M. Rodriguez ‘A Semi-Automated Approach to the Direct Calibration Method for Measurement of Equivalent Source Match’, ARMMS Conference, April 1999

Direct method - Mathematics

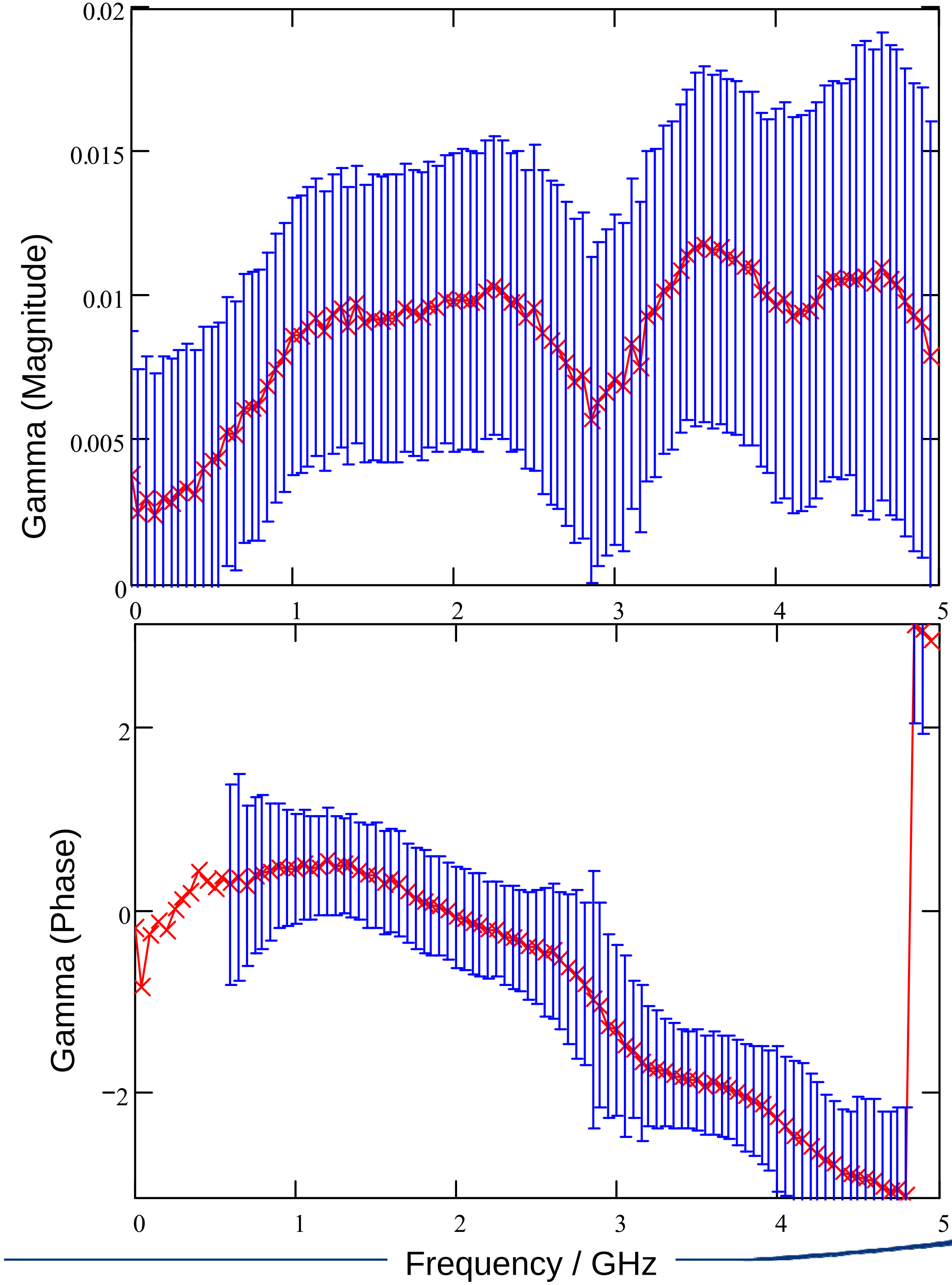
$$\begin{pmatrix} 1 & L\cdot\Gamma_L & \Gamma_L \\ 1 & O\cdot\Gamma_O & \Gamma_O \\ 1 & S\cdot\Gamma_S & \Gamma_S \end{pmatrix}\cdot\begin{pmatrix} E_{DF} \\ E_{SF} \\ E \end{pmatrix}=\begin{pmatrix} L \\ O \\ S \end{pmatrix}$$

where $E = E_{RF} - E_{DF}\cdot E_{SF}$

and $L/O/S = \left.\dfrac{S_{11,raw}}{S_{21,raw}}\right|_{with\cdot L/O/S\cdot attached}$



