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Uncertainty and traceability for measurement of reflection coefficient

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Abstract

The main aim of this report is to show, that the potentially dominant contribution to the uncertainty of the phase of reflection coefficient is often being neglected. The problem of traceability for reflection measurement is briefly described. Uncertainty calculation for two calibration methods utilizing matrix representation of the law of uncertainty propagation is presented.

Introduction

In recent times, the problems connected with evaluating and expressing uncertainty in complex S-parameter measurements have been often discussed and many articles on the theme have been published. In the centre of interest of this article is uncertainty of phase of the reflection coefficient. If the random contributions (e.g. cable flexure) are not taken into account, the uncertainty of phase is often calculated by equation (1).

$$u(\arg(\Gamma)) = \arcsin \frac{u(|\Gamma|)}{|\Gamma|} \tag{1}$$

This equation is valid only when the uncertainty region around the measured value is of the circular shape but this is not generally the case. Therefore, in this way calculated phase uncertainty can be sometimes significantly underestimated. This is demonstrated by uncertainty analysis for the OSL and TRL calibration methods using the matrix representation of the law of uncertainty propagation [1]. This report is a modified version of the text presented at the 20-th ANAMET meeting.

Traceability for reflection measurements

For measurements of reflection coefficient the error model can be represented as follows

$$\Gamma_M = D_R + \frac{T_R \Gamma_A}{1 - M_R \Gamma_A}, \tag{2}$$

where

Γ_M … reflection coefficient value indicated by the calibrated VNA
Γ_A … actual value of the reflection coefficient
D_R, T_R, M_R … residual error terms of the calibrated VNA.

The residual errors of the calibrated VNA can be evaluated by the ripple method. This procedure is described in the document EA-10/12. The traceability to SI units is established by the airline with electrical characteristics calculable from its geometrical dimensions. The line is here used only as a verification standard. The ripple method yields only the information about the magnitude of residual error terms D_R and M_R. These residual error terms cause the errors in both magnitude and phase measurement with sign + or – depending on the value of measurand while the residual error term T_R represents the systematic offset in both magnitude and phase. In order to evaluate the residual reflection tracking T_R, i.e. to make the measurements completely traceable to SI units, the high reflection standard with calculable or suitably certified electrical characteristics is needed. However, the magnitude of residual reflection tracking specified by the manufacturer is usually lower than several hundredths of dB. Hence, using such a high reflection standard is not unconditionally necessary for magnitude measurement.

A special case is the TRL (LRL) calibration. Similarly, the airline(s) serve as the impedance standard(s) with calculable electrical characteristics. The manner of establishing the traceability for reflection measurement is dependent on the type and sex of connectors at both test ports. The TRL algorithm allows to set the reference plane by the definition of the reflection standard (SET REF REFLECT option in AGILENT (HP) VNA's) or by the definition of the THRU standard (SET REF THRU option). When the SET REF REFLECT option is chosen for the TRL calibration, the phase of the residual reflection tracking is the same as the phase deviation of the reflection standard from the nominal value. The TRL method is based on the assumption that both reflection standards are identical. Hence, when the SET REF THRU option is used (and the electrical length of THRU exactly known), the phase of residual reflection tracking is a half of the difference between the phase deviations of both reflection standards. When the same reflection standard is used at both measurement ports, the phase of residual reflection tracking T_R is (in ideal case) 0.

The conclusions from the above are as follows:

- The traceability for the transmission medium with sexless connectors can be established with the TRL line.
- The traceability for the connectors with the same sex can be established with two (noninsertable) lines ('THRU' and 'LINE').
- The traceability for the connectors with different sex can be established with one TRL line and (at least) one calculable reflection standard.

Conversion between mag/phase and Re/Im form of uncertainty expression

The uncertainty of a complex quantity can be expressed in two forms: either in terms of its real and imaginary components, or as magnitude uncertainty and phase uncertainty. Sometimes, the first form of expression can be more convenient, sometimes the second one. Conversion between the two forms can be done utilizing the matrix representation of the law of uncertainty propagation presented in [1]. This law states:

$$V(\underline{Y}) = JV(\underline{X})J^T \tag{3}$$

where $V(\underline{X})$ is the covariance matrix of the input vector $\underline{X}$, $V(\underline{Y})$ is the covariance matrix of the output vector $\underline{Y}$ and J is the Jacobian matrix.

