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**DRAFT FOR COMMENT – PROPOSED REVISION TO THE
EA GUIDELINES ON EVALUATION OF VECTOR
NETWORK ANALYSERS (VNA) TO INCLUDE
UNCERTAINTY IN PHASE**

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DRAFT FOR COMMENT – PROPOSED REVISION TO THE
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INCLUDE UNCERTAINTY IN PHASE

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FOREWORD

This report contains a proposed revised version of the following EA guidance document¹:

“EA guidelines on the evaluation of vector network analysers (VNA)”, publication reference EA-10/12, May 2000.

The above original version of this document did not contain any information concerning methods for evaluating the uncertainty in phase for measurements of reflection and transmission coefficients. The proposed revised version of the document, presented in this report, contains sections dealing with uncertainty in phase for both reflection and transmission coefficients.

This proposed revision is therefore being made available to members of ANAMET specifically to invite comment on the methods presented for evaluating the uncertainty in phase measurements. This report is therefore labelled as a “Draft for Comment”. It is hoped that members of ANAMET will comment on the suitability, or otherwise, of the methods presented and that this will help with the formulation of a new edition of this document for consideration by EA.

Therefore, any comments concerning the methods presented in this document should be sent either to the Report’s author, Dr J P M de Vreede, or myself.

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August 2003

¹ EA stands for European co-operation for Accreditation. A copy of this guidance document can be downloaded from the EA web-site at www.european-accreditation.org.

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1. INTRODUCTION

- 1.0.1 This document has been produced by EA as a means of giving advice to laboratories using VNAs on how to evaluate some of their important characteristics. It describes the measurement procedures which may be carried out for an assessment of VNAs to meet the requirements of accreditation against EN45001. The principles given in this document apply to any frequency range for which VNAs can be used, and to any transmission medium; however, some of the techniques given for the assessment of uncertainties are only applicable to coaxial lines at frequencies above 500 MHz. The procedures specifically apply to measurements in coaxial line fitted with connectors having mating surfaces conforming to the relevant IEC Standard 457 [1] or to the IEEE Standard 287 [2] such as:

<u>Line Diameter</u>	<u>Connector</u>
14 mm	GR 900 (or equivalent)
7 mm	Type-N (precision N)
7 mm	PC 7 (or equivalent)
3.5 mm	PC 3.5

- 1.0.2 The procedure may have to be modified for other types of connector, including waveguide transmission line.
- 1.0.3 The calibration of a VNA must, by definition, cover phase as well as magnitude capabilities. In the first version the uncertainties as produced using this document were only applicable to magnitude quantities. In this version also uncertainty in phase is taken into account.

2. DOCUMENTATION

- 2.0.1 The laboratory should conform to the normal requirements of accreditation with respect to procedure documents that describe the calibration and use of the VNA, this should include at least the following:

- General description of equipment;
- List of reference standards and ancillary equipment;
- Operating instructions (reference to manufacturer's documents);
- Method of calibration (reference to this document);
- Uncertainty analysis and uncertainty budgets;
- Precautions for connector use;
- Requirements for environmental conditions;
- Other precautions to be taken.

3. REFERENCE STANDARDS

3.0.1 A number of standards will be required as follows:

3.1 A Calibration Kit

3.1.1 At least one calibration kit will be required for each connector type. The calibration kit supplied by the manufacturer will normally suffice. The calibration kit or kits used must be specified and uniquely identified in the procedure document. Each kit should contain a good short-circuit standard capable of supporting the inner-conductor of a beadless reference air line, one for each sex of connector.

3.2 A Set of Beadless Airlines

3.2.1 At least one beadless reference airline will be required for each type of connector, with appropriately dimensioned contacts at each end. The nominal length of the airline should be between 75 mm and 300 mm. Its physical dimensions should be recorded, together with their estimated uncertainties, using a method, which measurement results are traceable to national standards. The relevant dimensions are

- a) the length between the mating faces of the outer conductor
- b) the length of the inner conductor [for sexless connectors this should be measured from end to end. The reference surface for measurement of a female contact should be its end. That for a male contact should be the shoulder formed at the nominal inner-conductor diameter]
- c) the average outer diameter of the inner-conductor
- d) the average bore diameter of the outer-conductor.

3.2.2 A beadless airline supplied as part of a VNA manufacturer's verification kit may be used if its dimensions are suitably certified, i.e. traceable to national standards.

3.2.3 The geometrical length and ratio of bore and outer diameters can be used to determine both the phase shift introduced by the airline and the nominal characteristic impedance of the airline. A conversion between geometrical and electrical characteristics should be done using the following equation for impedance:

$$Z_0 = \sqrt{\frac{L'_0}{C_0}} = \sqrt{\frac{\mu_r \cdot \mu_0}{\varepsilon_r \cdot \varepsilon_0}} \cdot \left[\frac{\ln(b/a)}{2 \cdot \pi} \right] = \frac{\mu_0 \cdot c_0 \cdot \ln(b/a)}{2 \cdot \pi \cdot \sqrt{\varepsilon_r}} = 59,93916 \cdot \ln(b/a) \quad (1)$$

where b and a refer to the outer and inner diameters of the airline, respectively. The constants μ_0 and ε_0 are natural constants. For the air dielectric, $\mu_r = 1$ and $\varepsilon_r = 1.000645$ can be taken as standard values.

For the phase shift (in °) the following equation may be used:

$$\Phi_L(f) = 360 * L * f * \sqrt{\varepsilon_r * \mu_r} / c_0 = 1.2^\circ * L * f \quad \{1a\}$$

where L is the geometrical length of the airline (in mm) and f is the frequency (in GHz).

Hence the uncertainties in the geometrical quantities fully determine its electrical uncertainties.

For example, a manufacturer states for a 7 mm beadless airline (50 Ω impedance) an uncertainty of 0.05 % for the outer diameter and an uncertainty of 0.1 % for the inner diameter. Using these values to determine the maximum deviation in the calculation one obtains a deviation of about 0.17 % in the characteristic impedance of this airline.

In general the thermal expansion coefficient of metals is about $(1-2) \cdot 10^{-5}/K$. Usually geometrical measurements are done at 20 °C while electrical measurements are done at 23 °C, which means an adjustment of the geometrical dimensions by about $5 \cdot 10^{-5}$ when working on electrical quantities. A length measurement uncertainty of about 1.5 μm on an airline of about 100 mm is quite common. In this case it implies a relative uncertainty of $1.5 \cdot 10^{-5}$ in the phase measurement.