Direct method - Results



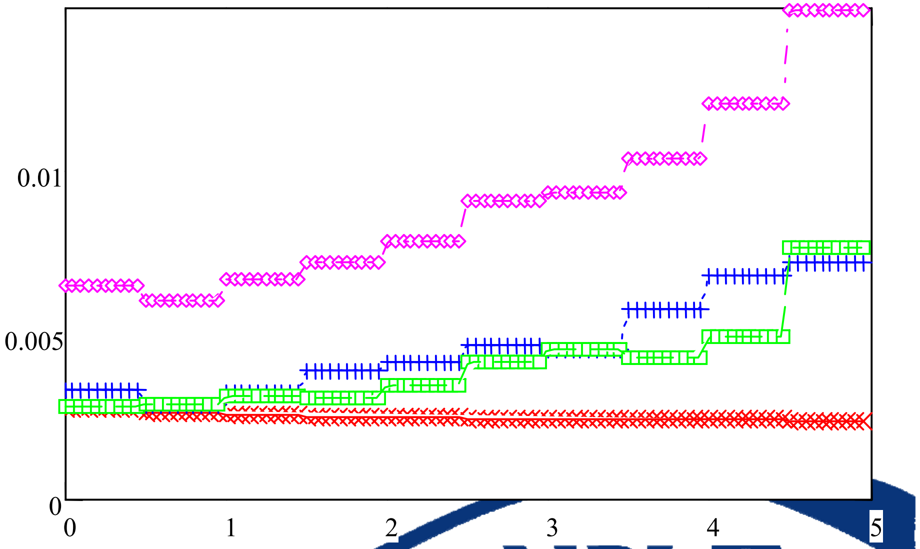
Example measurement of the equivalent output port mismatch of a Weinschel 1870A 2-resistor power splitter with type-N connectors

Direct method – Uncertainties

For the measurement of a well matched 2 resistor splitter with a Short, Open and Load as the known impedances the uncertainty is:

$$u_{Esf} \approx \sqrt{2u_L{}^2 + u_O{}^2 + u_S{}^2 + random + VNA}$$

i.e. the ***Load*** is an ***important contribution*** (although the uncertainty on this should be smaller than on either the Short or Open)



Problems

- Need access to ***all 3 ports*** of device
- This is not possible in many situations such as ***transfer standards*** or ***Tegam / Weinschel-style*** sensors
- How should a calibration laboratory characterise these devices?

Mathematics 1

Define Equations:

$$P_{DUT} = P_{TS} \cdot \frac{\left[1 - \left|\Gamma_{DUT}\right|^2\right] \cdot \left[1 - \left|S\right|^2\right]}{\left|1 - S \cdot \Gamma_{DUT}\right|^2}$$

Where *S* is the source match of the output port that we are trying to find

If you expand out the terms into their real and imaginary parts and use:

$$\left|o + j \cdot p\right|^2 = o^2 + p^2$$

Mathematics 2

Then you can rearrange the equation into the form:

$$S_r^2 + S_i^2 + a \cdot S_r + b \cdot S_i + c = 0$$

with:

$$a = \frac{-2 \cdot \Gamma_r}{\Gamma_r^2 + \Gamma_i^2 + k} \qquad b = \frac{-2 \cdot \Gamma_i}{\Gamma_r^2 + \Gamma_i^2 + k} \qquad c = \frac{1-k}{\Gamma_r^2 + \Gamma_i^2 + k}$$

$$k = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{R} \qquad R = \frac{P_{DUT}}{P_{TS}}$$

Using just the real parts of *a* and *b* this is the equation for a ***circle offset from the origin***

(actually equation in general is for a conic section but neither a hyperbola or parabola is possible)

Mathematics 3

A more recognisable form might be:

