Robust Calibration for Sigma-Delta (ΣΔ) Analogue-to-Digital Converters

M. Gani, J. Mckernan, F. Yang, D. Henrion, CDSPR (KCL)/KCL/Brunel/LAAS
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Contact: mahbub.gani@kcl.ac.uk
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Introduction

- Motivation: digital signal processing for broadband communication and audio systems; software radio
- Precision Conversion: Cost-effective, high speed and precision analog-to-digital (A/D) converters
- Potential architecture of choice: Sigma Delta modulator
Sigma Delta Modulator ADC

- Oversamples Input
- Integrates (Sigma)
- Quantises (typically 1-bit)
- Feedback to input (Delta)
- Decimates output (wordlength increases)
- Although non-linear, investigate just quantisation error, & model as linear.
In order to improve performance, high-order noise shaping filters are employed:
- Relax requirements on OSR
- Quantisation noise moved out of signal band
- Two ways to implement the high-order noise shaping filters:
  - High Order Single Loops
  - Cascaded Loops
High-Order Single loop

- The high-order single-loop SD modulator
- Only conditionally stable
Cascaded (Mash) Modulator

- Unconditionally stable.
- Need accurate matching of analog filters/amplifiers $H$ with digital noise cancellation filters $C$.
- Any mismatch produces leakage of unshaped quantisation noise and poor SNR performance.
Cascaded Modulator Model

If system known *precisely*, chose filters C1,C2 so that 1st Stage Quantisation Error e1 cancelled at Output y. In practice the system analog components vary.
Achieving Robustness

Two methods to deal with the analog circuit imperfections:

- Gain estimation and adaptive correction method
  - increases complexity of circuit implementation,
  - circuit real estate and costs.
- Robust correction method
  - proposed method
  - based on convex optimisation
  - no additional burden on implementation.
Convex Optimisation

- A set $S$ is convex if
  $$\{x_1, x_2 \in S\} \Rightarrow \{\alpha x_1 + (1-\alpha)x_2 \in S, \forall \alpha \in S\}$$

- A function $f : S \rightarrow \mathbb{R}$ is convex if $S$ is convex and $\forall x_1, x_2 \in S, \alpha \in (0,1)$
  $$f(\alpha x_1 + (1-\alpha)x_2) < \alpha f(x_1) + (1-\alpha)f(x_2)$$
Why Convex Optimisation?

- Every local optimal solution of $f$ is a global one
- We can test just vertices if $S$ is a convex hull
- Linear Matrix Inequalities (LMIs) represent convex constraints
- Many control problems can be cast as LMIs
  - feasibility – test LMI constraint feasible
  - optimization over LMI constraint

$$u^T F(x) u > 0, \forall u \neq 0$$

where $F(x) = F_0 + x_1 F_1 + \ldots + x_m F_m$

$x = (x_1, \ldots x_m)$ and $F_0, \ldots F_m$ are real symmetric matrices
System for Main Theorem
Main Theorem 1

- System in state-space form $A, B, C, D$
- $A, B, C, D$ vary with component uncertainty
- **Theorem**: For all $N$ vertices $(A(i), B(i), C(i), C1(i), D(i), D1(i))$ $(i=1, 2, \ldots, N)$. There exists a robust filter if the following optimisation problem is solvable:

$$\min_{R, V, W, Q_1, Q_2, Q_3, Q_4, P_1^{(i)}, P_2^{(i)}, P_3^{(i)}} \gamma$$

$s.t. P_1^{(i)} > 0, P_2^{(i)} > 0, P_3^{(i)} > 0, (i=1, 2, \ldots, N)$
Main Theorem 2

subject to LMI

\[
\begin{bmatrix}
P_1^{(i)} - V - V^T & P_2^{(i)} - R - V^T - W^T & V^T A^{(i)} & V^T A^{(i)} & V^T B^{(i)} & 0 \\
* & P_3^{(i)} - R^T - R & R^T A^{(i)} + Q_2 C^{(i)} + Q_1 & R^T A^{(i)} + Q_2 C^{(i)} & R^T B^{(i)} + Q_2 D^{(i)} & 0 \\
* & * & -P_1^{(i)} & -P_2^{(i)} & 0 & (C_1^{(i)} - Q_4 C^{(i)})^T - Q_3^T \\
* & * & * & -P_3^{(i)} & 0 & (C_1^{(i)} - Q_4 C^{(i)})^T \\
* & * & * & * & -\gamma I & (D_1^{(i)} - Q_4 D^{(i)})^T \\
* & * & * & * & * & -\gamma I \\
\end{bmatrix} < 0,
\]

for \( i = 1, 2, \ldots, N \). Filter given by:

\[
Q(z) = Q_3 W^{-1} (z I - Q_1 W^{-1})^{-1} Q_2 + Q_4
\]
Simulations

- Transfer functions of the analog nonideal integrators $H_j(z)$ ($j=1,2,3$) are given by

$$H_j(z) = \frac{1 - a_j}{1 - (1 - b_j) z^{-1}}$$

- Analog imperfections manifested as uncertainties in gains and poles with ranges:

$$0 \leq a_j \leq 0.05,$$

$$0 \leq b_j \leq 0.05$$
Simulation Results

- Plots of deviation in SNR as parameters varied for range of input levels
- Worst case SNR is higher for robust filter (right) than for nominal filter
- Also variation of SNR lower for robust filter

Fig. 5. The SNR performance change range with the parameters imperfections deviation from 0 to 0.05 using the nominal filter.

The SNR performance change range with the parameters imperfections deviation from 0 to 0.05 using the proposed robust filter.
Simulation Results

- Nominal filter better for small imperfections (left).
- Robust filter better for larger imperfections (right).
Future Work

- Component uncertainty represented as a polytope of open-loop system polynomials $i$

\[ T_i = \frac{a_i(z)}{b_i(z)}, i = 1\ldots16 \]

- Stability & robust performance over polytope required

\[ \max_i \left\| T_{E1, Y_i} \right\|_\infty < \gamma \]

- Design posed as a Linear Matrix Inequality (LMI) feasibility problem over Q polynomials

\[ Q = \frac{n(z)}{d(z)} \]

- Filter Order Preset in ‘Central Polynomial’
Results for Flexible Structure

- Controller Robustly Minimises Sensitivity $S$ over structured uncertainties (blue=closed, green=open-loop)
Conclusions

- Proposed approach suitable for performance A/D converters
  - No increase in complexity of underlying circuitry.
  - No careful design of analog noise-shaping filters.
- Technique an alternative design philosophy
  - Worst-case design approach.