When the uncertainty of reflection coefficient Γ in the form Re/Im is to be obtained, the vector of input quantities is

$$\underline{X} = (x_1, x_2) = (|\Gamma|, \phi_\Gamma) \tag{4}$$

where $\phi_\Gamma = \arg(\Gamma)$ and vector of output quantities is

$$\underline{Y} = (y_1, y_2) = (\mathrm{Re}(\Gamma), \mathrm{Im}(\Gamma)) \tag{5}$$

The Jacobian matrix is:

$$J = \begin{bmatrix} \cos\phi_\Gamma & -|\Gamma|\sin\phi_\Gamma \\ \sin\phi_\Gamma & |\Gamma|\cos\phi_\Gamma \end{bmatrix} \tag{6}$$

When the uncertainty in the form mag/phase is to be obtained, the vector of input quantities is

$$\underline{X} = (x_1, x_2) = (\mathrm{Re}(\Gamma), \mathrm{Im}(\Gamma)) \tag{7}$$

and vector of output quantities is

$$\underline{Y} = (y_1, y_2) = (|\Gamma|, \phi_\Gamma) \tag{8}$$

The Jacobian matrix is:

$$J = \begin{bmatrix} \cos\phi_\Gamma & \sin\phi_\Gamma \\ -\dfrac{\sin\phi_\Gamma}{|\Gamma|} & \dfrac{\cos\phi_\Gamma}{|\Gamma|} \end{bmatrix} \tag{9}$$

Note that the above procedure yields reasonable results only when the magnitude of the complex quantity is significantly greater than the uncertainty.

Uncertainty for OSL calibration

Using the law of uncertainty propagation in matrix form the uncertainties for one-port calibration can be analysed. The vector of input quantities is

$$\underline{X} = (\mathrm{Re}(\Gamma_1), \mathrm{Im}(\Gamma_1), \mathrm{Re}(\Gamma_2), \mathrm{Im}(\Gamma_2), \mathrm{Re}(\Gamma_3), \mathrm{Im}(\Gamma_3)) \tag{10}$$

where Γ_1, Γ_2, Γ_3 are the reflection coefficients of the calibration standards. The two-dimensional vector of output quantities is

$$\underline{Y} = (\mathrm{Re}(\Gamma), \mathrm{Im}(\Gamma)) \tag{11}$$

where Γ is the measured value of reflection coefficient. The covariance matrix of the input vector is

$$V(\underline{X}) = \begin{bmatrix} u^2(\mathrm{Re}(\Gamma_1)) & u(\mathrm{Re}(\Gamma_1),\mathrm{Im}(\Gamma_1)) & 0 & 0 & 0 & 0 \\ u(\mathrm{Im}(\Gamma_1),\mathrm{Re}(\Gamma_1)) & u^2(\mathrm{Im}(\Gamma_1)) & 0 & 0 & 0 & 0 \\ 0 & 0 & u^2(\mathrm{Re}(\Gamma_2)) & u(\mathrm{Re}(\Gamma_2),\mathrm{Im}(\Gamma_2)) & 0 & 0 \\ 0 & 0 & u(\mathrm{Im}(\Gamma_2),\mathrm{Re}(\Gamma_2)) & u^2(\mathrm{Im}(\Gamma_2)) & 0 & 0 \\ 0 & 0 & 0 & 0 & u^2(\mathrm{Re}(\Gamma_3)) & u(\mathrm{Re}(\Gamma_3),\mathrm{Im}(\Gamma_3)) \\ 0 & 0 & 0 & 0 & u(\mathrm{Im}(\Gamma_3),\mathrm{Re}(\Gamma_3)) & u^2(\mathrm{Im}(\Gamma_3)) \end{bmatrix} \tag{12}$$

where the off-diagonal elements represent covariance terms. Covariance terms for individual standards are

$$u(\mathrm{Re}(\Gamma_i),\mathrm{Im}(\Gamma_i)) = u(\mathrm{Im}(\Gamma_i),\mathrm{Re}(\Gamma_i)) = u(\mathrm{Re}(\Gamma_i))u(\mathrm{Im}(\Gamma_i))r(\mathrm{Re}(\Gamma_i),\mathrm{Im}(\Gamma_i)) \qquad (13)$$