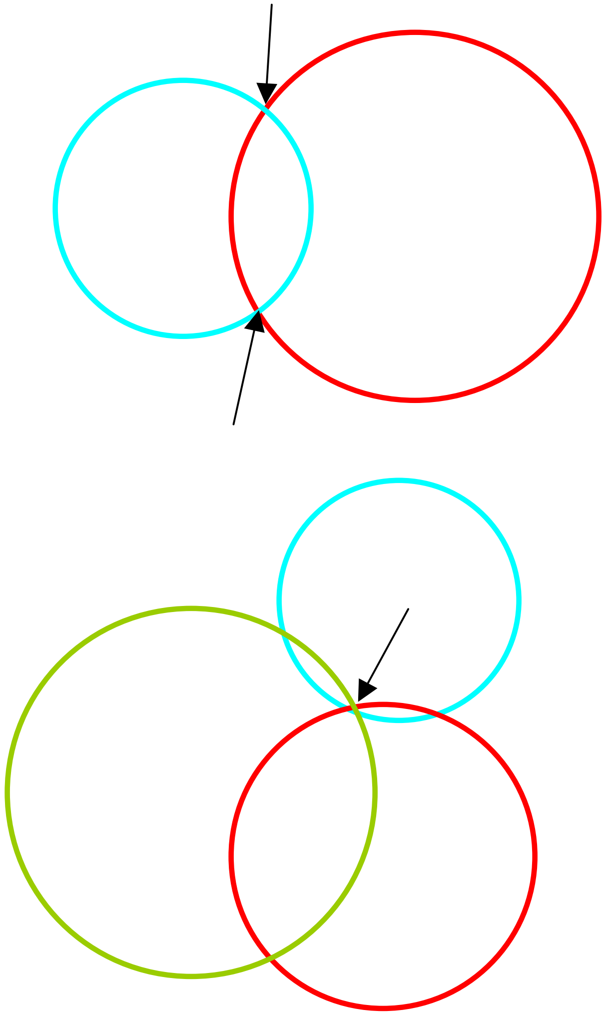
$$\left(S_r + d\right)^2 + \left(S_i + e\right)^2 = f^2$$

with:

$$d = \frac{a}{2} \qquad e = \frac{b}{2} \qquad f^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c$$

So from ***one*** measurement of ***power ratio*** with a sensor of ***known VRC*** we define a ***circle of possible source match values*** (this does not correspond to knowing the magnitude and not knowing the phase though!)

Circles

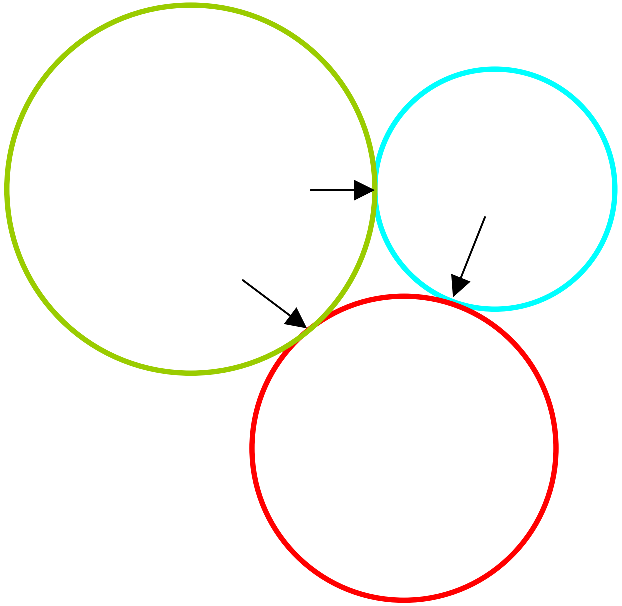


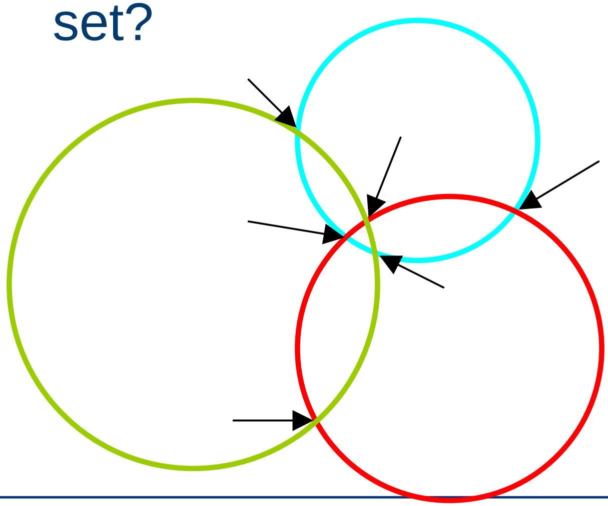
Once we have done a second measurement the circles should cross at (1 or) 2 points. If they don't cross at all then there has probably been a mistake in the measurements.

