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An Algorithm for the Evaluation of the
Residual Directivity "Ripple" Trace

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An algorithm for the evaluation of the residual directivity “ripple” trace

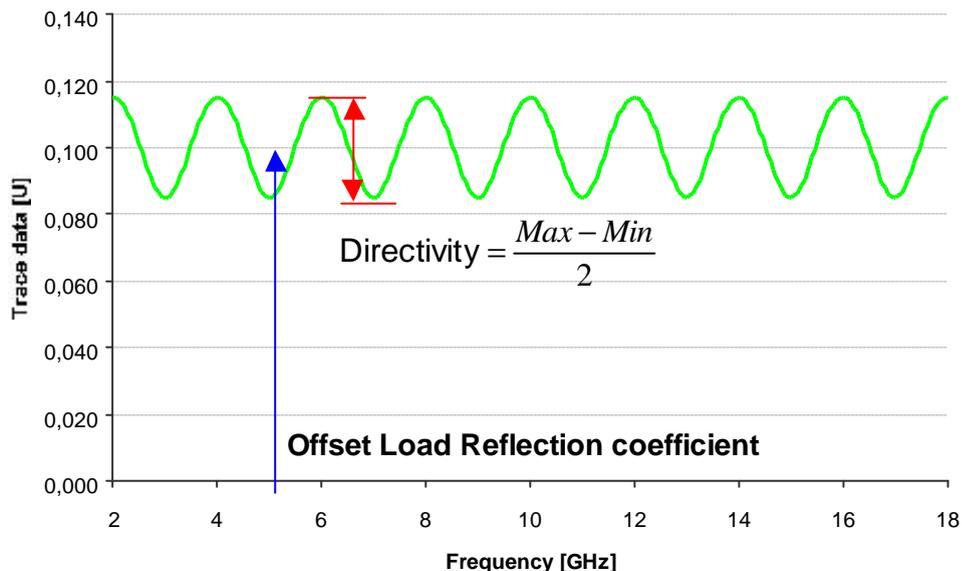
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1 Classical airline “ripple” technique

The idea of the classical airline "ripple" technique is to connect a long airline to the test port under test and to connect an offset load at the other end of the airline. The length of the airline will cause the reflection coefficient vector of the offset load to spin around the directivity vector when the frequency is increased. This will produce a ripple in the magnitude trace in the frequency domain and the directivity can be found as half of the difference between the maximum magnitude and minimum magnitude of the rippled trace.

Figure 1 Classical airline "ripple" technique directivity trace.



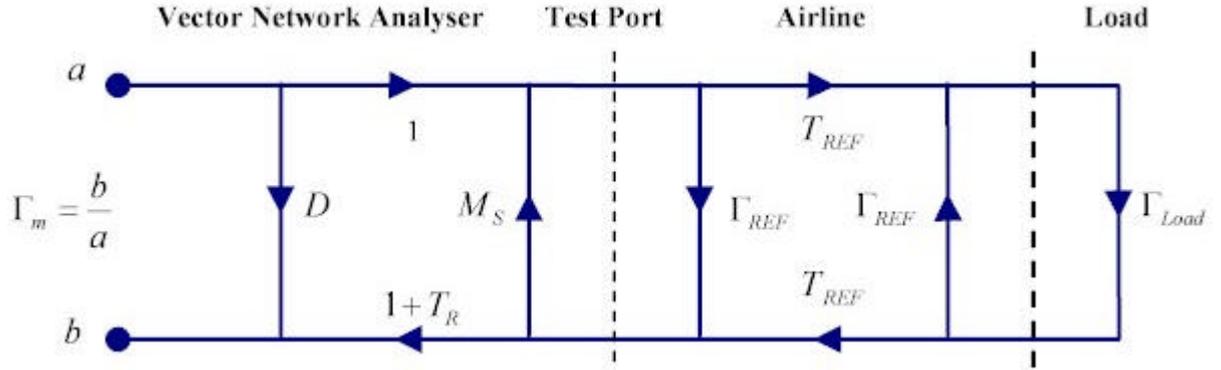
This is a pure scalar measurement, where we only see the sum of the two vectors. The magnitude of the offset load reflection coefficient should be large enough to be easily distinguished from the directivity but small enough to minimise the influence of source match.