3.3 A calibrated step attenuator or a set of calibrated fixed attenuators

3.3.1 These are used to verify the linearity of the VNA with respect to the national standards for RF attenuation. A series of calibrated fixed attenuators (e.g. 3, 6, 10, 20, 30, 30 dB attenuators), traceable to national standards, can be used to generate steps up to 90 dB or higher at the measuring frequencies. However, a step attenuator will generally provide a more repeatable measurement, particularly if a programmable one is used. The step attenuator need not normally be calibrated at more than one frequency, although traceability to national standards will be required at this frequency. Its purpose is to check the linearity of both channels of the VNA by carrying out full-two-port calibrated S_{21} and S_{12} measurements. The measurements are normally carried out at one frequency in the range of approval. The frequency should be toward the bottom end of the frequency range where the attenuator has better repeatability, normally below 1000 MHz for a VNA with 45 MHz to 18 GHz frequency range. The attenuator, or attenuators should be capable of covering the full dynamic range for which accreditation is sought.

3.4 A Traceability Kit

3.4.1 The purpose of this kit is to demonstrate traceability to national standards on a continuing basis. The contents of this kit will have to be agreed with the accreditation body. For one of the connector types at a given frequency range for which accreditation is sought, a kit similar to a manufacturer's verification kit will be needed, augmented by additional components. These additional components will comprise at least one fixed attenuator and at least one broadband mismatch, both to be measured at a number of frequencies covering the accredited frequency range. For subsequent connector types (in the same frequency range) a kit similar to a VNA manufacturer's verification kit will suffice with suitable traceability.

4. MATHEMATICAL MODELS AND CALIBRATION

4.0.1 Different mathematical models and associated calibration measurements are used to cause a VNA to meet full operational specifications (the so-called calibration of the VNA). It is up to the laboratory which one is the most suitable method for its

purposes. In most cases, it will not become clear to the user of a VNA how the mathematical model of each type of calibration is exactly implemented in the VNAs software. Hence the effect of imperfections in the calibration items and in the measurements themselves will not be self-evident.

- 4.0.2 The laboratory always should follow its own procedures of calibrating and operating its VNA. This procedure usually will deviate from the assessment procedure outlined below, especially for the set of frequency points. Obtaining full confidence that the assessment procedure given below is valid for the normal practice of the user means that the assessment procedure should be carried out using both set-ups. It should lead to a consistent set of measurements.
- 4.0.3 The usual precautions concerning connector care should be taken: first check and clean all connectors to be used. Second check that all relevant connectors meet the required inspection criteria. It is advisable to use torque wrenches at all time to improve connector repeatability. The correct setting of the torque wrench depends on the specific connector.

5. UNCERTAINTY EVALUATION

- 5.0.1 Instead of trying to implement all possible models, a blackbox approach is used for uncertainty assessment. An analysis is necessary to assure that such an approach is adequate for a specific VNA set-up.
- 5.0.2 The evaluation is based upon the following philosophy. If a VNA is calibrated correctly and an ideal lossless transmission line is inserted between a device used during calibration and the relevant port, the VNA should measure the same values of S-parameters as inserted during calibration, however with a phase shift. If the calibration is not perfect, residual error terms will exist. These will be visible due to the changing phase relations between the residual vector (located at the measuring port) and the now in phase shifted characteristics of the calibration device.
- 5.0.3 E.g. due to the use of a non-ideal 50Ω load in determining the directivity (as part of a calibration routine using the Open-Short-Load method) an incorrect vector will be assigned as directivity term to compensate for the non-zero reflection. Remeasuring *without* airline will result in a zero reflection, remeasuring *with* an ideal airline will show the interaction of both vectors (the “directivity” and the “ideal load”) separated by a fixed length of airline. In a frequency scan this leads to a continuously changing phase in the measurand, the so-called ripple. The periodicity Δf of this ripple is dependent on the length of the airline used: $\Delta f = c / 2L$ with L as length of the airline (e.g. 1500 MHz for 10 cm).
- 5.0.4 Information can be obtained from the magnitude of the ripple if the magnitudes of the vectors (i.e. directivity and reflection of the load) involved only change slowly with respect to the periodicity of the ripple. A long airline will produce fast ripples, necessitating a high resolution scan.
- 5.0.5 At low frequencies, about below 500 MHz, the available airlines become less ideal (deviating from the 50Ω characteristics as calculated from their dimensions). Also very long (more than 30 cm) airlines are necessary, making it almost impossible to

obtain one complete ripple period.

- 5.0.6 Evaluation as given below should be carried out for all relevant S-parameters, and for other connector types (i.e. for each used coaxial system & connector), where appropriate. In the description given below, typical values are given for the sources of uncertainties; they will be used in the examples given in the Annex.
- 5.0.7 Calibrate the VNA in accordance with the procedure document for the relevant measurements across the full frequency range of accreditation. Use sufficient frequency points for obtaining reliable measurement data, e.g. about 10 points per ripple period. Check to see that the calibration is valid by measuring a previously known standard. The ports of the VNA are located at the connector interface where the calibration items are connected during calibration.

6. **UNCERTAINTY EVALUATION FOR ONE-PORT MEASUREMENTS, U_{VRC}**

- 6.0.1 For measurements of S_{11} and S_{22} , the error model for a VNA can be represented using only the major error terms as follows (eq. 2):

$$U_{VRC} = D + T + M\Gamma^2 + R_{VRC} \quad (2)$$

where

Γ	is the Measured Voltage Reflection Coefficient
D	is the Measured Effective Directivity
T	is the Estimated Overall Effect of Tracking and Non-linearity
M	is the Measured Effective Test Port Match
R_{VRC}	represents all the Random contributions

Note: unless specifically mentioned, the symbols refer to the magnitude of the quantity.

- 6.0.2 The above expression is not intended as a definitive model of all error terms. The values for the terms are not used as corrections, i.e. they all have an assumed value of zero, but for each one there will appear an associated uncertainty. These values should be based on measured data, where this is possible and other sources, such as the manufacturer's specifications, where measurements cannot provide a good estimate of the uncertainty. The random contributions will include at least the following:

- System repeatability (Resolution and Noise)
- Connector repeatability
- Effects of cable flexure
- Effects of ambient conditions

The term effective, as in effective directivity, is used to denote the residual directivity after a full calibration of the relevant ports using the specified calibration kit has been carried out.

- 6.0.3 The following procedures can be used to obtain data on the uncertainty contributions mentioned above (for simplicity, it is assumed that the measurements take place at port x, where x is either 1 or 2). Where relevant, the procedures below should be carried out for both ports.

