ANAMET Report 035
May 2002

ANAMET REPORT

Pass or Fail: With Which Probability?

Manuel Rodriguez

NPL
National Physical Laboratory

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Comments on this report should be sent to :

Mail: ANAMET
CETM
National Physical Laboratory
Queens Road
Teddington
Middlesex
TW11 0LW

Fax: 020 8943 6098 (UK)
+ 44 20 8943 6098 (International)

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PASS OR FAIL: WITH WHICH PROBABILITY?

Manuel Rodríguez
INTA, Spain

1. INTRODUCTION

Pass-or-Fail tests are often defined in order to assess whether a given piece of equipment meets specifications or not. They are not in principle intended for use within a metrological environment, and normally rely on one single measurement. There are normally two types of tests: Single (in which a unique threshold or limiting value is defined, either maximum or minimum) and Double (in which two limiting values are specified, maximum and minimum).

In a Single-Limit Test, it is normally agreed that the result of the test is “Pass” when the measured value lies well below the maximum value or well above the minimum value. In a Double-Limit Test, “Pass” is defined as the measured value lying below the maximum value and above the minimum value.

The question sometimes arises as whether measured values which lie close to the limiting values are to be “accepted” (Pass) or “rejected” (Fail). The question becomes more difficult to answer if the measured value is the result of one single experiment. One would normally repeat the measurement in order to decide whether to accept or to reject the test result, but in occasions the same results is obtained repeatedly, which does not help making a decision.

The problem still remains whenever the margin between the measured result and the specification limit is less than the measurement uncertainty [1], [2]. Our aim is to express the results of Pass-or-Fail Tests in terms of probability. With this in mind, we shall follow a metrological approach, in which concepts such as repeatability, number of effective degrees of freedom, level of confidence, coverage factors and expanded uncertainty will play a role.

2. STATISTICAL BACKGROUND

In a general case, a coverage factor k_p should be derived for the expression of the uncertainty related to a measured quantity, where p is the probability or level of confidence in percent [3], [4], [5]. This coverage factor is based on a Student´s t Distribution in case of unreliable input quantities. The expanded uncertainty is then given by:

$$u_{\exp} = k_p \cdot u_c(y)$$

Where $u_c(y)$ is the combined uncertainty. For determination of k_p, the number of effective degrees of freedom, u_{eff}, of the combined standard uncertainty, has to be estimated. This is made using the Welch-Satterwaite equation, based on the degrees of freedom u_i of the individual uncertainty contributions $u_i(y)$:

$$u_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{u_i}}$$

The degrees of freedom u_i of Type A contributions is n-1, n being the number of measurements. The degrees of freedom u_i of Type B contributions can be assumed to be infinite. The coverage factor as a function of the confidence level and the number of effective degrees of freedom is then given by the inverse *t* Distribution:

$$k(p, u_{eff}) = Inverse\ tDistribution(100 - p(\%); u_{eff})$$

3. THE EXPRESSION OF UNCERTAINTY IN A MEASUREMENT RESULT

Having obtained the expanded uncertainty related to the measurement of a given quantity, the result is reported as:

$$y \pm U$$

Which could be expressed in words as follows: "The measurand is estimated to lie within the interval [y–U, y+U] with a level of confidence of p(%). The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor of k".

Note that the information about the number of effective degrees of freedom is redundant, since the customer can deduce it from k and p (alternatively, a simpler although perhaps less intuitive solution could be to provide u_{eff} and let the customer obtain k for any probability p he or she might wish!).

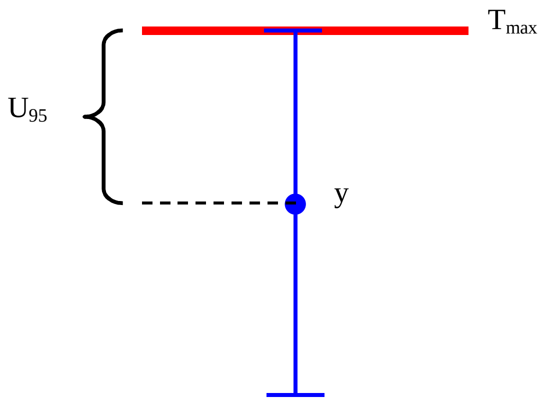
Note also that, assuming a symmetrical probability density function around the measured value, the following statements:

- The measurand is estimated to lie outside the interval [y–U, y+U] with a level of confidence of 100–p
- The measurand is estimated to be greater than y+U with a level of confidence of (100–p)/2
- The measurand is estimated to be less than y–U with a level of confidence of (100–p)/2

...are also true!

4. THE EXPRESSION OF UNCERTAINTY IN A SINGLE PASS-OR-FAIL TEST

Let us define a Pass-or-Fail Test in which a maximum threshold T_{max} is defined. We measure the quantity y (the measurand) below the threshold, with an expanded uncertainty U, and for a confidence level of p=95%. Let us assume that the difference between T_{max} and y is exactly the measurement uncertainty, U.

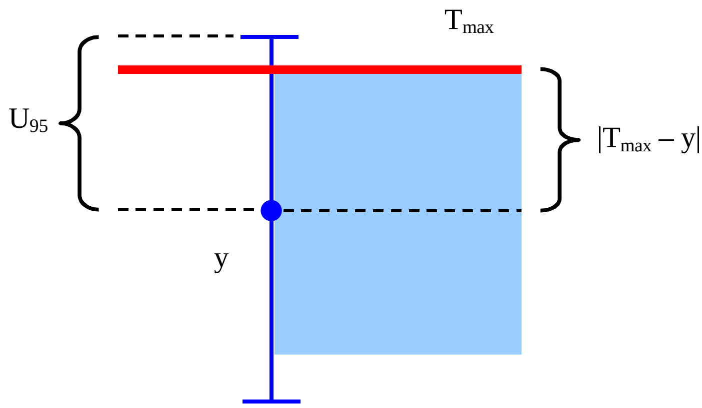


Let us recall one of the above statements: “The measurand is estimated to be greater than $y+U$ with a level of confidence of (100–95)/2=2.5%”. Which is equivalent to any of the following:

- The measurand is estimated to Fail the Test with a probability or level of confidence of 2.5%
- The measurand is estimated to Pass the Test with a probability or level of confidence of 97.5%

4.1 Probability of "just-in-margin"

Let us now define a new coverage factor which we can call the “just-in-margin” factor. It can be defined as the coverage factor which exactly includes the threshold limit. As it can be seen in the figure, this “just-in-margin” factor is related to the absolute difference $|T_{max} - y|$. We have denoted it with the subindex p, because our next step will be to obtain the probability or level of confidence associated to this coverage factor.