The fundamental assumption that has to be fulfilled in the ripple technique is that both the load reflection coefficient and the directivity are constant, both in magnitude and phase, in the frequency interval used to evaluate the measurement. The frequency interval needed to get one ripple period depends on the length of the airline. One big disadvantage with this scalar technique is that only two measurement points in the frequency range in question, the maximum and minimum value, is used to evaluate the trace.

2 The “vector ripple” technique

When trying to find the residual directivity of a calibrated vector network analyser system we will use a reference airline and a matched load instead of an offset load to improve the classical airline "ripple" technique. The figure below shows a flow graph representation of the residual directivity “ripple” measurement.

Figure 2 Flow graph representation of the residual directivity measurement.



Γ_m	Measured reflection coefficient
D	Residual directivity
T_R	Residual reflection tracking
M_S	Residual source match
Γ_{REF}	Reference airline reflection coefficient
T_{REF}	Reference airline transmission coefficient
Γ_{Load}	Load reflection coefficient

After a calibration the residual systematic errors would be relatively small so we can use a simple flow graph analysis of the figure above:

$$\Gamma_m = T_{REFm}^2 \cdot \Gamma_{Loadm} + D_m = D + \Gamma_{REF} + T_{REF}^2 \cdot \Gamma_{Load} + T_R \cdot T_{REF}^2 \cdot \Gamma_{Load} + M_S \cdot \Gamma_{REF} + M_S \cdot T_{REF}^4 \cdot \Gamma_{Load}^2 \quad \text{Equation 1}$$

T_{REFm}	Measured reference airline transmission coefficient
Γ_{Loadm}	Measured load reflection coefficient
D_m	Measured residual directivity
M_S	Residual source match

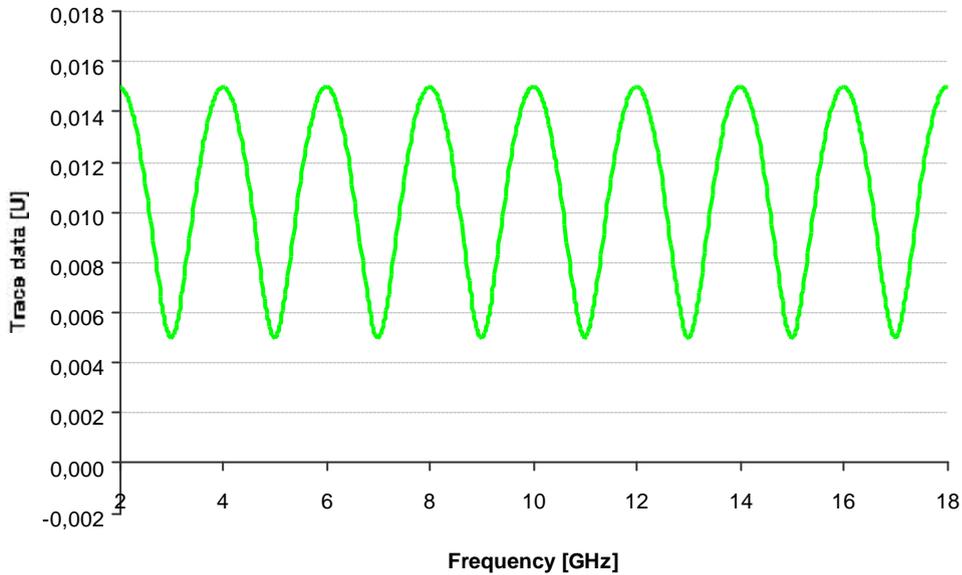
Since we use a matched load the influence of residual reflection tracking and residual source match can be neglected:

$$\Gamma_m = T_{REFm}^2 \cdot \Gamma_{Loadm} + D_m \approx D + \Gamma_{REF} + T_{REF}^2 \cdot \Gamma_{Load} \quad \text{Equation 2}$$

This is the equation that describes the ripple trace. The transmission coefficient T_{REF} of the airline will cause the reflection coefficient vector Γ_{Load} of the matched load to spin around a constant vector, that consists of the directivity D plus the reference airline reflection coefficient Γ_{REF} , when the frequency is increased. This will produce the ripple in the