6.1 Uncertainty in Magnitude

6.1.1 Measurement of Effective Directivity

6.1.1.1 Connect the traceable beadless airline to the measurement port and terminate it with a suitable "matched" load (a load with Voltage Reflection Coefficient (VRC) in the range 0.1 to 0.2 is most suitable). Make a measurement of S_{11} and display 'linear' magnitude against frequency on the VNA screen. Use 'autoscale' or some other means to provide suitable axis scaling.

6.1.1.2 The display should show a discernible sinusoidal 'ripple' superimposed on the VRC plot of the load itself, see Fig. 1, p. 16. Compute the effective directivity, D , as a function of frequency from the magnitude of the ripple (eq. 3):

$$D = \frac{\text{Maximum Ripple Amplitude}}{2} \quad (3)$$

6.1.1.3 The maximum ripple amplitude should be obtained from adjacent peaks and troughs with adjustment made for any slope caused by the variation with frequency of the VRC of the terminating load. The plot gives some indication of the variation of effective directivity with frequency and will normally show a worse directivity at the higher frequencies. As indicated in paragraph 5 more discrimination, i.e. more ripples, will be obtained in proportion to the length of the airline, a 300 mm airline will give ripples with a period of 500 MHz and a 100 mm line 1500 MHz. The line length will clearly limit the lowest frequency at which a meaningful result can be obtained, it should be possible to make a reasonable estimate of the effective directivity at frequencies down to 1 GHz using a 300 mm airline. For purposes of assigning a value for D , it is recommended that the frequency range is not subdivided into more than three ranges, e.g. 1 GHz to 8 GHz; 8 GHz to 12 GHz; 12 GHz to 18 GHz.

6.1.1.4 In the case of a calibration using a broadband 50 Ω this load can be used as an alternative: the same equation applies. The use of another, calibrated 50 Ω , might lead to problems as its reflection coefficient might be of the same order as the effective directivity. Note: if the magnitude of its VRC is smaller than D , then the ripple amplitude could be $2|\Gamma|$, not $2D$.

6.1.1.5 The procedure may be repeated with other airlines to check for consistency and should in any case be repeated several times using the same airline (both with and without performing a **new** calibration) so that a good estimate can be made of the random variations in the value for effective directivity.

At low frequencies the DC value or a very low frequency value (e.g. at 1 kHz) of the calibration load impedance (in relation to 50 Ω) can be used to estimate the effective directivity. The maximum of these values and that obtained from the ripple technique at about 1 GHz is suggested to be used in the assessment.

6.1.1.6 The magnitude of the ripple is directly influenced by the exact value of the characteristic impedance of the airline. Its deviation from nominal and its uncertainty should be included in the final value of the ripple and hence the effective directivity (e.g. by root-sum-of-squares). The effective directivity should normally be in the range 0.002 to 0.02 (-54 dB to -34 dB). In the examples values of 0.01 and 0.015 are

used in the lower and upper frequency band.

6.1.2 Measurement of Effective Test Port Match

6.1.2.1 This procedure is identical to 6.1.1 except that the short circuit (section 3.1) is used in place of the “matched “load to terminate the airline.

6.1.2.2 In this case, the VNA display (again in linear magnitude mode, after suitable scaling) should show that the loss of the line increases regularly with frequency, but superimposed upon this there will be a ripple which typically increases in amplitude with frequency (though not necessarily monotonically), see Fig. 2, p. 17. Compute the 'effective test port match', M , as a function of frequency from the magnitude of the ripple (eq. 4):

$$M = \frac{\text{Maximum Ripple Amplitude}}{2} \quad (4)$$

6.1.2.3 This result is influenced by the effective directivity D since the ripple obtained for this measurement will include both effects. It is only possible to obtain an approximate value for M applicable to the complete frequency range. However, since the uncertainty due to the test port match is obtained by multiplying by Γ^2 its effect will only be apparent when measuring relatively large values of Γ . The value of D gives an estimate of the uncertainty of the effective test port match M .

6.1.2.4 The effective port match should normally be in the range 0.005 to 0.02 (-46 dB to -34 dB). In the examples, values of 0.01 and 0.02 are used for the lower and upper frequency range, respectively.

6.1.3 Measurement of Linearity

6.1.3.1 The uncertainty due to linearity is obtained from the procedures given in 7.2.1.

Based upon these typical values (of 0.002 dB/dB) its contribution to the uncertainty in VRC of 0.2 will be of the order of 0.0006. This estimate is based upon the fact that a VRC of 0.2 is equivalent with a power level of 14 dB below the reference level. A rectangular distribution is assumed.

6.1.4 Tracking

6.1.4.1 It is considered that the effect on the overall uncertainty for reflection measurements caused by imperfect tracking between the incident and reflected signals will be relatively small. An experimental estimate might be obtained from the repeatability of measuring high reflective devices. Usually it is satisfactory to use the manufacturer's value for this contribution, e.g. a relative uncertainty of 0.001 as half interval of a rectangular distribution.

6.1.5 System Repeatability

6.1.5.1 The system repeatability can be divided between repeatability using the same

calibration and repeatability after recalibration. Tests should be conducted to determine the standard deviation of a series of readings using the same calibration without reconnecting the device being calibrated. This test gives a measure of the basic repeatability due to resolution and noise of the system and should be performed for several values of reflection coefficient at a number of frequencies, the internal averaging figure used should be recorded. Tests should also be made of the typical repeatability after recalibration, again this should be performed at a several values of reflection coefficient and several frequencies.

6.1.5.2 Typical values to be expected are between 0.001 and 0.01. In the examples, a value of 0.010 (gaussian with $k = 2$) is used.

6.1.6 Connector Repeatability

6.1.6.1 The test described in 6.1.5, when a recalibration is performed, will include contributions from connectors on the test ports and the calibration standards and it is difficult to separate out these effects. Normally, the item being calibrated will be the dominant contribution to connector repeatability, however, it will not always be practical to make repeat measurements on this device, although this is recommended. One approach is to make a series of measurements of "typical" devices by simply reconnecting the device without a recalibration, but with rotation in steps of 120° , and to use the standard deviation of these results as a basis for estimating a global figure for connector repeatability. Separate assessments will be required for different connector types and over a range of frequencies. The more devices used for this assessment the more reliable will be the estimate of the uncertainty and it is recommended that this data is added to on a continuing basis.

6.1.6.2 For quality connectors used in standards laboratories values of not more than 0.010 (gaussian with $k = 2$) are expected.