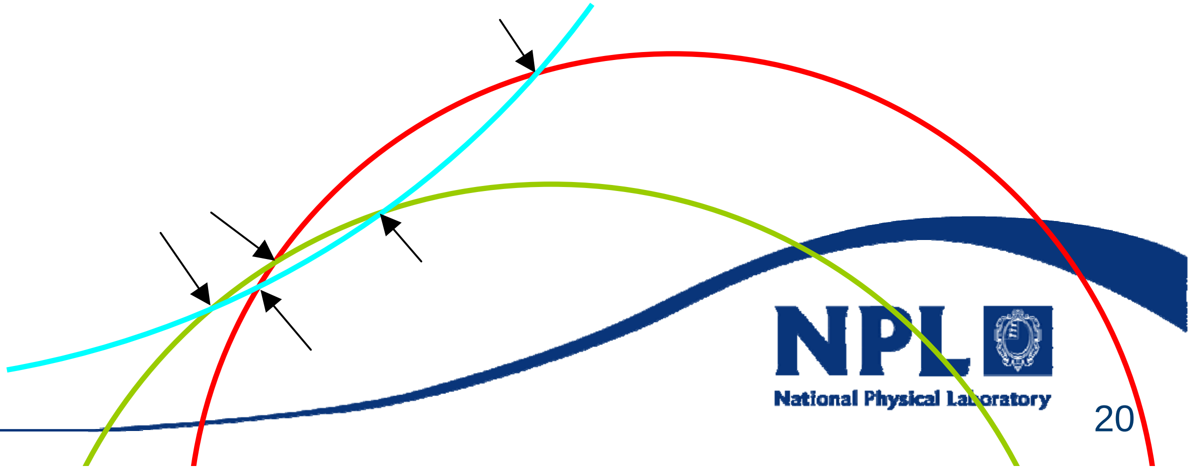
Once we have done 3 measurements then all 3 circles should cross at 1 point which we then need to find.

The problem

- Finding the intersection of 3 circles is not tricky if they do all actually cross at a single point
- As there will be some error associated with the circle centres and radii then they may instead meet each other at 0,1 or 2 points
- Giving 0-6 potential crossing points
- How do we decide which are the best set?