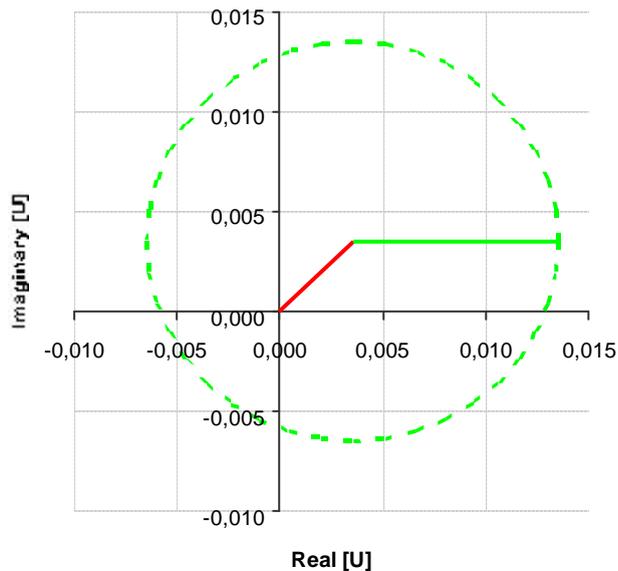
magnitude trace in the frequency domain, see the figure below. It is also clear that we can't distinguish between the residual directivity D that we want to measure and the reflection coefficient Γ_{REF} of the reference airline used.

Figure 3 Directivity ripple in the magnitude trace in the frequency domain.



Since we are concerned with a vector network analyser system, we can look at the ripple trace in the complex plane, see the figure below. The constant vector, that consists of the directivity D plus the reference airline reflection coefficient Γ_{REF} , is the centre of the circle and the radius is the magnitude of the reflection coefficient vector Γ_{Load} of the matched load that turns around the constant vector when the frequency is increased.

Figure 4 Directivity ripple circle in the complex plane.



The figure above indicates that we can evaluate the ripple trace by fitting a circle to it in the complex plane. The ripple trace can be divided into frequency intervals that cover about a period of the ripple each. These relatively short frequency intervals will help us to fulfil the fundamental assumption that both the load reflection coefficient and the directivity is

constant, both in magnitude and phase, in the frequency interval used to evaluate the measurement. One big advantage with this technique is that all measurement points in the trace will be used to evaluate the trace.

We will use the matrix form of the least squares fit to fit a circle to the ripple trace in the complex plane. The equation of a circle can be written as:

$$\begin{aligned} (X - X_0)^2 + (Y - Y_0)^2 &= r^2 \\ r^2 + 2 \cdot X \cdot X_0 + 2 \cdot Y \cdot Y_0 - X_0^2 - Y_0^2 &= X^2 + Y^2 \\ C_1 + C_2 \cdot X + C_3 \cdot Y &= X^2 + Y^2 \\ C_1 &= r^2 - X_0^2 - Y_0^2 \\ C_2 &= 2 \cdot X_0 \\ C_3 &= 2 \cdot Y_0 \end{aligned} \tag{Equation 3}$$

(X, Y) Data point on the circle

(X₀, Y₀) Centre of the circle

r Radius of the circle

This can be re-formulated as a matrix equation:

$$A \times C = B$$

$$A = \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \\ \dots & \dots & \dots \\ 1 & x_n & y_n \end{pmatrix} \quad B = \begin{pmatrix} x_1^2 & y_1^2 \\ x_2^2 & y_2^2 \\ x_3^2 & y_3^2 \\ \dots & \dots \\ x_n^2 & y_n^2 \end{pmatrix} \quad C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} \tag{Equation 4}$$

(X_i, Y_i) Measured data point in the ripple trace

n Number of measured data points in the ripple trace

The matrix form of the least squares fit can now be applied:

$$\begin{aligned} A \times C &= B \\ A^T \times A \times C &= A^T \times B \\ C &= (A^T \times A)^{-1} \times A^T \times B \end{aligned} \tag{Equation 5}$$

The solution is a circle with the following parameters:

$$\begin{aligned} X_0 &= \frac{C_2}{2}, \quad Y_0 = \frac{C_3}{2} \\ r &= \sqrt{C_1 + \frac{C_2^2}{4} + \frac{C_3^2}{4}} \end{aligned} \tag{Equation 6}$$

The measured residual directivity can be estimated as the centre (X₀, Y₀) of the fitted circle. One big advantage with this technique is that it can distinguish between the residual directivity vector and the matched load vector.