6.1.7 Cable Flexure

6.1.7.1 Where cables are used to connect the device being calibrated there is a possibility of errors if the cable is moved after the calibration has been performed. A series of repeat measurements should be undertaken to determine the change in readings when only the cable position is changed in a defined way, e.g. bend through an angle of 90° (only for flexible cable types!!). It is important to describe and record the way the cable position is changed so that the measurements can be repeated at a later date. Several typical cable positions should be assessed so that a global figure for the uncertainty can be assigned. The measurement should be performed over the accredited frequency range.

6.1.7.2 Typical values of 0,004 (gaussian with $k = 2$) have been obtained.

6.1.8 Ambient Conditions

6.1.8.1 Ideally, VNAs should be operated in typical laboratory conditions, e.g. 18°C to 25°C and RH 20% to 60% and if this is the case the manufacturer's specification can be used as a basis for estimating this uncertainty. Other than the contribution of the device being calibrated, which will have to be assessed at the time of measurement, the most sensitive devices are probably the airlines used in the calibration process but provided

they are used within the above conditions their contribution will be negligible. It is often rapid changes in ambient temperature that can affect results, especially if there is a significant change between when the calibration is performed and when the device is calibrated, and this should be taken into account when assessing the uncertainty due to ambient conditions.

6.1.8.2 In the examples, a value of 0.002 will be used (half interval of a rectangular distribution).

6.2 Uncertainty in Phase

Usually the measuring port is connected via a cable to the test set. Then flexure of the cable will influence the magnitude and phase of the signals transmitted over the cable. Also changes in ambient conditions will change the length of the cable, and therefore introduce a phase change. Hence in [5] the following equation is proposed for use to describe the uncertainty in phase:

$$u(\arg(\Gamma)) = \arcsin\left(\frac{U_{|\Gamma|}}{|\Gamma|}\right) * \frac{180}{\pi} + K * f \quad \{4a\}$$

where:

U_{Γ} = uncertainty in the reflection measurement and

$K*f$ = uncertainty due to cable flexure.

In the latter term effect of flexing and temperature change (not more than 1 K) are both taken into account. A worst case approach is followed in this uncertainty estimate by just adding the contributions.

Typical values for a 60 cm cable are: $K = 0.05^{\circ}/\text{GHz}$

Note:

- only the first term is relevant, if the measurement is done directly on the VNA port.
- In a reflection measurement the signal travels twice over the cable.

6.3 Effective Load Match, Γ_L

6.3.1 During the calibration of the VNA for two-port operation (in the case of Open-Short-Load-Through calibration: the so-called 12-term calibration) the reflection coefficient of the other test port is determined. In this case, it is the first reflection measurement of the calibrated VNA. The reflection coefficient (the so-called “load match”) itself might be of the order of 0.2. After calibration the *effective* load match Γ_L can be represented by the uncertainty estimate given in example 1 of the Annex.

6.3.2 A value of 0.02 (with $k = 2$) will be used in the Examples 3 through 5.

7 UNCERTAINTY EVALUATION FOR TWO-PORT MEASUREMENTS

7.1 Reflection U_{VRC}

- 7.1.1 For measurements of $\Gamma = S_{11}$ and $\Gamma = S_{22}$ of two-port devices, a similar uncertainty model can be used as for one-port measurements, but taking into account also the Effective Load Match Γ_L (the other test port) and the nominal attenuation S_{21} of the device under test, according to eq. 5 :

$$U_{VRC} = D + T\Gamma + M\Gamma^2 + R_{VRC} + S_{21}^2\Gamma_L \quad (5)$$

where Γ is the Measured Voltage Reflection Coefficient (as defined in equation 2).
Note: for small attenuations (e. g. 3dB) a term $2\cdot\Gamma\cdot M\cdot\Gamma_L\cdot S_{12}^2$ might be added.

- 7.1.2 For a 3 dB attenuator ($S_{21} = \sqrt{2}$), the contribution due to Γ_L is hence reduced with a factor of 0,5.

7.2 Transmission U_{TM}

- 7.2.1 For measurements of S_{12} and S_{21} , the error model for a VNA can be represented using only the major error terms as follows (eq. 6), assuming a logarithmic representation (in dB):

$$U_{TM} = L + M_{TM} + I + R_{dB} \quad (6)$$

where L is the measured System Deviation from linearity;
 M_{TM} is the calculated error term due to Mismatch, see eq. 7;
 I is the estimated or measured Cross-Talk; dA, see eq. 8
 R_{dB} represents all the Random Contributions.

- 7.2.2 As with reflection measurements, the above expression is not the definitive error model but indicates how the major terms contribute to the uncertainty. Contributions should be based on measured results wherever possible. The random contributions will be from similar sources as reflection measurements:

System repeatability (Resolution and Noise);
Connector repeatability;
Effects of cable flexure;
Effects of ambient conditions

and may be assessed in a similar way as described in paragraphs 6.1.6 through 6.1.8.

7.2.1 Measurement of Linearity

- 7.2.1.1 The VNA should be calibrated according to the accredited procedure for S_{21} measurements. The procedure then consists of measuring the step attenuator or a set of calibrated attenuators (section 3.3), at the frequency for which traceability to national standards has been established, with reference to the zero setting of the attenuator. No other attenuators should be included in the circuit and the source level should be set to the level specified in the measurement procedure, this is to ensure that the level at the

detectors is approximately the same as it would be for normal measurements. It is recommended that results are obtained at not greater than 5 dB steps over the range for which accreditation is required and, if possible, at 10 dB greater than this range, e.g. if the accredited range is 0 to 70 dB results for the range 0 to 80 dB should be obtained. It is important that random effects are minimised therefore the results should be obtained from a sufficient number of repeat measurements, particularly at the high levels of attenuation. The internal averaging figure should always be adequate for the attenuation step.

7.2.1.2 It is not normal practice to correct for the linearity errors in VNAs and the results obtained from the measurements mentioned above provide an estimate of the uncertainty that should be assigned to this contribution, in the form $\Delta\text{dB/dB}$. A plot of the difference between the VNA result and the calibration value against the reference attenuator value is the recommended method for estimating a reliable figure for the linearity uncertainty. The linearity should be assessed from the results between 0 dB and 50 dB since beyond this setting the effects of imperfect isolation will contribute to the errors (see also section 7.2.3).