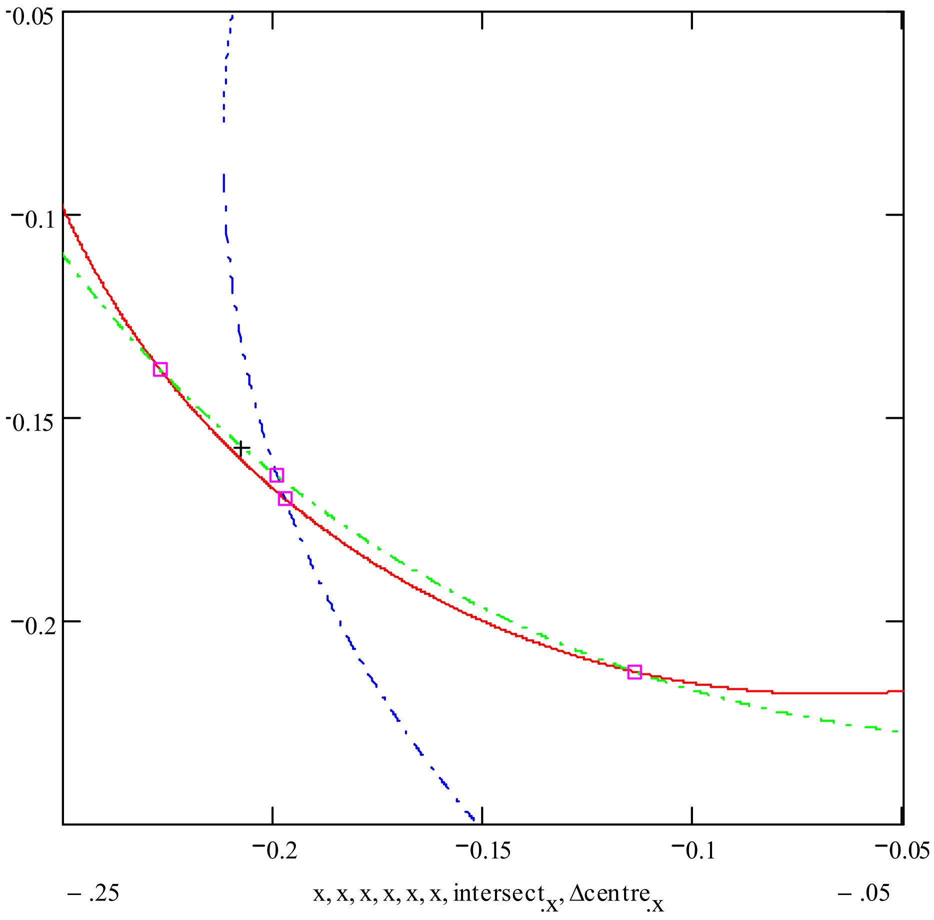




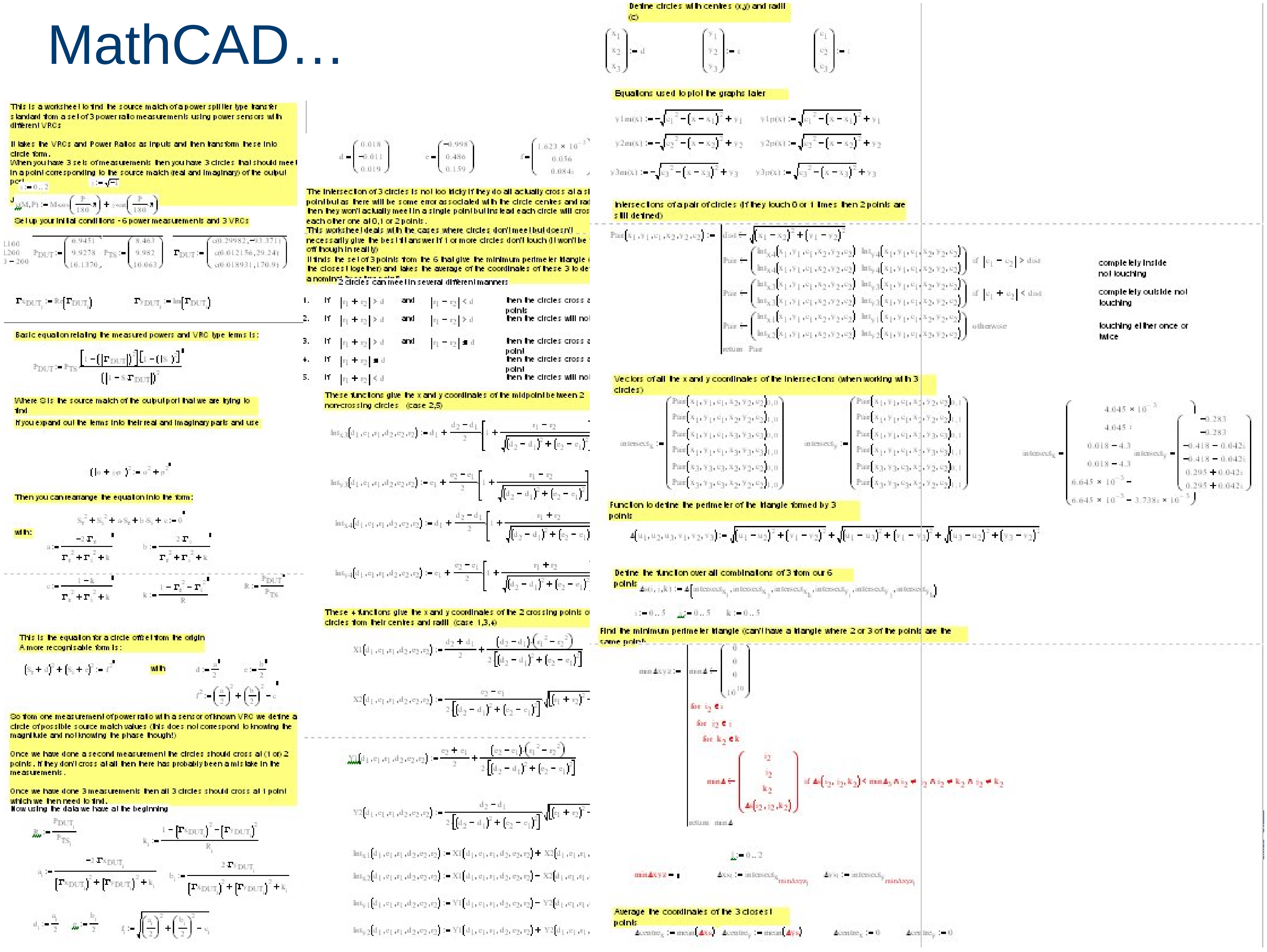


Finding A Robust Solution

- Often the correct solution will be obvious ***to the eye*** such as a set of 3 points forming a small triangle
- Sometimes it will be less obvious, for example the situation to the right
- What we really have here is a crossing ***area***, however it is useful to define a ***single point***
- Several methods were tried and 1 that was fairly simple and worked in most cases tried
- It finds the ***set of 3 points*** from the 6 that give the ***minimum perimeter triangle*** (i.e. the closest together) and takes the average of the coordinates of these 3 to define a nominal "meeting point"



MathCAD…



This is a worksheet to find the source match of a power splitter type transfer standard from a set of 3 power ratio measurements using power sensors with different VRCs

It takes the VRCs and Power Ratios as inputs and then transform these into circle form.
When you have 3 sets of measurements then you have 3 circles that should meet in a point corresponding to the source match (real and imaginary) of the output port

$i := 0..2$ $\quad$ $j := \sqrt{-1}$

$J(M,P) := M\cos\left(\frac{P}{180}\cdot\pi\right) + j\sin\left(\frac{P}{180}\cdot\pi\right)$

Set up your initial conditions - 6 power measurements and 3 VRCs

$P_{DUT} := \begin{pmatrix} 6.9451 \\ 9.9278 \\ 10.1370 \end{pmatrix}$ $\quad P_{TS} := \begin{pmatrix} 8.463 \\ 9.982 \\ 10.063 \end{pmatrix}$ $\quad \Gamma_{DUT} := \begin{pmatrix} J(0.29982, -93.3711) \\ J(0.012156, 29.241) \\ J(0.018931, 170.91) \end{pmatrix}$