The so-defined “just-in-margin” factor is given by:

$$k_p = \frac{|T_{max} - y| \cdot k_{95}}{U_{95}}$$

And the probability p related to this coverage factor is:

$$p(\%) = 100 - tDistribution(k_p; u_{eff})$$

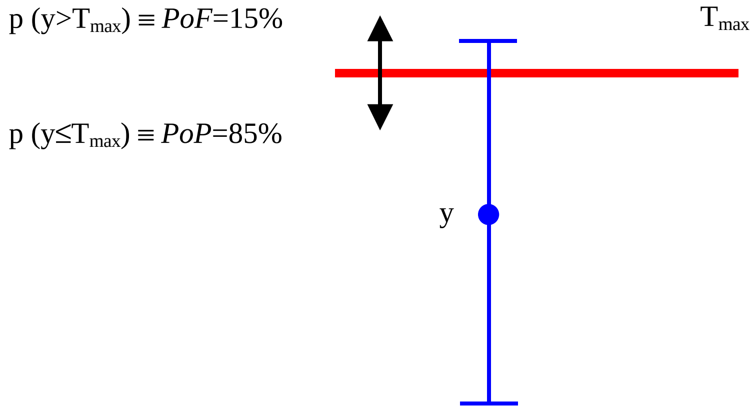
4.2 Probability of Pass and Probability of Fail

This “just-in-margin” coverage factor serves us to state things as follows: “The measurand is estimated to lie within the interval $[y-|T_{max} - y|, T_{max}]$ with a level of confidence of p”.

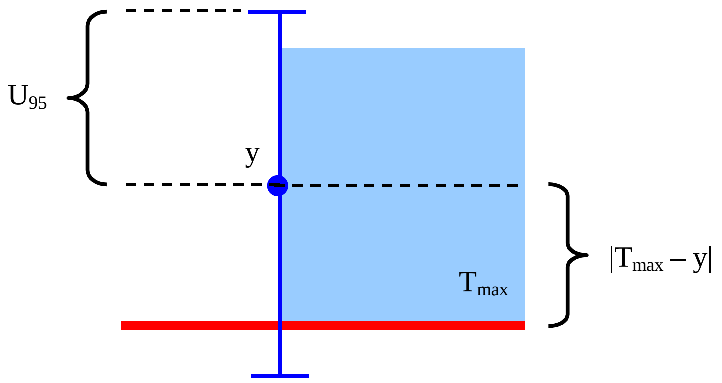
Of course this is a mere intermediate step in order to compute the following probabilities, expressed as any of our usual statements:

- The measurand is estimated to Fail the Test with a probability of $(100-p)/2$
- The measurand is estimated to Pass the Test with a probability of $(100+p)/2$

In the above example, if p=70%, the Probability of Fail would be PoF=(100–70)/2 and the Probability of Pass PoP=(100+70)/2:



In the following example we assume that the measured value exceeds the maximum limit.



In this case the measurand is estimated to lie within the "just-in-margin" interval [T_{max}, $y+|T_{max}-y|$] with a level of confidence of p. The expressions of the Probability of Pass and the Probability of Fail are reversed:

- The measurand is estimated to Fail the Test with a level of confidence of (100+p)/2
- The measurand is estimated to Pass the Test with a level of confidence of (100–p)/2

If the probability of "just-in-margin" is the same as in the previous example p=70%, in this case PoF=85% and PoP=15%.

5. SOME EXAMPLES OF SINGLE PASS-OR-FAIL TESTS

5.1. Example I

In Figure 1 several cases of test results are shown, together with their associated Probability of Pass. Hereinafter, the probability level considered is p=95.45%. The threshold value T_{max} is 1 and the measurement uncertainty U is 0.1. As it can be seen, as the measured value approaches the limit, the probability of pass decreases.

From the comparison between these particular cases, it can be seen that Meas #1 and Meas #7 are paired, i.e. their position with respect to the threshold value is reversed. This is also the case for Meas #2 and #6 and for Meas #3 and #5. The Probability of Pass and Fail are reversed within each pair, just as probably our common sense would tell us.

Meas #2 and Meas #6 are particular cases, since the "just-in-margin" factor is exactly k_{95}. This makes *PoP* and *PoF* independent on the number of effective degrees of freedom, which can be verified as compared with Meas #2 and Meas #6 in Examples II and III.

Meas #4 is also a particular case. Our common sense is also happy to know that when the measured value exactly coincides with the threshold value, there is no means to assign a greater probability to the event "Pass" or to the event "Fail".

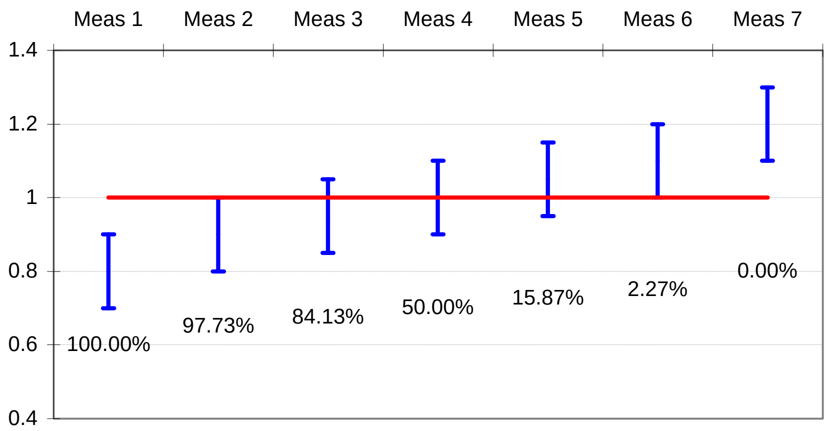


Figure 1. T_{max}=1, U=0.1, Number of effective degrees of freedom u_{eff}=10^6

	Value	Uncertainty	k_{95}	k_p	p(%) "In-margin"	p(%) of Pass	p(%) of Fail
Meas 1	0.8	0.1	2.00	4.00	99.99	100.00	0.00
Meas 2	0.9	0.1	2.00	2.00	95.45	97.73	2.27
Meas 3	0.95	0.1	2.00	1.00	68.27	84.13	15.87
Meas 4	1	0.1	2.00	0.00	0.00	50.00	50.00
Meas 5	1.05	0.1	2.00	1.00	68.27	15.87	84.13
Meas 6	1.1	0.1	2.00	2.00	95.45	2.27	97.73
Meas 7	1.2	0.1	2.00	4.00	99.99	0.00	100.00