7.2.1.3 Typical values for linearity are of the order of 0.002 dB/dB ($k=2$ with a gaussian distribution).

7.2.2 Mismatch

7.2.2.1 There will be an **uncertainty due to mismatch** arising from the residual source and load reflection coefficients and the input and output reflection coefficients of the item being measured, according to eq. 24 in [3], this is (in dB)

$$M_{\text{TM}} = 20\log_{10} \frac{1 + (|MS_{11}| + |\Gamma_L S_{22}| + |M\Gamma_L S_{11} S_{22}| + |M\Gamma_L S_{21} S_{12}|)}{1 - |M||\Gamma_L|} \quad (7)$$

where M is the Effective Test Port Match

Γ_L is the Effective Load Match

$S_{11}, S_{22}, S_{12}, S_{21}$ are the Scattering coefficients of device being measured.

Note: for clarity, the modulus signs along the quantities are shown to make evident that a calculation on basis of magnitudes should be done.

7.2.2.2 Often this will be the dominant term in the uncertainty budget.

7.2.3 Isolation (Cross-talk)

7.2.3.1 The effects of imperfect isolation after calibration (cross-talk) between the two ports can be based on the manufacturer's specification figure but this should be checked using the linearity test results at readings above 50 dB. The isolation uncertainty will not be a linear function of the measured attenuation and can be calculated for each measured value of attenuation. If A is the attenuation value and I the cross-talk, then according to [4] the uncertainty dA is (eq. 8):

$$dA = \pm 20 \cdot \log \left[1 + 10^{-\frac{(I-A)}{20}} \right] \text{dB} \quad (8)$$

7.2.3.2 However, it is usually more convenient to express the uncertainty as a linear function of the measured attenuation, broken down into ranges of, say, 5 dB. With $I = 90$ dB then dA (the uncertainty in A):

A_a	dA_a
65 dB	0,48 dB
70 dB	0,83 dB
75 dB	1,42 dB
80 dB	2,39 dB

7.2.3.3 The effects of isolation will vary with frequency, therefore measurements should be made at different frequencies over the accredited range, using attenuators with adequate isolation, in order to check the validity of the manufacturer's specification figures. In the examples a cross-talk of 90 dB is assumed as half interval of a rectangular distribution.

7.2.4 System repeatability, resolution

7.2.4.1 Resolution concerns the reproducibility of relative large signals: it means checking at low attenuation. In essence, this is described in section 6.1.5.

7.2.5 System repeatability, noise floor

7.2.5.1 The contribution from noise will have a relatively greater impact on transmission measurements and should therefore be assessed for this parameter. The contribution from system noise can be assessed using the step attenuator by making a series of measurements at various levels of attenuation, without recalibration of the VNA. The uncertainty is obtained from the standard deviation of these repeat measurements and should be assessed at different internal averaging settings. The noise contribution will be dependent on the level of signal reaching the detector and will therefore be dependent on source level and attenuator setting, it is therefore important that these values are recorded. The noise contribution will change significantly with frequency due mainly to the reduction in available signal level at the detectors as the frequency increases. The effect of noise must therefore be evaluated at various frequencies over the range of operation.

7.3 Phase uncertainty in Transmission Measurements

The measuring ports are connected via cables to the test set. Flexure of the cables will influence the magnitude and phase of the transmitted signals. Changes in ambient conditions will change the length of the cables, and therefore introduce a phase change. The uncertainty in the phase of a transmitted signal may be evaluated using the geometrical dimensions of a calibrated airline.

Hence in [5] the following equation is proposed for use to describe the uncertainty in phase:

$$u(\arg(S_{12})) = \arcsin(10^{\frac{U}{20}} - 1) * \frac{180}{\pi} + K * f + L_{\phi} * f + I \quad \{9\}$$

Where

U = uncertainty in the attenuation measurement (dB)

K*f = uncertainty due the cable flexure, and

L_φ*f = uncertainty in the phase standard (airline)

I = 0.1+0.0025*A+(0.03+0.00035*A)*f (°): In [5] the author introduced this additional uncertainty contribution based upon an analysis of an international comparison [6];

A = attenuation (dB),

f = frequency (GHz).

Again a worst case estimate is used

Typical values for the cables (each 60 cm of length) are:

K = 0.05°/GHz

L_φ = 1.2°*u(L) with u(L) the uncertainty in the length L of the standard (in mm) (see clause 3.2.3)

A systematic error usually will occur if no temperature correction is carried out: this will be of the order of 5*10⁻⁵. (see also 3.2.3).

8 CALCULATION OF UNCERTAINTY

8.0.1 The uncertainty contributions should be combined in accordance with EA recommendations to provide an expanded uncertainty based on a coverage factor of $k = 2$ (a level of confidence of approximately 95%). The following points should be considered:

- While most of the contributions can be considered to be uncorrelated, there is a possibility that the effective directivity and effective test port match will be correlated to some extent, due to the common factors in the calibration process of a VNA. It is beyond the scope of this document to provide guidance for the evaluation of the correlation coefficients of these contributions and it has not been possible to find any references for such evaluation. It is therefore recommended that, in the absence of reliable information, a correlation coefficient of +1 is assumed, which means that the uncertainty contributions for directivity and test port match should be added together before being combined with the other contribution in the usual way, i.e. root-sum-of-squares.
- The contributions that are dependent on the relative phase of two vectors, such as directivity and test port match will have a probability distribution which is a U-shaped form. The standard uncertainty for these contribution is obtained by dividing the limit value by $\sqrt{2}$. However, when making transmission measurements, the uncertainty contribution calculated using the expression given

in section 7.2.2 will be the contribution of three U-shaped contributions and it may be argued that a rectangular probability distribution is to be assumed for this contribution. A normal distribution can be assumed for the uncertainties derived from measured data (type A evaluations). It is recommended that a rectangular distribution is assumed for the effects of ambient conditions and manufacturer's data, unless there is evidence that the distribution is normal. The choice of the form of the probability distribution for the smaller contributions will, in general, have little effect on the expanded uncertainty and it is therefore not necessary to make a detailed study of the likely probability distribution for these contributions.

9 EXPERIMENTAL VERIFICATION

The traceability kit, as mentioned in paragraph 3.4, is the link to establish traceability to national standards. It should be used to verify experimentally that the calibrated network analyser yields similar results as obtained from an accredited laboratory. The uncertainty calculated in the previous sections gives, at least in the case of the so-called 12-term calibration, an acceptable estimate of the uncertainty contributions: certain system uncertainty sources might still be overlooked. The now calculated uncertainty is the additional contribution to the uncertainty when performing a measurement on the VNA.