$\Gamma x_{DUT_i} := \mathrm{Re}(\Gamma_{DUT_i})$ $\qquad \Gamma y_{DUT_i} := \mathrm{Im}(\Gamma_{DUT_i})$

Basic equation relating the measured powers and VRC type terms is:

$$P_{DUT} := P_{TS}\cdot\frac{\left[1-(|\Gamma_{DUT}|)^2\right]\left[1-(|S|)^2\right]}{(|1-S\,\Gamma_{DUT}|)^2}$$

Where S is the source match of the output port that we are trying to find

If you expand out the terms into their real and imaginary parts and use

$$(|o + j\,p|)^2 := o^2 + p^2$$

Then you can rearrange the equation into the form:

$$S_r^2 + S_i^2 + a\,S_r + b\,S_i + c := 0$$

with:

$$a := \frac{-2\,\Gamma_r}{\Gamma_r^2 + \Gamma_i^2 + k} \qquad b := \frac{2\,\Gamma_i}{\Gamma_r^2 + \Gamma_i^2 + k}$$

$$c := \frac{1-k}{\Gamma_r^2 + \Gamma_i^2 + k} \qquad k := \frac{1-\Gamma_r^2-\Gamma_i^2}{R} \qquad R := \frac{P_{DUT}}{P_{TS}}$$

This is the equation for a circle offset from the origin
A more recognisable form is:

$$(S_r + d)^2 + (S_i + e)^2 := f^2 \quad \text{with} \quad d := \frac{a}{2} \quad e := \frac{b}{2}$$

$$f^2 := \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c$$

So from one measurement of power ratio with a sensor of known VRC we define a circle of possible source match values (this does not correspond to knowing the magnitude and not knowing the phase though!)

Once we have done a second measurement the circles should cross at (1 or) 2 points. If they don't cross at all then there has probably been a mistake in the measurements.

Once we have done 3 measurements then all 3 circles should cross at 1 point which we then need to find.

Now using the data we have at the beginning

$$R_i := \frac{P_{DUT_i}}{P_{TS_i}} \qquad k_i := \frac{1-(\Gamma x_{DUT_i})^2-(\Gamma y_{DUT_i})^2}{R_i}$$

$$a_i := \frac{-2\,\Gamma x_{DUT_i}}{(\Gamma x_{DUT_i})^2 + (\Gamma y_{DUT_i})^2 + k_i} \qquad b_i := \frac{2\,\Gamma y_{DUT_i}}{(\Gamma x_{DUT_i})^2 + (\Gamma y_{DUT_i})^2 + k_i}$$

$$d_i := \frac{a_i}{2} \qquad e_i := \frac{b_i}{2} \qquad f_i := \sqrt{\left(\frac{a_i}{2}\right)^2 + \left(\frac{b_i}{2}\right)^2 - c_i}$$

$d = \begin{pmatrix} 0.018 \\ -0.011 \\ 0.019 \end{pmatrix}$ $\quad e = \begin{pmatrix} -0.998 \\ 0.486 \\ 0.159 \end{pmatrix}$ $\quad f = \begin{pmatrix} 1.623\times10^{-3} \\ 0.056 \\ 0.084 \end{pmatrix}$

The intersection of 3 circles is not too tricky if they do all actually cross at a si
point but as there will be some error associated with the circle centres and rad
then they won't actually meet in a single point but instead each circle will cros
each other one at 0,1 or 2 points.
This worksheet deals with the cases where circles don't meet but doesn't
necessarily give the best fit answer if 1 or more circles don't touch (I won't be
off though in reality)
It finds the set of 3 points from the 6 that give the minimum perimeter triangle (
the closest together) and takes the average of the coordinates of these 3 to de
a nomin

2 circles can meet in several different manners

1. If $|r_1 + r_2| > d$ and $|r_1 - r_2| < d$ then the circles cross a
points
2. If $|r_1 + r_2| > d$ and $|r_1 - r_2| > d$ then the circles will not
3. If $|r_1 + r_2| > d$ and $|r_1 - r_2| = d$ then the circles cross a
point
4. If $|r_1 + r_2| = d$ then the circles cross a
point
5. If $|r_1 + r_2| < d$ then the circles will not