We have now separated the two parts of the ripple trace so the equation from the vector network analyser flowgraph can be rewritten to get a basis for a model function to the residual directivity measurement:

$$T_{REFm}^2 \cdot \Gamma_{Loadm} + D_m = D + \Gamma_{REF} + T_{REF}^2 \cdot \Gamma_{Load}$$

$$1) \quad T_{REFm}^2 \cdot \Gamma_{Loadm} = T_{REF}^2 \cdot \Gamma_{Load}$$

$$|\Gamma_{Load}| = \frac{|T_{REFm}^2|}{|T_{REF}^2|} \cdot |\Gamma_{Loadm}|$$

Equation 7

$$2) \quad D_m = D + \Gamma_{REF}$$

$$D = D_m - |\Gamma_{REF}| = D_m - r_{REF}$$

If we add other errors in the measurement, we can obtain a model function for the residual directivity measurement:

$$D = D_m - (r_{REF} + C_R + R_R) - (A + N_H + C_D) \cdot D_m - N_L$$

Equation 8

D Measurand, the "true" residual directivity

D_m Measured residual directivity

r_{REF} Magnitude of the reference airline reflection coefficient

C_R Reflection stability of the test port cable

R_R Reflection repeatability of the test port connector interface

A Dynamic accuracy

N_H High level noise

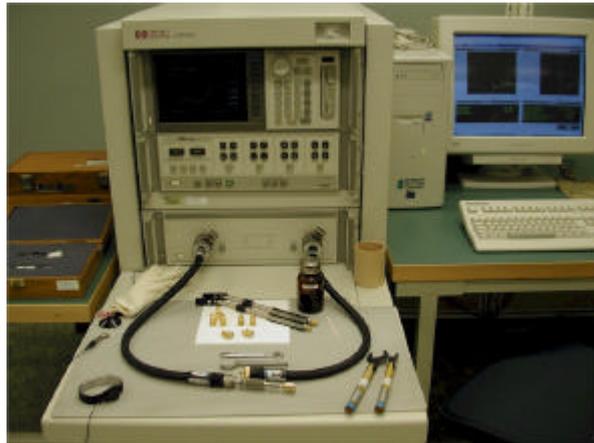
C_D Drift since system calibration

N_L Low level reflection noise

3 Experimental verification of the “vector ripple” technique

The measurements for the experimental verification of the “vector ripple” technique are made with a Hewlett Packard 8510C network analyser system which is controlled by a computer with an in-house developed software.

Figure 5 Hewlett Packard 8510C network analyser system.



The reference airline selected for the measurements is a beadless 3.5 mm 50 Ω Hewlett Packard 85052B Option K15 airline. Two Hewlett Packard terminations, one broadband load with good high frequency performance and one lowband load with poorer high frequency performance, are used to terminate the airline.

The diameters of the airline has been measured in an Mercer 122LM air-gauging system and the theoretical value of the reflection coefficient of the airline have been calculated from these measured dimensions and material constants.

Figure 6 Mercer 122LM air-gauging system.



3.1 Residual directivity of a broadband load calibration with a “known” load

The network analyser system is calibrated with the “known” broadband load so the residual directivity obtained with the “vector ripple” technique should be equal to the “known” reflection coefficient of the load. The reflection coefficient of the broadband load is “known” since it has been measured on the network analyser system after a TRL calibration with a Hewlett Packard 85052C calibration kit.

The reference airline is connected to the calibrated test port and terminated with a good load and the computer with the in-house developed software is used to measure and evaluate the residual directivity of the broadband load calibration.

Figure 7 Set-up for the residual directivity measurement.



Figure 8 The measurement results presented on the computer screen by the software.

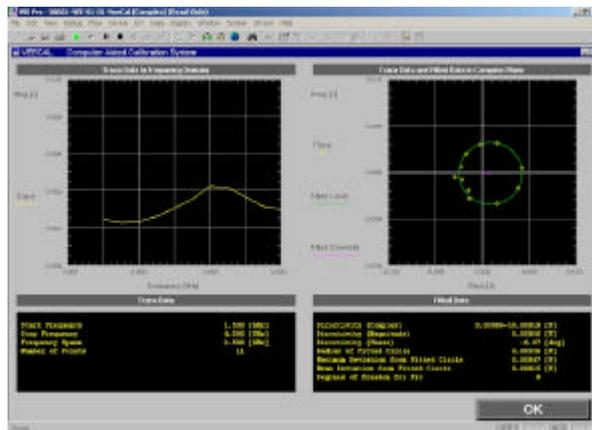


Figure 9 A closer look at the trace data in the frequency domain and the trace data and fitted data in the complex plane.