Table 1. Probability of "just-in-margin", Probability of Pass and Probability of Fail

5.2. Example II

In Figure 2 the same cases as in Example I are shown. The threshold value and the measurement uncertainty remain the same. The only difference is the number of effective degrees of freedom.

If we take a look at the different measurements and compare them with those in Example I, we notice that Meas #2 and #6 show the same *PoP* and *PoF*. This is due to the fact that the "just-in-margin" factor is exactly k_{95}, regardless of the number of degrees of freedom.

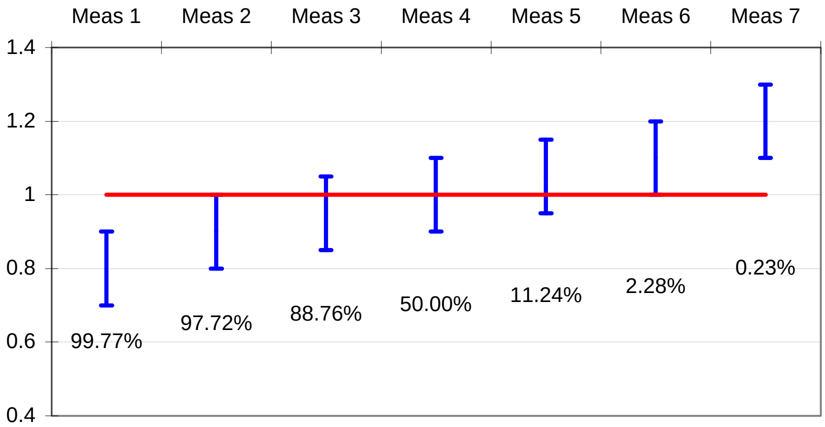


Figure 2. T_{max}=1, U=0.1. Number of effective degrees of freedom u_{eff}=4

	Value	Uncertainty	k_{95}	k_p	p(%) "In-margin"	p(%) of Pass	p(%) of Fail
Meas 1	**0.8**	**0.1**	**2.87**	**5.74**	**99.54**	**99.77**	**0.23**
Meas 2	**0.9**	**0.1**	**2.87**	**2.87**	**95.45**	**97.72**	**2.28**
Meas 3	**0.95**	**0.1**	**2.87**	**1.43**	**77.53**	**88.76**	**11.24**
Meas 4	**1**	**0.1**	**2.87**	**0.00**	**0.00**	**50.00**	**50.00**
Meas 5	**1.05**	**0.1**	**2.87**	**1.43**	**77.53**	**11.24**	**88.76**
Meas 6	**1.1**	**0.1**	**2.87**	**2.87**	**95.45**	**2.28**	**97.72**
Meas 7	**1.2**	**0.1**	**2.87**	**5.74**	**99.54**	**0.23**	**99.77**

Table 2. Probability of "just-in-margin", Probability of Pass and Probability of Fail.

5.3. Example III

Again, Meas #2 and #6 show the same *PoP* and *PoF* as in previous examples. Note that the coverage factors become larger as the number of degrees of freedom is decreased.

In Examples I, II and III, we have fixed the expanded uncertainty for didactical purposes. However, in real life, a decrease in the number of degrees of freedom usually leads to a larger measurement uncertainty (see Examples IV and V in paragraphs 8.1 and 8.2 below).

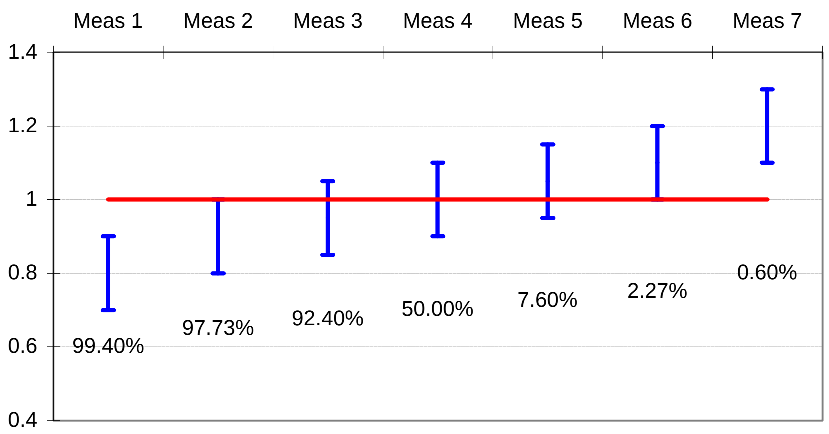


Figure 3. T_{max}=1, U=0.1. Number of effective degrees of freedom u_{eff}=2

	Value	Uncertainty	k_{95}	k_p	p(%) "In-margin"	p(%) of Pass	p(%) of Fail
Meas 1	**0.8**	**0.1**	**4.53**	**9.05**	**98.80**	**99.40**	**0.60**
Meas 2	**0.9**	**0.1**	**4.53**	**4.53**	**95.45**	**97.73**	**2.27**
Meas 3	**0.95**	**0.1**	**4.53**	**2.26**	**84.81**	**92.40**	**7.60**
Meas 4	**1**	**0.1**	**4.53**	**0.00**	**0.00**	**50.00**	**50.00**
Meas 5	**1.05**	**0.1**	**4.53**	**2.26**	**84.81**	**7.60**	**92.40**
Meas 6	**1.1**	**0.1**	**4.53**	**4.53**	**95.45**	**2.27**	**97.73**
Meas 7	**1.2**	**0.1**	**4.53**	**9.05**	**98.80**	**0.60**	**99.40**

Table 3. Probability of "just-in-margin", Probability of Pass and Probability of Fail.