In the unlikely case of insufficient consistency between measured values and calibration data (i.e. the two sets of data differ by more than the combined (root-sum-of-squares) uncertainties) further investigation is necessary to obtain the source(s) of this inconsistency. Until an explanation is found, the uncertainty should be increased with the observed difference.

10 REFERENCES:

- [1] IEC 60457 Series of standards, Rigid coaxial lines and their associated connectors", International Electrotechnical Commission, Geneva, (1974 and later)
- [2] IEEE Standard 287, "Precision coaxial connectors", Institute of Electrical and Electronic Engineers, New York, USA, (1968, revised in 1988)
- [3] e.g. H. Bayer, "An error analysis for the RF-Attenuation measuring equipment of the PTB applying the power method", *Metrologia* **11** (1975), pp.43-51
- [4] F.L. Warner, "Microwave Attenuation Measurements", IEE Monograph 19, Peter Peregrinus (UK), (1977)
- [5] D. Stumpe. "Streuparametermessungen mit Netzwerkanalysatoren", pp.53-84 in Hans Bachmair and Ulrich Stumper: "Aktuelle Probleme bei der Weitergabe von HF-Meßgrößen", PTB-E-58,(1998), ISBN 3-89701-173-5
- [6] NPL Report CEM 15, July 1999 (describes GT-RF 83-4 comparison on S-parameters in PC7 transmission lines)

FIGURES

Fig. 1

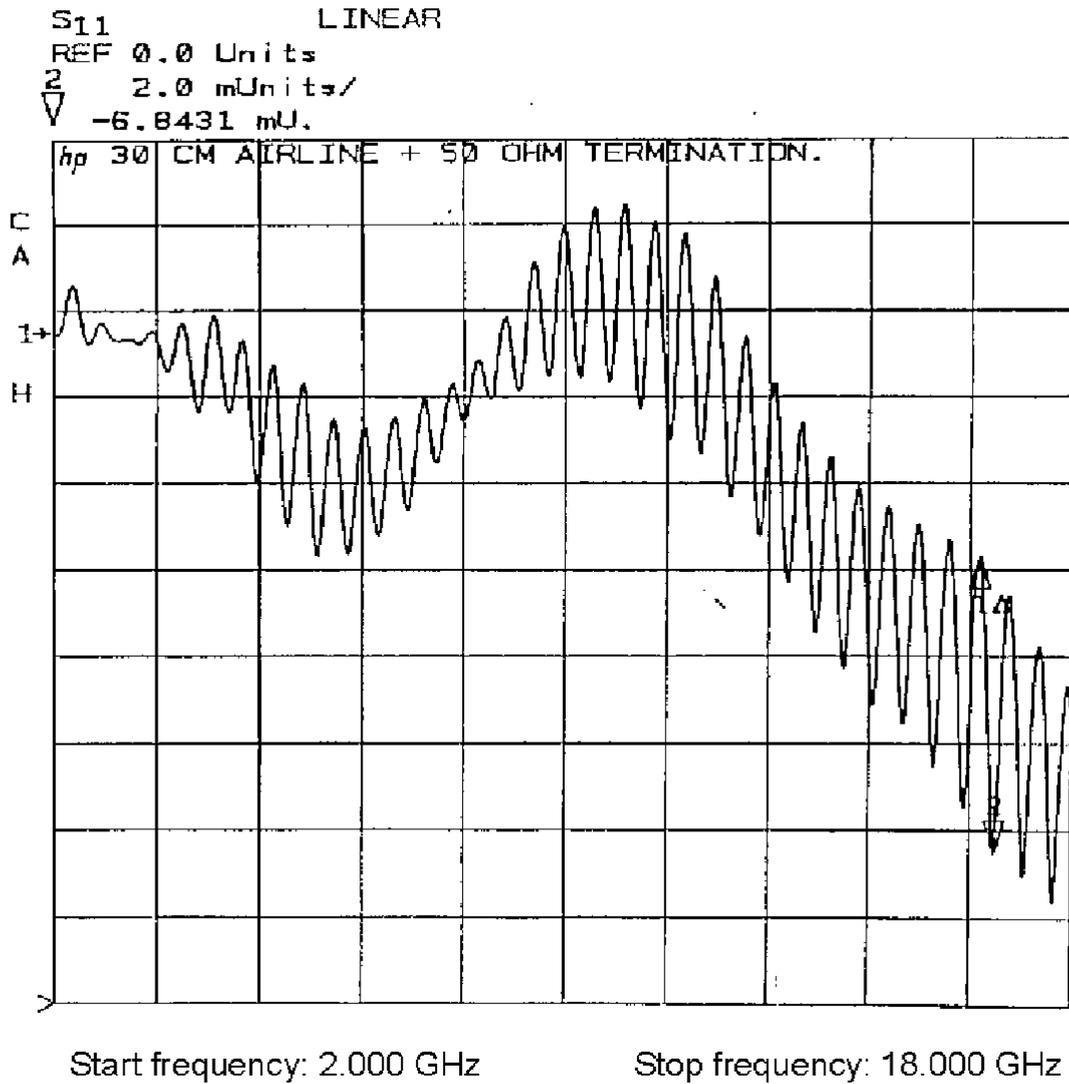


Figure 1: A non-perfect 50Ω load is connected to a 30 cm airline. The other side of the airline is attached to port 1. This figure shows which kind of typical response will be obtained measuring S11 as function of frequency. From the ripple an effective directivity can be determined.