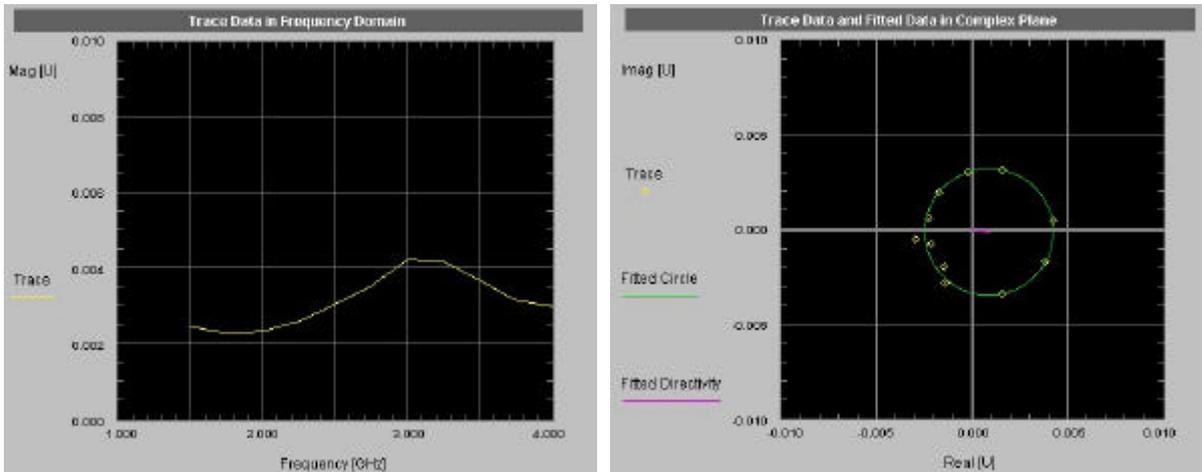
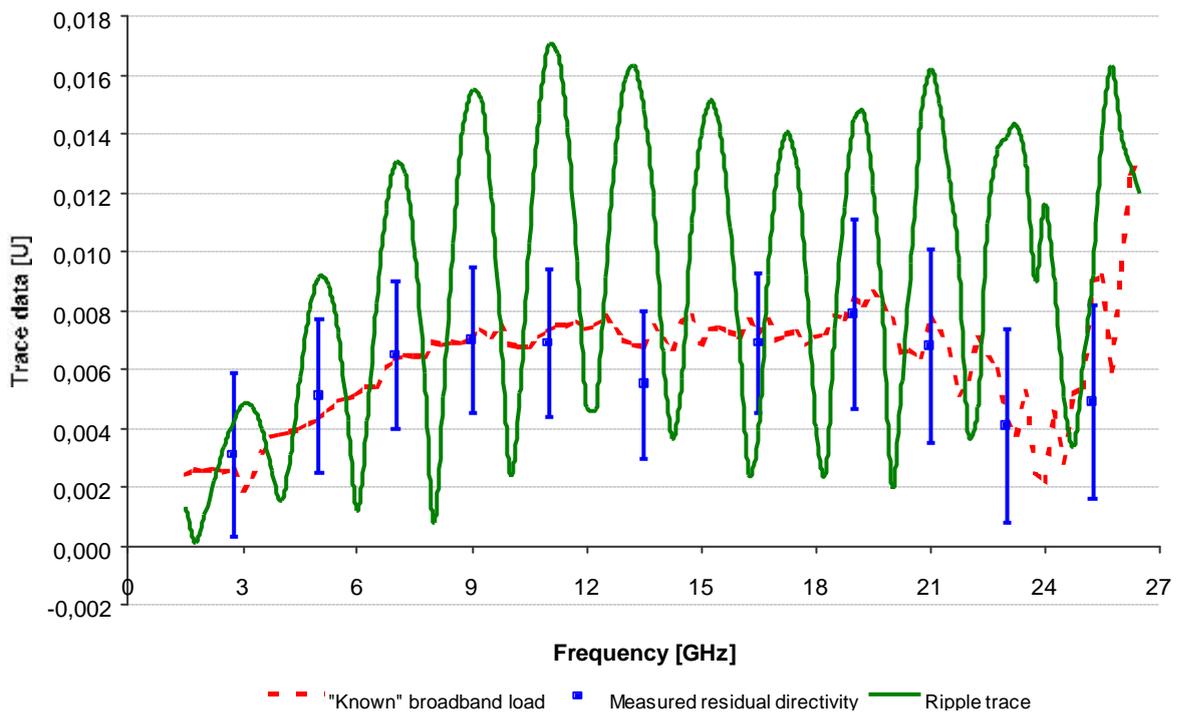