5.4. Probability of Pass as a function of the measured value

Having seen different examples for different numbers of effective degrees of freedom, let us now examine how the Probability of Pass is changed as the measured value approaches the threshold limit, T_{max}.=1. In Figure 4 the three previous examples are represented. The measurement uncertainty is U=0.1.

As it can be seen, there are three probability levels at which all curves cross each other: 50% (where the measured value coincides with the threshold), 97.725% (where the expanded uncertainty exactly comprises the maximum limit and the measured value lies below the threshold) and 2.275% (where the expanded uncertainty exactly comprises the maximum limit and the measured value exceeds the threshold). This is the same effect already observed in the examples.

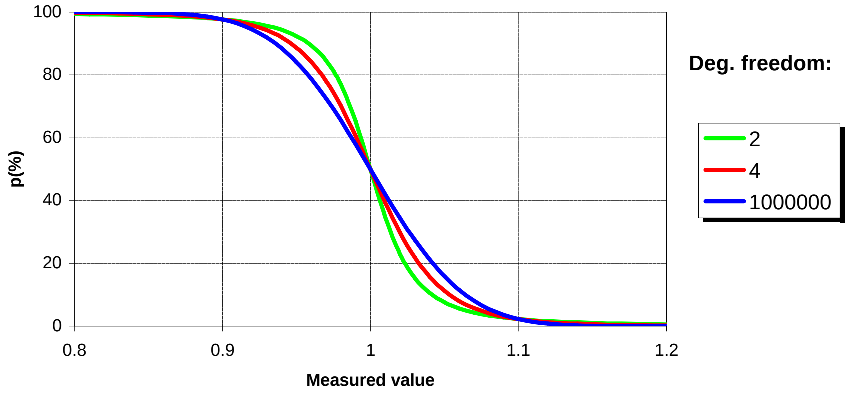


Figure 4. Probability of Pass as a function of the measured value. T_{max}=1. U=0.1

6. SOME STATISTICAL CONSIDERATIONS

6.1. Coverage factors and Level of Confidence

In the next two Figures the probability or level of confidence is shown as a function of the coverage factor k, for different values of u_{eff} . The greater the effective number of degrees of freedom, the less relative weight Type A contributions have within the overall uncertainty budget, and thus the more confident one can be with respect to the quality of the obtained experimental results. This is consistent with the observation that, for a given k, the probability p increases as the number of degrees of freedom does.

In Figure 6 we have represented k normalised with respect to k_{95}, that is the coverage factor for a level of confidence of 95.45%. For $k_{norm}<1$ the level of confidence seems to decrease for a given k as u_{eff} increases, but this effect is due to the normalisation made.

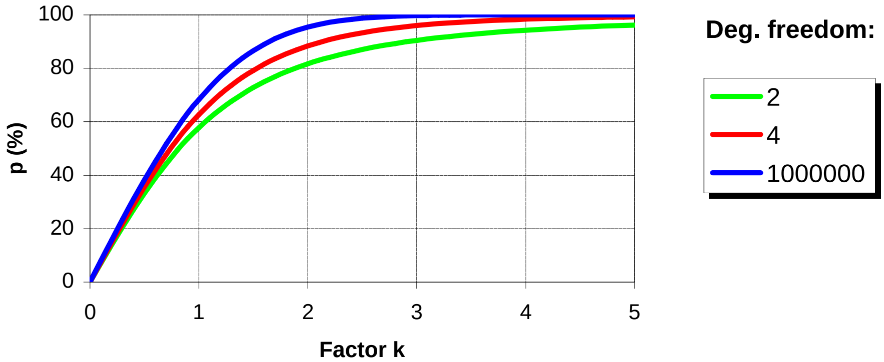


Figure 5. Probability p(%) as a function of the coverage factor k

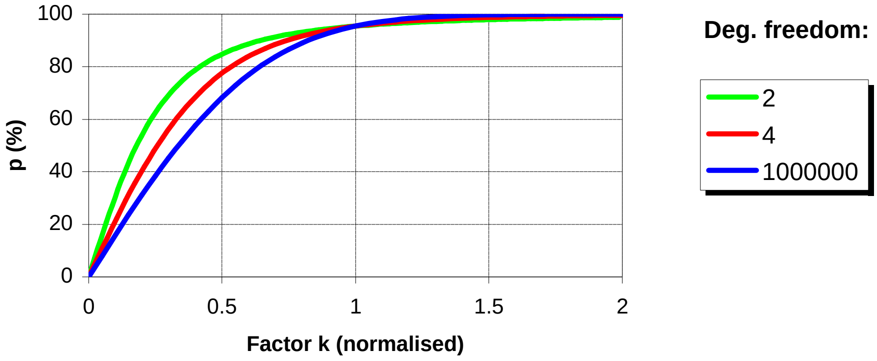


Figure 6. Probability $p(\%)$ as a function of the coverage factor k normalised to k_{95}

6.2. Probability density functions for different Student´s *t* distributions

By definition, the probability density function for the Student´s *t* Distribution is given by:

$$p.d.f.(y;u_{eff}) = \frac{\partial\, tDistribution(y;u_{eff})}{\partial\, y} = lim_{\Delta y \to 0} \frac{tDistribution(y+\Delta y;u_{eff}) - tDistribution(y;u_{eff})}{\Delta y}$$

As any probability density function, it has to satisfy the following condition:

$$\int_{y=-\infty}^{\infty} p.d.f.(y;u_{eff}) = 1$$

The integral between –*k* and *k* gives us the probability for the measurand to lie within the interval [–*k*, *k*]:

$$\int_{y=-k}^{k} p.d.f.(y;u_{eff}) = 1 - tDistribution(k;u_{eff})$$

In the next two Figures different probability density functions are shown as a function of *y* and *y* normalised with respect to k_{95}. They are all symmetrical around 0, so we have represented them for positive values of *y* exclusively.