These functions give the x and y coordinates of the midpoint between 2 non-crossing circles (case 2,5)

$$\mathrm{Int}_{x3}(d_1, e_1, r_1, d_2, e_2, r_2) := d_1 + \frac{d_2-d_1}{2}\left[1 + \frac{r_1 - r_2}{\sqrt{(d_2-d_1)^2 + (e_2-e_1)^2}}\right.$$

$$\mathrm{Int}_{y3}(d_1, e_1, r_1, d_2, e_2, r_2) := e_1 + \frac{e_2-e_1}{2}\left[1 + \frac{r_1 - r_2}{\sqrt{(d_2-d_1)^2 + (e_2-e_1)^2}}\right.$$

$$\mathrm{Int}_{x4}(d_1, e_1, r_1, d_2, e_2, r_2) := d_1 + \frac{d_2-d_1}{2}\left[1 + \frac{r_1 + r_2}{\sqrt{(d_2-d_1)^2 + (e_2-e_1)^2}}\right.$$

$$\mathrm{Int}_{y4}(d_1, e_1, r_1, d_2, e_2, r_2) := e_1 + \frac{e_2-e_1}{2}\left[1 + \frac{r_1 + r_2}{\sqrt{(d_2-d_1)^2 + (e_2-e_1)^2}}\right.$$

These 4 functions give the x and y coordinates of the 2 crossing points o
circles from their centres and radii (case 1,3,4)

$$X1(d_1, e_1, r_1, d_2, e_2, r_2) := \frac{d_2+d_1}{2} + \frac{(d_2-d_1)\cdot(r_1^2-r_2^2)}{2\left[(d_2-d_1)^2+(e_2-e_1)^2\right]}$$

$$X2(d_1, e_1, r_1, d_2, e_2, r_2) := \frac{e_2-e_1}{2\left[(d_2-d_1)^2+(e_2-e_1)^2\right]}\cdot\sqrt{\left[(r_1+r_2)^2 -\right.}$$

$$Y1(d_1, e_1, r_1, d_2, e_2, r_2) := \frac{e_2+e_1}{2} + \frac{(e_2-e_1)\cdot(r_1^2-r_2^2)}{2\left[(d_2-d_1)^2+(e_2-e_1)^2\right]}$$

$$Y2(d_1, e_1, r_1, d_2, e_2, r_2) := \frac{d_2-d_1}{2\left[(d_2-d_1)^2+(e_2-e_1)^2\right]}\cdot\sqrt{\left[(r_1+r_2)^2 -\right.}$$

$\mathrm{Int}_{x1}(d_1, e_1, r_1, d_2, e_2, r_2) := X1(d_1, e_1, r_1, d_2, e_2, r_2) + X2(d_1, e_1, r_1,$

$\mathrm{Int}_{x2}(d_1, e_1, r_1, d_2, e_2, r_2) := X1(d_1, e_1, r_1, d_2, e_2, r_2) - X2(d_1, e_1, r_1,$

$\mathrm{Int}_{y1}(d_1, e_1, r_1, d_2, e_2, r_2) := Y1(d_1, e_1, r_1, d_2, e_2, r_2) - Y2(d_1, e_1, r_1,$

$\mathrm{Int}_{y2}(d_1, e_1, r_1, d_2, e_2, r_2) := Y1(d_1, e_1, r_1, d_2, e_2, r_2) + Y2(d_1, e_1, r_1,$

Define circles with centres (x,y) and radii (c)

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} := d$ $\quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} := e$ $\quad \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} := f$

Equations used to plot the graphs later

$y1m(x) := -\sqrt{c_1^2 - (x - x_1)^2} + y_1$ $\qquad y1p(x) := \sqrt{c_1^2 - (x - x_1)^2} + y_1$

$y2m(x) := -\sqrt{c_2^2 - (x - x_2)^2} + y_2$ $\qquad y2p(x) := \sqrt{c_2^2 - (x - x_2)^2} + y_2$

$y3m(x) := -\sqrt{c_3^2 - (x - x_3)^2} + y_3$ $\qquad y3p(x) := \sqrt{c_3^2 - (x - x_3)^2} + y_3$

Intersections of a pair of circles (if they touch 0 or 1 times then 2 points are still defined)

$$\mathrm{Pint}(x_1, y_1, c_1, x_2, y_2, c_2) := \left| \begin{array}{l} dist \leftarrow \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \\ \mathrm{Pint} \leftarrow \begin{pmatrix} \mathrm{Int}_{x4}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y4}(x_1, y_1, c_1, x_2, y_2, c_2) \\ \mathrm{Int}_{x4}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y4}(x_1, y_1, c_1, x_2, y_2, c_2) \end{pmatrix} \text{ if } |c_1 - c_2| > dist \\ \mathrm{Pint} \leftarrow \begin{pmatrix} \mathrm{Int}_{x3}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y3}(x_1, y_1, c_1, x_2, y_2, c_2) \\ \mathrm{Int}_{x3}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y3}(x_1, y_1, c_1, x_2, y_2, c_2) \end{pmatrix} \text{ if } |c_1 + c_2| < dist \\ \mathrm{Pint} \leftarrow \begin{pmatrix} \mathrm{Int}_{x1}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y1}(x_1, y_1, c_1, x_2, y_2, c_2) \\ \mathrm{Int}_{x2}(x_1, y_1, c_1, x_2, y_2, c_2) & \mathrm{Int}_{y2}(x_1, y_1, c_1, x_2, y_2, c_2) \end{pmatrix} \text{ otherwise} \\ \text{return } \mathrm{Pint} \end{array} \right.$$