where $r(\mathrm{Re}(\Gamma_i),\mathrm{Im}(\Gamma_i))$ are the correlation coefficients. It is assumed that Γ_1, Γ_2 and Γ_3 are uncorrelated. In [2], [4] the expressions relating the measurement errors to the errors of calibration standards have been shown. Utilizing these expressions the Jacobian matrix can be written:

$$J = \left[\begin{matrix} \mathrm{Re}\left(\frac{(\Gamma-\Gamma_2)(\Gamma-\Gamma_3)}{(\Gamma_1-\Gamma_2)(\Gamma_1-\Gamma_3)}\right) & -\mathrm{Im}\left(\frac{(\Gamma-\Gamma_2)(\Gamma-\Gamma_3)}{(\Gamma_1-\Gamma_2)(\Gamma_1-\Gamma_3)}\right) & \mathrm{Re}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_3)}{(\Gamma_2-\Gamma_1)(\Gamma_2-\Gamma_3)}\right) \\ \mathrm{Im}\left(\frac{(\Gamma-\Gamma_2)(\Gamma-\Gamma_3)}{(\Gamma_1-\Gamma_2)(\Gamma_1-\Gamma_3)}\right) & \mathrm{Re}\left(\frac{(\Gamma-\Gamma_2)(\Gamma-\Gamma_3)}{(\Gamma_1-\Gamma_2)(\Gamma_1-\Gamma_3)}\right) & \mathrm{Im}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_3)}{(\Gamma_2-\Gamma_1)(\Gamma_2-\Gamma_3)}\right) \end{matrix} \right. \cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots \left. \begin{matrix} -\mathrm{Im}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_3)}{(\Gamma_2-\Gamma_1)(\Gamma_2-\Gamma_3)}\right) & \mathrm{Re}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_2)}{(\Gamma_3-\Gamma_1)(\Gamma_3-\Gamma_2)}\right) & -\mathrm{Im}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_2)}{(\Gamma_3-\Gamma_1)(\Gamma_3-\Gamma_2)}\right) \\ \mathrm{Re}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_3)}{(\Gamma_2-\Gamma_1)(\Gamma_2-\Gamma_3)}\right) & \mathrm{Im}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_2)}{(\Gamma_3-\Gamma_1)(\Gamma_3-\Gamma_2)}\right) & \mathrm{Re}\left(\frac{(\Gamma-\Gamma_1)(\Gamma-\Gamma_2)}{(\Gamma_3-\Gamma_1)(\Gamma_3-\Gamma_2)}\right) \end{matrix} \right] \qquad (14)$$

For several locations of the measured reflection coefficient in the complex plane the covariance matrices were calculated. As an example, a type-N open-short-sliding load calibration at 18 GHz was considered. Results are presented in Tab. 1.

For the calculation, the following input data were used:

OPEN
phase of reflection coefficient -103.3°
reflection coefficient uncertainty:
phase: 1.5° (manufacturer’s specification)
magnitude: 0.003 (verified using TRL calibration)
no correlation between magnitude and phase assumed

SHORT
phase of reflection coefficient 82.2°
reflection coefficient uncertainty:
phase: 1.0° (manufacturer’s specification)
magnitude: 0.003 (verified using TRL calibration)
no correlation between magnitude and phase assumed

LOAD
reflection coefficient uncertainty:
real part: 0.008 (manufacturer’s specification)
imaginary part: 0.008 (manufacturer’s specification)
no correlation between real and imaginary part assumed