As it has been said, the following condition has to be satisfied for all curves shown:

$$2 \cdot \int_{y=-0}^{\infty} p.d.f.(y;u_{eff}) = 1$$

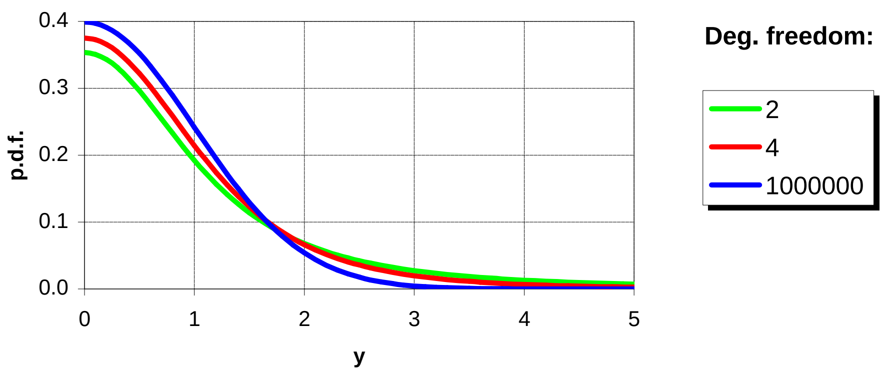


Figure 7. *p.d.f.* as a function of *y*

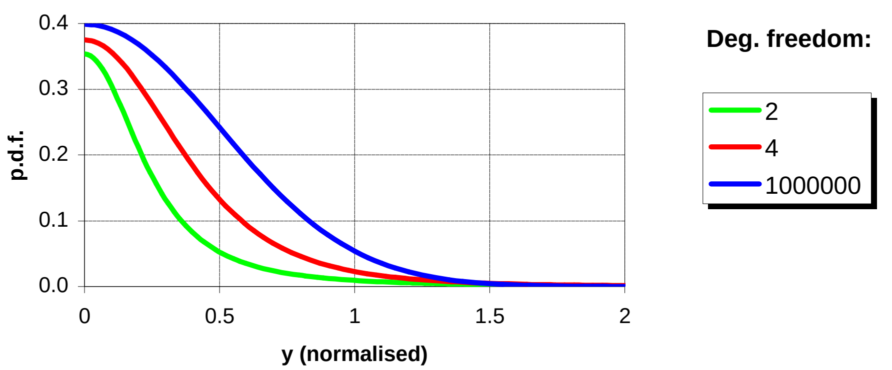


Figure 8. *p.d.f.* as a function of y normalised to k_{95}

6.3. Gaussian probability density function

In Figure 7, the p.d.f. for the Student´s *t* Distribution with $u_{eff}=10^6$ corresponds to the Normal or Gaussian Distribution with mean value 0 and standard deviation 1:

$$f(y) = \frac{1}{\sqrt{2 \cdot p}} \cdot e^{\frac{-y^2}{2}}$$

As a matter of verification, one could assess by integration the following levels of confidence for integer number of times the standard deviation:

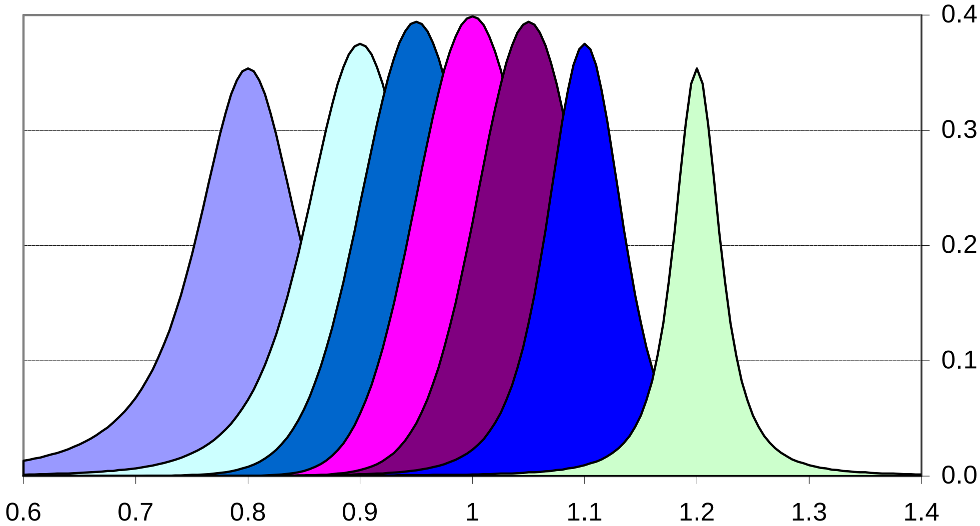
$$2 \cdot \int_{y=0}^{1} f(y) = 0.6827 \qquad 2 \cdot \int_{y=0}^{2} f(y) = 0.9545 \qquad 2 \cdot \int_{y=0}^{3} f(y) = 0.9973$$

Integration can be of course applied to any of the curves shown in Figures 7 and 8.

6.4. Probability density functions in Pass-or-Fail tests

In Figure 9 and Table 4 several examples of probability density functions are shown. The threshold value considered is again T_{max}=1.

As a matter of verification, one could integrate each of the curves below and above T_{max} and compare the results obtained with *PoP* and *PoF* in Table 4.

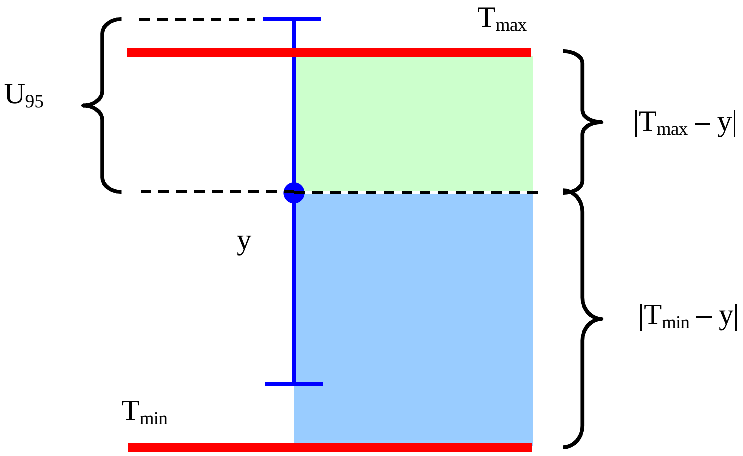


	Value	Uncertainty	95	freedom	p	margin”	p(%) of Pass	of Fail
Meas 1		0.226	4.53		4.00	94.28		2.86
Meas 2		0.143	2.87		2.00	88.39		5.81
Meas 3		0.107	2.13		1.00	67.07		16.46
Meas 4		0.100	2.00		0.00	0.00		50.00
Meas 5		0.100	2.13		1.07	70.11	14.94	
Meas 6	1.1		2.87	4		95.45	2.28	
Meas 7	1.2		4.53	2		98.80	0.60	