Fig. 2

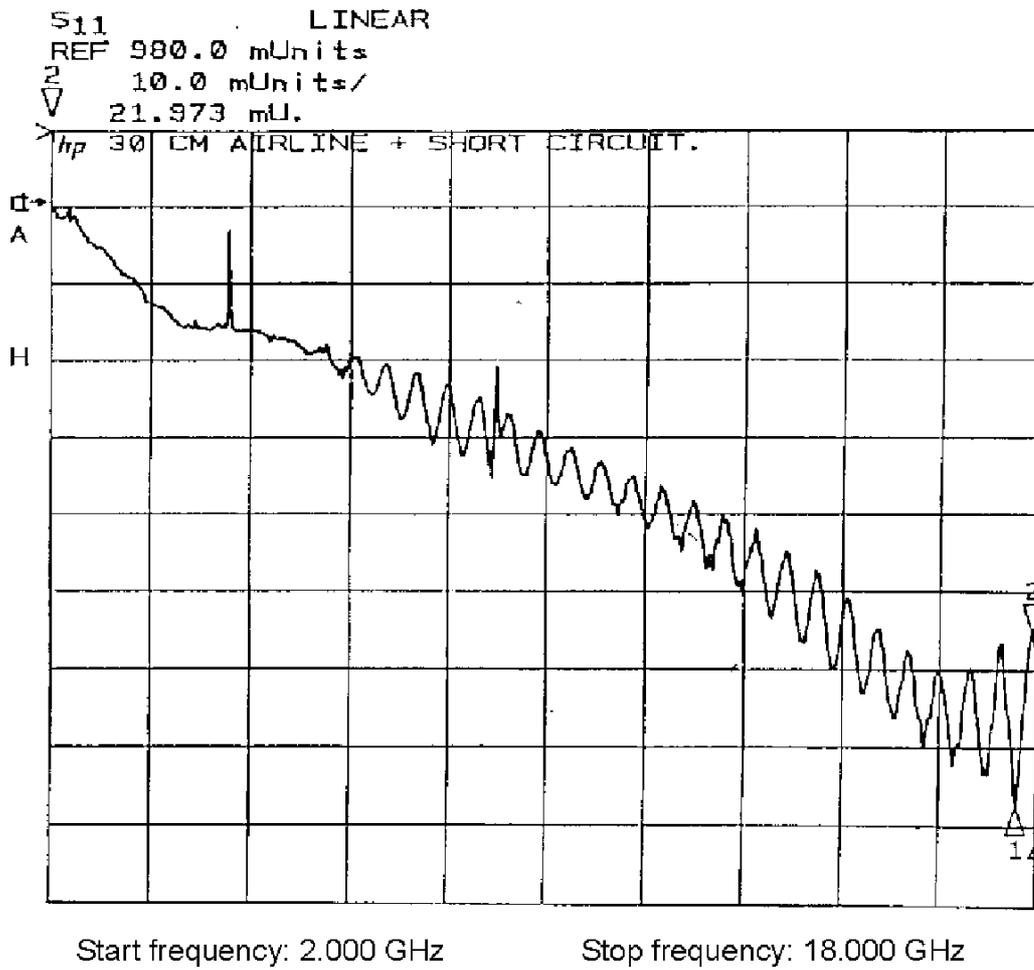


Figure 2: A short is connected to a 30 cm airline. The other side of the airline is attached to port 1.
This figure shows which kind of typical response will be obtained measuring S11 as function of frequency. From the ripple an effective source match can be determined.

ANNEX A: EXAMPLES OF UNCERTAINTY BUDGETS OF VECTOR NETWORK ANALYSERS

A1 GENERAL

A1.1 The uncertainty budgets given in this document are not intended to imply any mandatory requirement. They provide guidance on what would be expected for the general approach to documenting the uncertainties derived from using the methods given in this document. These examples do not cover the complete capabilities of typical VNAs.

A1.2.1 Typical examples are given for uncertainty in magnitude:

- low frequency, low reflection in a one-port measurement;
- high frequency, high reflection in a one-port measurement;
- low frequency, medium reflection in a two port measurement;
- low frequency, low loss transmission measurement;
- high frequency, high loss transmission measurement.

A1.2.2 Typical examples are given for uncertainty in phase:

- one port reflection measurement
- two port transmission measurement

A1.3 All values concerning the VNA itself are assumed to be obtained using the procedure outlined above. Random uncertainties, such as repeatability and cable flexure, are assumed to be obtained from actual measurements as described above, leading to larger values at higher frequencies.

The mentioned values of linearity, effective directivity, effective test port match and cross-talk are valid for all examples:

Deviation of Linearity	= 0,002 dB/dB
Effective directivity	= 0,010, increasing to 0,015 at higher frequencies
Effective test port match	= 0,010, increasing to 0,020 at higher frequencies
Cross-talk	= 90 dB.

A1.4 In the examples, the contributions are referenced to the relevant section of the main document. Only in example 1 the uncertainty in the dimensions of the airline are included.

Note: The reflection coefficient is a dimensionless quantity, as similar the transmission coefficient (however the latter is often presented as attenuation using a logarithmic scale: dB). The values for the random contributions for reflection and transmission are not related.

A2 EXAMPLE 1

One-port Reflection Measurements (magnitude of voltage reflection coefficient: VRC) based upon equation 2 at low frequencies

Contribution	Estimate of Contribution	Uncertainty For VRC = 0,2	Distribution	Divisor	Standard Uncertainty
Airline reflection (3.2.3)	0.0017	0.0017	rectangular	$\sqrt{3}$	0.0010
Ripple (for eff. D) (6.1.1)	0.010	0.010	U-shaped	$\sqrt{2}$	0.0071
Effective Directivity (calculated)		0.0101	U-shaped	$\sqrt{2}$	0.0071
Effective Test Port match (6.1.2)	0.01	0.0004	U-shaped	$\sqrt{2}$	
Sum of correlated quantities		0.0105	U-shaped	$\sqrt{2}$	0.0074
Tracking (6.1.4)	0.001	0.0002	rectangular	$\sqrt{3}$	0.00011
Linearity (6.1.3.1)	0.002 dB/dB	0.00064	rectangular	$\sqrt{3}$	0.0004
System Repeatability (6.1.5)	0.010	0.002	gaussian	2	0.001
Cable Flexure (6.1.7)	0.004	0.0008	gaussian	2	0.0004
Ambient Conditions (6.1.8)	0.002	0.0004	rectangular	$\sqrt{3}$	0.0002
Connector Repeatability (6.1.6)	0.010	0.010	gaussian	2	0.005
Combined Standard Uncertainty					0.0090
Expanded Uncertainty ($k = 2$)					0.018

A3 EXAMPLE 2

One-port Reflection Measurements (magnitude of voltage reflection coefficient) based upon equation 2 -- at high frequencies

Contribution	Estimate	Uncertainty for VRC = 0,8	Distribution	Divisor	Standard Uncertainty
Effective Directivity (6.1.1)	0.015	0.015	U-shaped	$\sqrt{2}$	
Effective Test Port Match (6.1.2)	0.02	0.0128	U-shaped	$\sqrt{2}$	
Sum of correlated quantities		0.0278	U-shaped	$\sqrt{2}$	0.0197
Tracking (6.1.4)	0.001	0.0008	rectangular	$\sqrt{3}$	0.0005
Linearity (6.1.3.1)	0.002 dB/dB	0.00036	rectangular	$\sqrt{3}$	0.0002
System Repeatability (6.1.5)	0.010	0.008	gaussian	2	0.004
Cable Flexure (6.1.7)	0.008	0.0064	gaussian	2	0.0032
Ambient Conditions (6.1.8)	0.002	0.0016	rectangular	$\sqrt{3}$	0.0009
Connector Repeatability (6.1.6)	0.02	0.02	gaussian	2	0.01
Combined Standard Uncertainty					0.0227
Expanded Uncertainty ($k = 2$)					0.045

A4 EXAMPLE 3

Reflection measurement (magnitude of voltage reflection coefficient) on a two-port device; calculation based upon equation 5

A4.1 A 3 dB attenuator is used with S_{11} and S_{22} not larger than 0,05.