reflection coefficient		uncertainties & correlation coefficients					
magnitude	phase (°)	$u(\mathrm{Re}(\Gamma))$	$u(\mathrm{Im}(\Gamma))$	$r(\mathrm{Re}(\Gamma),\mathrm{Im}(\Gamma))$	$u(\|\Gamma\|)$	$u(\phi_\Gamma)$ (°)	$r(\|\Gamma\|,\phi_\Gamma)$
1	0	0.023	0.022	-0.10	0.023	1.28	-0.10
1	45	0.015	0.019	-0.30	0.014	1.09	0.25
1	90	0.018	0.004	0.27	0.004	1.00	-0.27
1	135	0.018	0.019	0.10	0.018	1.11	-0.10
1	180	0.021	0.023	0.27	0.021	1.31	0.27
1	225	0.016	0.023	-0.69	0.012	1.46	0.51
1	270	0.026	0.006	0.49	0.006	1.51	-0.49
1	315	0.018	0.027	0.26	0.020	1.45	-0.41
0.5	0	0.011	0.013	-0.07	0.011	1.47	-0.07
0.5	45	0.009	0.010	-0.25	0.008	1.22	0.01
0.5	90	0.009	0.006	0.01	0.006	1.08	-0.01
0.5	135	0.010	0.010	0.21	0.009	1.29	0.02
0.5	180	0.010	0.013	0.10	0.010	1.44	0.10
0.5	225	0.009	0.011	-0.44	0.008	1.38	0.13
0.5	270	0.012	0.006	0.10	0.006	1.35	-0.10
0.5	315	0.011	0.012	0.25	0.010	1.48	-0.15
0.1	0	0.008	0.008	0.00	0.008	4.76	0.00
0.1	90	0.008	0.008	0.00	0.008	4.58	0.00
0	0	0.008	0.008	0.00	0.008	-	-

Tab.1 Uncertainties for OSL calibration

Reflection coefficient uncertainty for TRL calibration

The law of uncertainty propagation in matrix form can be used for the analysis as well. For simplicity, only two main causes of errors are considered. The first one is the reflection coefficient of the airline and the second one is the difference between reflection standards (shorts) used at both test ports. The vector of input quantities is

$$\underline{X} = \left(|\Gamma_1|, \arg(\Gamma_1), |\Gamma_2|, \arg(\Gamma_2), \mathrm{Re}(\Gamma_{LINE}), \mathrm{Im}(\Gamma_{LINE})\right) \tag{15}$$

where Γ_1, Γ_2 are the reflection coefficients of reflection standards (SHORTs) used for calibration, and Γ_{LINE} is the reflection coefficient of the TRL line. The vector of output quantities is

$$\underline{Y} = \left(|\Gamma|, \arg(\Gamma)\right) \tag{16}$$

The measured value of reflection coefficient can be expressed

$$\Gamma_M \cong D_R + T_R\Gamma + T_R M_R \Gamma^2 \tag{17}$$

where Γ is the actual value of measured reflection coefficient and D_R, M_R, T_R are residual error terms. The airline serves as an impedance standard, in a similar way as the load in an OSL calibration. The residual error terms D_R and M_R were calculated analogically as for OSL calibration in [4].

$$D_R = -M_R = -\Gamma_{LINE} \tag{18}$$

Then

$$\Gamma_M = T_R\Gamma + \left(T_R\Gamma^2 - 1\right)\cdot\Gamma_{LINE} \tag{19}$$

and for $T_R \approx 1$ the partial derivative with respect to Γ_{LINE} is

$$\frac{\partial\Gamma_M}{\partial\Gamma_{LINE}} = \Gamma^2 - 1 \tag{20}$$

The correctness of the assumption that $M_R = -D_R$ for TRL calibration is still to be analyzed. However, in the presented example the error term M_R doesn't affect significantly the phase uncertainty.

The partial derivatives with respect to parameters of reflection standards for the Jacobian matrix were stated in the following way. In [5] (appendix VIII: Self-Calibration Procedure) the mathematical solution for the TRL calibration procedure is described. This solution is based on the assumption that both high reflection standards are identical, but the solution can be modified (equations 33, 35, 36, ...) for non-identical two standards. The analysis was performed with the following result. With changes of the assumed reflection coefficients of standards the error terms are changing so that the corrected value of measured reflection coefficient is changing proportionally to the value of $\sqrt{\Gamma_{A1}/\Gamma_{A2}}$, where Γ_{A1},Γ_{A2} are the values of reflection coefficient of the standards. Thus, the sensitivity coefficients for Jacobian matrix are equal to 0.5 and -0.5.