completely inside not touching

completely outside not touching

touching either once or twice

Vectors of all the x and y coordinates of the intersections (when working with 3 circles)

$$intersect_x := \begin{pmatrix} \mathrm{Pint}(x_1,y_1,c_1,x_2,y_2,c_2)_{0,0} \\ \mathrm{Pint}(x_1,y_1,c_1,x_2,y_2,c_2)_{1,0} \\ \mathrm{Pint}(x_1,y_1,c_1,x_3,y_3,c_3)_{0,0} \\ \mathrm{Pint}(x_1,y_1,c_1,x_3,y_3,c_3)_{1,0} \\ \mathrm{Pint}(x_3,y_3,c_3,x_2,y_2,c_2)_{0,0} \\ \mathrm{Pint}(x_3,y_3,c_3,x_2,y_2,c_2)_{1,0} \end{pmatrix} \quad intersect_y := \begin{pmatrix} \mathrm{Pint}(x_1,y_1,c_1,x_2,y_2,c_2)_{0,1} \\ \mathrm{Pint}(x_1,y_1,c_1,x_2,y_2,c_2)_{1,1} \\ \mathrm{Pint}(x_1,y_1,c_1,x_3,y_3,c_3)_{0,1} \\ \mathrm{Pint}(x_1,y_1,c_1,x_3,y_3,c_3)_{1,1} \\ \mathrm{Pint}(x_3,y_3,c_3,x_2,y_2,c_2)_{0,1} \\ \mathrm{Pint}(x_3,y_3,c_3,x_2,y_2,c_2)_{1,1} \end{pmatrix}$$

$$intersect_x = \begin{pmatrix} 4.045\times10^{-3} \\ 4.045 \\ 0.018-4.3 \\ 0.018-4.3 \\ 6.645\times10^{-3}- \\ 6.645\times10^{-3}-3.738i\times10^{-3} \end{pmatrix} \quad intersect_y = \begin{pmatrix} -0.283 \\ -0.283 \\ -0.418-0.042i \\ -0.418-0.042i \\ 0.295+0.042i \\ 0.295+0.042i \end{pmatrix}$$

Function to define the perimeter of the triangle formed by 3 points

$$\Delta(u_1,u_2,u_3,v_1,v_2,v_3) := \sqrt{(u_1-u_2)^2+(v_1-v_2)^2} + \sqrt{(u_1-u_3)^2+(v_1-v_3)^2} + \sqrt{(u_3-u_2)^2+(v_3-v_2)^2}$$

Define the function over all combinations of 3 from our 6 points

$$\Delta s(i,j,k) := \Delta(intersect_{x_i}, intersect_{x_j}, intersect_{x_k}, intersect_{y_i}, intersect_{y_j}, intersect_{y_k})$$

$i := 0..5 \quad j := 0..5 \quad k := 0..5$

Find the minimum perimeter triangle (can't have a triangle where 2 or 3 of the points are the same point)

$$min\Delta xyz := \left| \begin{array}{l} min\Delta \leftarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 10^{10} \end{pmatrix} \\ \text{for } i_2 \in i \\ \quad \text{for } j_2 \in j \\ \quad\quad \text{for } k_2 \in k \\ \quad\quad\quad min\Delta \leftarrow \begin{pmatrix} i_2 \\ j_2 \\ k_2 \\ \Delta s(i_2, j_2, k_2) \end{pmatrix} \text{ if } \Delta s(i_2,j_2,k_2) < min\Delta_3 \wedge i_2 \ne j_2 \wedge i_2 \ne k_2 \wedge j_2 \ne k_2 \\ \text{return } min\Delta \end{array} \right.$$

$j := 0..2$

$min\Delta xyz = \blacksquare \qquad \Delta xs_j := intersect_{x_{min\Delta xyz_j}} \qquad \Delta ys_j := intersect_{y_{min\Delta xyz_j}}$

Average the coordinates of the 3 closest points

$\Delta centre_x := \mathrm{mean}(\Delta xs) \quad \Delta centre_y := \mathrm{mean}(\Delta ys) \qquad \Delta centre_x = 0 \qquad \Delta centre_y = 0$

Conclusions

- Power splitters can be measured in a variety of ways
- Measuring power splitters can be tricky without access to all ports!