Contribution	Estimate	Uncertainty for 3 dB and $S_{11}=0,05$	Distribution	Divisor	Standard Uncertainty
Effective Directivity (6.1.1)	0.01	0.01	U-shaped	$\sqrt{2}$	
Effective Test Port Match (6.1.2)	0.01	0.000025	U-shaped	$\sqrt{2}$	
Sum of correlated quantities		0.010025	U-shaped	$\sqrt{2}$	0.0071
Tracking (6.1.4)	0.001	0.000025	rectangular	$\sqrt{3}$	0.0000
Linearity (6.1.3.1)	0.002 dB/dB	0.0003	rectangular	$\sqrt{3}$	0.0002
System Repeatability (6.1.5)	0.010	0.000025	gaussian	2	0.0000
Cable Flexure (6.1.7)	0.004	0.0001	gaussian	2	0.0000
Ambient Conditions (6.1.8)	0.002	0.000005	rectangular	$\sqrt{3}$	0.0000
Connector Repeatability (6.1.6)	0.010	0.010	gaussian	2	0.0050
Effective Load Match (6.3)	0.018	0.009	U-shaped	$\sqrt{2}$	0.0064
Combined Standard Uncertainty					0.0108
Expanded Uncertainty ($k = 2$)					0.022

A5 EXAMPLE 4

Transmission Measurements (attenuation magnitude); calculation based upon equation 6:

A5.1 A 20 dB attenuator is used with S_{11} and S_{22} not larger than 0,05.

Contribution	Estimate dB	Uncertainty for 20 dB	Distribution	Divisor	Standard Uncertainty dB
Linearity (7.2.1)	0.002 dB/dB	0.040	gaussian	2	0.020
Test port Match ($M \cdot S_{11}$)	0.01 U	0.0005			
Load Match ($\Gamma_L \cdot S_{22}$)	0.02 U	0.001			
Test port * Load Match ($M \cdot \Gamma_L$)	0.0002 U	0.0002			
Mismatch calculated (7.2.2)		0.0148	gaussian	$\sqrt{2}$	0.0104
Cross-talk (7.2.3)	90	0.0027	rectangular	$\sqrt{3}$	0.0016
System Repeatability (7.2.4)	0.002	0.002	gaussian	2	0.0010
Noise (7.2.5)	0.004	0.004	gaussian	2	0.0020
Cable Flexure (6.1.7)	0.010	0.010	gaussian	2	0.0050
Ambient Conditions (6.1.8)	0.002	0.002	rectangular	$\sqrt{3}$	0.0012
Connector Repeatability (6.1.6)	0.02	0.02	gaussian	2	0.0100
Combined Standard Uncertainty					0.0254
Expanded Uncertainty ($k = 2$)					0.051

A6 EXAMPLE 5

Transmission Measurements (attenuation magnitude); calculation based upon equation 6

A6.1 A 70 dB attenuator is used with S_{11} and S_{22} not larger than 0,05.

Contribution	Estimate dB	Uncertainty for 70 dB	Distribution	Divisor	Standard Uncertainty dB
Linearity (7.2.1)	0.002 dB/dB	0.140	gaussian	2	0.070
Test port Match ($M \cdot S_{11}$)	0.01 U	0.0005			
Load Match ($\Gamma_L \cdot S_{22}$)	0.02 U	0.001			
Test port * Load Match ($M \cdot \Gamma_L$)	0.0002 U	0.0002			
Mismatch calculated (7.2.2)		0.0148	gaussian	$\sqrt{2}$	0.0104
Cross-talk (7.2.3)	90	0.8686	rectangular	$\sqrt{3}$	0.5015
System Repeatability (7.2.4)	0.002	0.002	gaussian	2	0.001
Noise (7.2.5)	0.04	0.04	gaussian	2	0.02
Cable Flexure (6.1.7)	0.010	0.010	gaussian	2	0.005
Ambient Conditions (6.1.8)	0.002	0.002	rectangular	$\sqrt{3}$	0.0012
Connector Repeatability (6.1.6)	0.02	0.02	gaussian	2	0.01
Combined Standard Uncertainty					0.5070
Expanded Uncertainty ($k = 2$)					1.01

A7 EXAMPLE 6

A7.1 One-port Reflection Measurements (magnitude of voltage reflection coefficient) based upon equation 4a

Cable length 60 cm; $f = 10$ GHz;

Contribution	Parameter Value	Uncertainty	Parameter influence	Uncertainty in degrees	Distribution	Divisor	Standard Uncertainty
Magnitude S_{11} (example 1)	VRC= 0.2	0.009	0.045	2.58	gaussian	1	2.58
Cable Flexure (6.2)	Length (2*60 cm)	0.05	2				
	Frequency 10 GHz		10	1.00	gaussian	1	1.00
Combined Standard Uncertainty							3.58
Expanded Uncertainty ($k = 2$)							7.2

A8 EXAMPLE 7

Two port Transmission Measurements (based upon equation 9)

A8.1 A 20 dB attenuator is used with S_{11} and S_{22} not larger than 0,05

Device electrical about 0.5 times calibrated airline; $f = 10$ GHz; 2 cables of 60 cm length;

Contribution	Parameter Value	Uncertainty	Parameter influence	Uncertainty in degrees	Distribution	Divisor	Standard Uncertainty
Magnitude S_{21} (dB) (example 4)	20	0.0254	0.00127	0.17	gaussian	1	0.17
Uncertainty in phase standard (Airline)		1.2 °/mm/GHz					
Length standard (100 mm)			0.0015				
Phase shift relative to standard (0.5)			0.5				
Frequency (10 GHz)			10	0.009	gaussian	1	0.01
Cable flexure (6.2) Length 2*60 cm		0.05	2				
Frequency (GHz)		1	10	1.00	gaussian	1	1.00
Uncertainty ILC (7.3)	Constant	0.1	1	0.52	gaussian	1	0.52
	Attenuation (dB)	20	0.0025				
	Frequency (GHz)	10	0.037				
Combined Standard Uncertainty							1.70
Expanded Uncertainty ($k = 2$)							3.4