The Jacobian matrix is

$$J = \begin{bmatrix} 0.5 & 0 & -0.5 & 0 & \mathrm{Re}\left(\Gamma^2-1\right)\cdot\cos\phi_\Gamma + \mathrm{Im}\left(\Gamma^2-1\right)\cdot\sin\phi_\Gamma & -\mathrm{Im}\left(\Gamma^2-1\right)\cdot\cos\phi_\Gamma + \mathrm{Re}\left(\Gamma^2-1\right)\cdot\sin\phi_\Gamma \\ 0 & 0.5 & 0 & -0.5 & -\frac{1}{|\Gamma|}[\mathrm{Re}\left(\Gamma^2-1\right)\cdot\sin\phi_\Gamma - \mathrm{Im}\left(\Gamma^2-1\right)\cdot\cos\phi_\Gamma] & \frac{1}{|\Gamma|}[\mathrm{Im}\left(\Gamma^2-1\right)\cdot\sin\phi_\Gamma - \mathrm{Re}\left(\Gamma^2-1\right)\cdot\cos\phi_\Gamma] \end{bmatrix} \tag{21}$$

As an example, uncertanties for 3.5 mm connectors at a frequency of about 20 GHz were calculated. It is assumed that the TRL option SET REF THRU and connectors with opposite sex at test ports are used.

For the calculation, the following input data were used:

TRL LINE
return loss of the line 50 dB (manufacturer's specification)

SHORTS
reflection coefficient uncertainty:

phase	1.75°	(manufacturer's specification)
magnitude	0.0009	(experimentally verified connector repeatability)

No correlation between parameters of standards is assumed.

Results are presented in Tab. 2.

reflection coefficient		uncertainties & correlation coefficients					
magnitude	phase (°)	$u(\mathrm{Re}(\Gamma))$	$u(\mathrm{Im}(\Gamma))$	$r(\mathrm{Re}(\Gamma),\mathrm{Im}(\Gamma))$	$u(\|\Gamma\|)$	$u(\phi_\Gamma)$ (°)	$r(\|\Gamma\|,\phi_\Gamma)$
1	0	0.001	0.022	0.00	0.001	1.24	0.00
1	45	0.016	0.016	-0.93	0.004	1.26	0.00
1	90	0.022	0.006	0.00	0.006	1.28	0.00
1	135	0.016	0.016	0.93	0.004	1.26	0.00
1	180	0.001	0.022	0.00	0.001	1.24	0.00
1	225	0.016	0.016	-0.93	0.004	1.26	0.00
1	270	0.022	0.006	0.00	0.006	1.28	0.00
1	315	0.016	0.016	0.93	0.004	1.26	0.00
0.5	0	0.002	0.011	0.00	0.002	1.26	0.00
0.5	45	0.008	0.008	-0.85	0.003	1.29	0.00
0.5	90	0.011	0.004	0.00	0.004	1.31	0.00
0.5	135	0.008	0.008	0.85	0.003	1.29	0.00
0.5	180	0.002	0.011	0.00	0.002	1.26	0.00
0.5	225	0.008	0.008	-0.85	0.003	1.29	0.00
0.5	270	0.011	0.004	0.00	0.004	1.31	0.00
0.5	315	0.008	0.008	0.85	0.003	1.29	0.00
0.1	0	0.003	0.004	0.00	0.003	2.10	0.00
0.1	90	0.004	0.003	0.00	0.003	2.13	0.00
0.01	0	0.003	0.003	0.00	0.003	17.23	0.00

Tab. 2 Uncertainties for TRL calibration

Conclusions

The law of propagation of uncertainty in matrix form has been applied to calculate the uncertainty for one-port measurement by using the OSL and TRL calibration methods. Only the systematic errors caused by imperfect standards were taken into account. Results obtained for TRL calibration can be used to demonstrate that the phase uncertainty values calculated by equation (1) can be significantly underestimated. For magnitude of reflection coefficient of approximately 1, the phase uncertainty is more than three times greater than the value calculated by equation (1). The reason is that the phase of residual reflection tracking T_R is ignored in the uncertainty calculation. This contribution can be evaluated either by measurement of additional high reflection standard(s) with known phase response or by using specified phase uncertainties for high reflection calibration standards. Although the underestimating of phase uncertainty for OSL calibration is less probable, the mentioned contribution should be taken into account as well.

References:

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