Probability of "just- -margin", and *PoF*

7. DOUBLE PASS-OR-FAIL TESTS

In a "Double" Pass-or-Fail Test two limiting values are defined. We shall thus define two "just-in-margin" coverage factors related to $|T_{max} - y|$ and $|T_{min} - y|$. They are in general not the same, as it can be seen in the following figure. Therefore the associated levels of confidence p_1 and p_2 are also different in a general case.



The two "just-in-margin" factors and their related probabilites p_1 and p_2 are given by:

$$k_{p1} = \frac{|T_{max} - y| \cdot k_{95}}{U_{95}} \qquad k_{p2} = \frac{|T_{min} - y| \cdot k_{95}}{U_{95}}$$

$$p_1(\%) = 100 - tDistribution(k_{p1}; u_{eff}) \qquad p_2(\%) = 100 - tDistribution(k_{p2}; u_{eff})$$

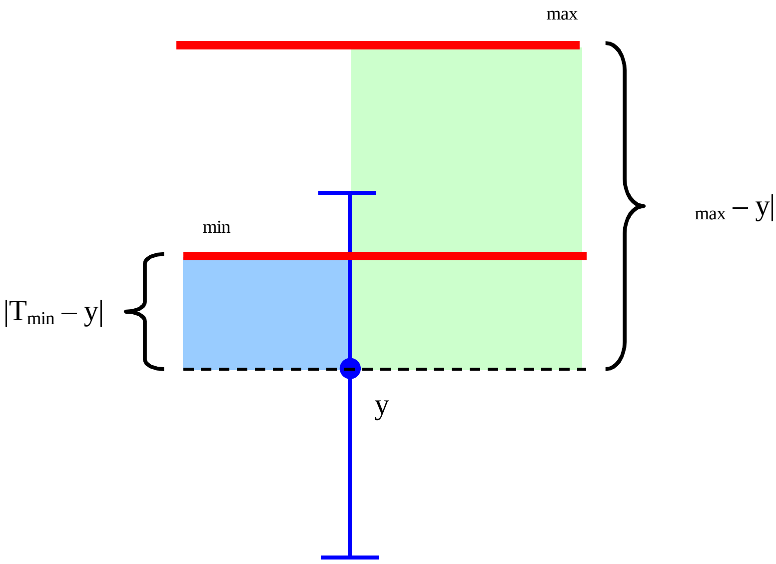
7.1. The measured value lies between the two limits

For the maximum limit: the measurand is estimated to lie within the interval **[y, T_{max}]** with a probability **$p_1/2$**. For the minimum limit: the measurand is estimated to lie within the interval **[T_{min}, y]** with a probability **$p_2/2$**. See figure above.

Therefore, the probability for the measurand to lie within the interval [T_{min}, T_{max}] is $(p_1+p_2)/2$. In other words:

- The measurand is estimated to <u>Pass the Test</u> with a probability $(p_1+p_2)/2$
- The measurand is estimated to <u>Fail the Test</u> with a probability $100 - (p_1+p_2)/2$

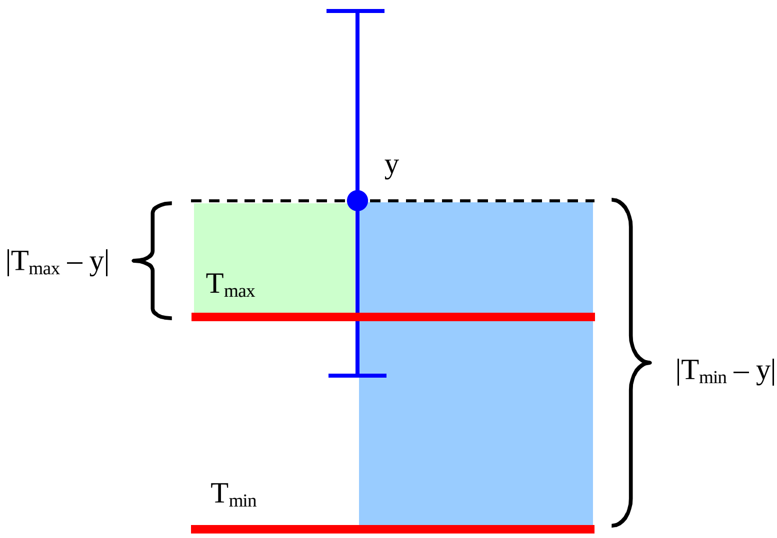
7.2. The measured value lies below the two limits



For the maximum limit: the measurand is estimated to lie within the interval **[, *T*]** with a probabilit **p_1** . For the minimum limit: th **[*y* T_{min}** with a probability **p_2** .

Therefore, the probability for the measurand to lie within the interval [*T* , *T*] is (p – p_2 words:

- Pass the Test with a p_1– p_2)/2
- The measurand is estimated to Fail the Test with a probability 100 – ($_1$ $_2$)/2



[$_{max}$,] ty **p /2.** **[*T* , *y*** with a probability **p_2** .

Therefore, the probability for the measurand to lie within the interval [T_{min}, T_{max}] is ($p_2 - p_1$)/2. In other words:

- The measurand is estimated to Pass the Test with a probability ($p_2 - p_1$)/2
- The measurand is estimated to Fail the Test with a probability 100 – ($p_2 - p_1$)/2

8. SOME EXAMPLES OF DOUBLE PASS-OR-FAIL TESTS

8.1. Example IV

In Figure 10 and Table 5 several cases for a Double Pass-or-Fail test are shown. As usual, the probability level considered is p=95.45%. The maximum threshold value T_{max} is 1 and the minimum limit T_{min}=0.6. The measurement uncertainty U is 0.1 times the coverage factor k_{95}. This is perhaps more representative of real-life measurements, where the expanded uncertainty increases as the number of effective degrees of freedom decreases.

From Figure 10 it can be seen that the Probability of Pass reaches a maximum as the measured value enters the zone between the two limits. Interestingly enough, Meas #5 confirms us that the Probability of Pass is exactly p=95.45%, just because the expanded uncertainty exactly comprises both limits.

There are no "paired" cases as in previous examples, although common sense also tells us that the Probabilities of Pass and Fail would repeat under certain circumstances. For instance, if the measured value were 1.2, *PoP* and *PoF* would be exactly the same as in Meas #1.

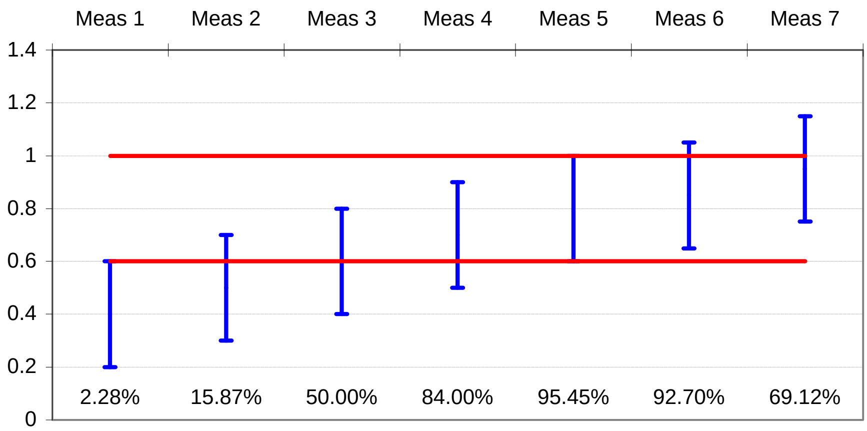


Figure 10. T_{max}=1, T_{min}=0.6, U=0.1 times k_{95}. Number of effective degrees of freedom u_{eff}=10^6

	Value	Uncertainty	k_{95}	k_{p1} sup	p_1(%)	k_{p2} inf	p_2(%)	PoP(%)	PoF(%)
Meas 1	0.4	0.2	2.00	6.00	100.00	2.00	95.45	2.28	97.73
Meas 2	0.5	0.2	2.00	5.00	100.00	1.00	68.27	15.87	84.13
Meas 3	0.6	0.2	2.00	4.00	99.99	0.00	0.00	50.00	50.00
Meas 4	0.7	0.2	2.00	3.00	99.73	1.00	68.27	84.00	16.00
Meas 5	0.8	0.2	2.00	2.00	95.45	2.00	95.45	95.45	4.55
Meas 6	0.85	0.2	2.00	1.50	86.64	2.50	98.76	92.70	7.30
Meas 7	0.95	0.2	2.00	0.50	38.29	3.50	99.95	69.12	30.88

Table 5. Probabilities of "just-in-margin", Probability of Pass and Probability of Fail.

8.2. Example V

Again, several cases of measured values are shown, together with *PoP* *PoF*. From comparison with re no particular cases for which the probabilities remain unchanged. This is due to the fact that the measurement uncertainty is not constant, but dependent on

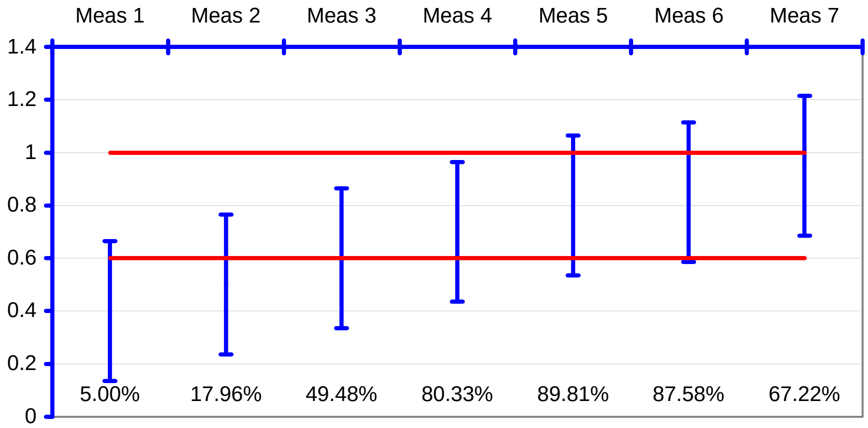


Figure 11. T_{max}=1, T_{min}=0.6, U=0.1 times k_{95}. Number of effective degrees of freedom u_{eff}=5

	Value	Uncertainty	k_{95}	k_{p1} sup	p_1(%)	k_{p2} inf	p_2(%)	PoP(%)	PoF(%)
Meas 1	0.4	0.265	2.65	6.00	99.82	2.00	89.81	5.00	95.00
Meas 2	0.5	0.265	2.65	5.00	99.59	1.00	63.68	17.96	82.04
Meas 3	0.6	0.265	2.65	4.00	98.97	0.00	0.00	49.48	50.52
Meas 4	0.7	0.265	2.65	3.00	96.99	1.00	63.68	80.33	19.67
Meas 5	0.8	0.265	2.65	2.00	89.81	2.00	89.81	89.81	10.19
Meas 6	0.85	0.265	2.65	1.50	80.61	2.50	94.55	87.58	12.42
Meas 7	0.95	0.265	2.65	0.50	36.17	3.50	98.27	67.22	32.78

Table 6. Probabilities of "just-in-margin", Probability of Pass and Probability of Fail.

8.3. Probability of Pass as a function of the measured value

In Figure 12, is represented as a function of the measured value in a Double Pass- -Fail test with Both examples are included in the blue and red curves, respectively. The measurement uncertainty is 0.1 times $_{95}$.

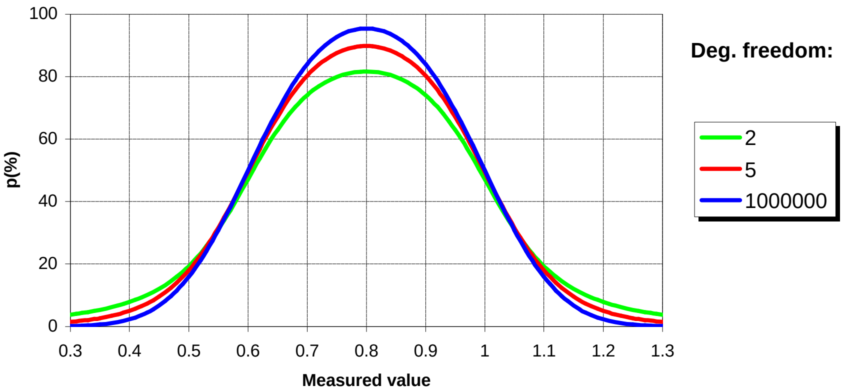


Figure 12. Probability of Pass as a function of the measured value. $_{max}$=1. $_{min}$=0.6. =0.1 times *k*

We can try maintaining *U* in Figure 13 are obtained. Apparently, there are now three probability levels at which all curves cross

measured value lies within both limits), 50% (where the measured value coincides with one of the threshold limits) and 2.275% (where the expanded uncer measured value lies outside the limits). This is only approximate, though, and depends on the ratio between and | $_{max} - T_{min}$|.

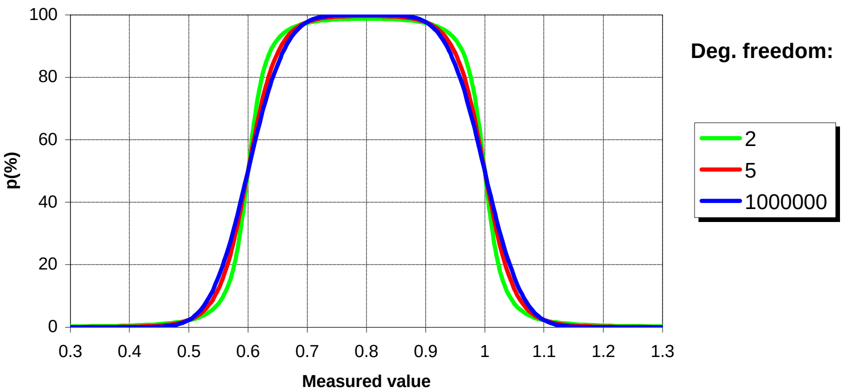


Figure 13. Probability of Pass as a function of the measured value. T_{max}=1. T_{min}=0.6. U=0.1

Finally, we can fix the number of effective degrees of freedom and see the effect of the measurement *PoP* is represented as a function of the measured value, for an expanded *U* r 6.

The effect of a greater uncertainty – in relation to the absolute difference $|T_{max} - {}_{min}|$ – can be seen to be a certain "spread" of the curves. As the uncertainty decreases, the tends to follow a square

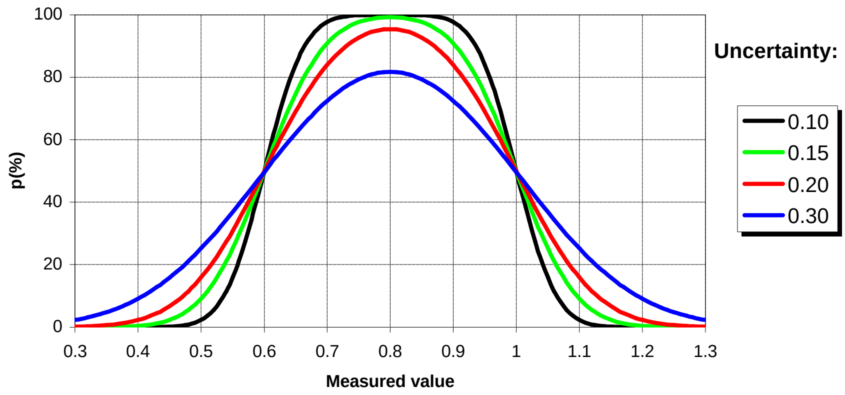


Figure 14. Probability of Pass as a function of the measured value. T_{max}=1. T_{min}=0.6. $u_{eff}=10^6$

9. THE EXPRESSION OF PASS-OR-FAIL RESULTS IN A CALIBRATION CERTIFICATE

There are in principle two ways of expressing the result of a Pass-or-Fail test in a metrological environment.

Option A) Measured Value + Measurement uncertainty + Probability of Pass

$$y \pm U$$

"The measurand is estimated to lie within the interval [y–U, y+U] with a level of confidence of 95.45%(1). The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor of k. The probability for the measurand to Pass the Test(2) is *PoP*(%)".

Option B) Measured Value + Probability of Pass + Nr. of effective degrees of freedom

$$y$$

"The measurand is estimated to Pass the Test(2) with a level of confidence or Probability of Pass *PoP*(%). The number of effective degrees of freedom is u_{eff}".

(1) Or any other agreed probability level
(2) Either: to lie below T_{max}
to lie above T_{min}
to lie within the interval [T_{min}, T_{max}]

In the author´s opinion, option B is preferred. It is a self-contained formula which contains all relevant information with a minimum of parameters.

10. PASS OR FAIL?

The author would suggest to define “compliance with specification” in relation to the computed Probability of Pass. For example, it seems reasonable to assess that the product complies with the required limits whenever the Probability of Pass is greater than 95.45%. (Of course this should be made application-dependent. One could state a limit of 99% for critical applications in the aerospace industry, whereas for other applications a limit of 66% could suffice).

11. CONCLUSIONS

We have presented a metrological approach to the popular “Pass-or-Fail” Tests, which deals with measurement uncertainties, coverage factors and levels of confidence.

We have considered different types of tests, depending on whether a maximum limit, a minimum limit or a double threshold condition are specified, and have examined them in the light of the computed Probability of Pass and Probability of Fail.

We have made some considerations about probability density functions which may help us understand the probability for the measurand to lie within a given interval around the measured (or most probable) value.

Finally the discussion about the most convenient way to incorporate such results into a calibration certificate remains open. In particular, the question about whether the product under test complies with specification or not, seems difficult to answer.

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