Figure 10 Measured residual directivity after a broadband load system calibration with a "known" broadband load.



The figure above shows a good agreement between the measured residual directivity with the "vector ripple" technique and the "known" broadband load that has been used to calibrate the network analyser system.

3.2 Residual directivity of a sliding load calibration

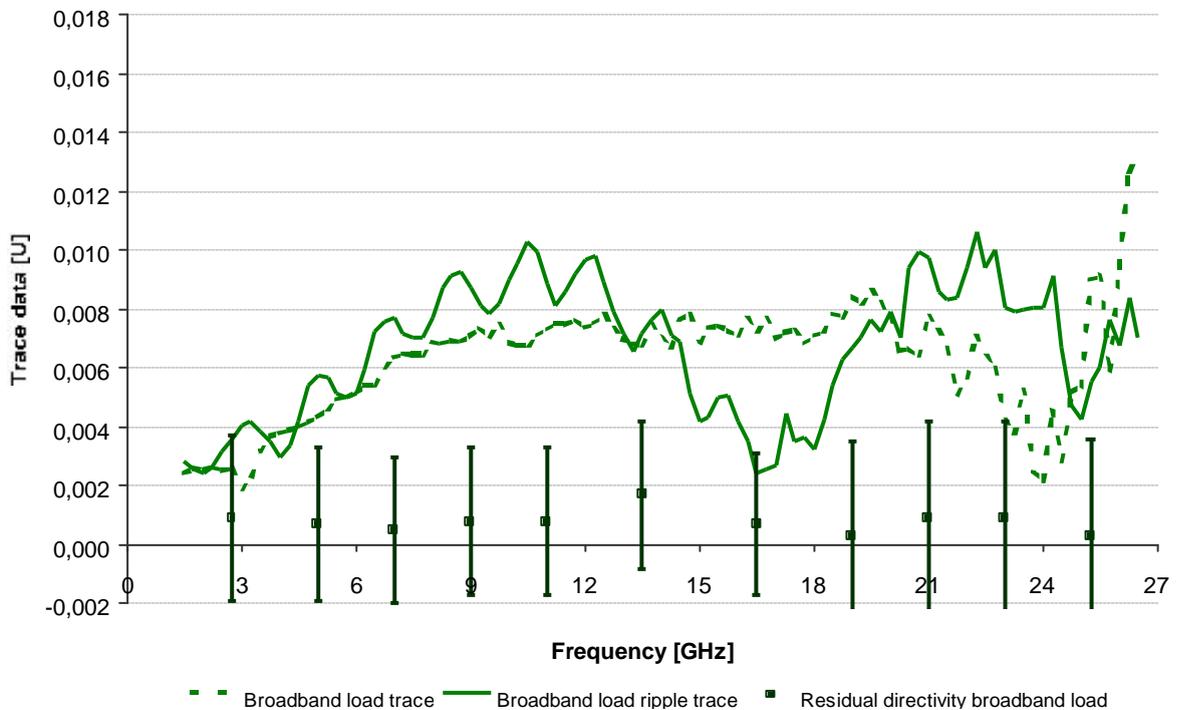
The network analyser system has now been calibrated with a Full 2-Port sliding load calibration. The residual directivity of this calibration will be measured using three different loads to terminate the reference airline. The calibration kit is a Hewlett Packard 85052B and the sliding loads used in the calibration are “known” to be very good so the residual directivity obtained with the “vector ripple” technique should be fairly small.

First the residual directivity of the sliding load calibration is measured with the reference airline terminated with the “good” broadband load.

Figure 11 Set-up for the broadband load residual directivity measurement.



Figure 12 Broadband load measured data of a Full 2-Port sliding load calibration.

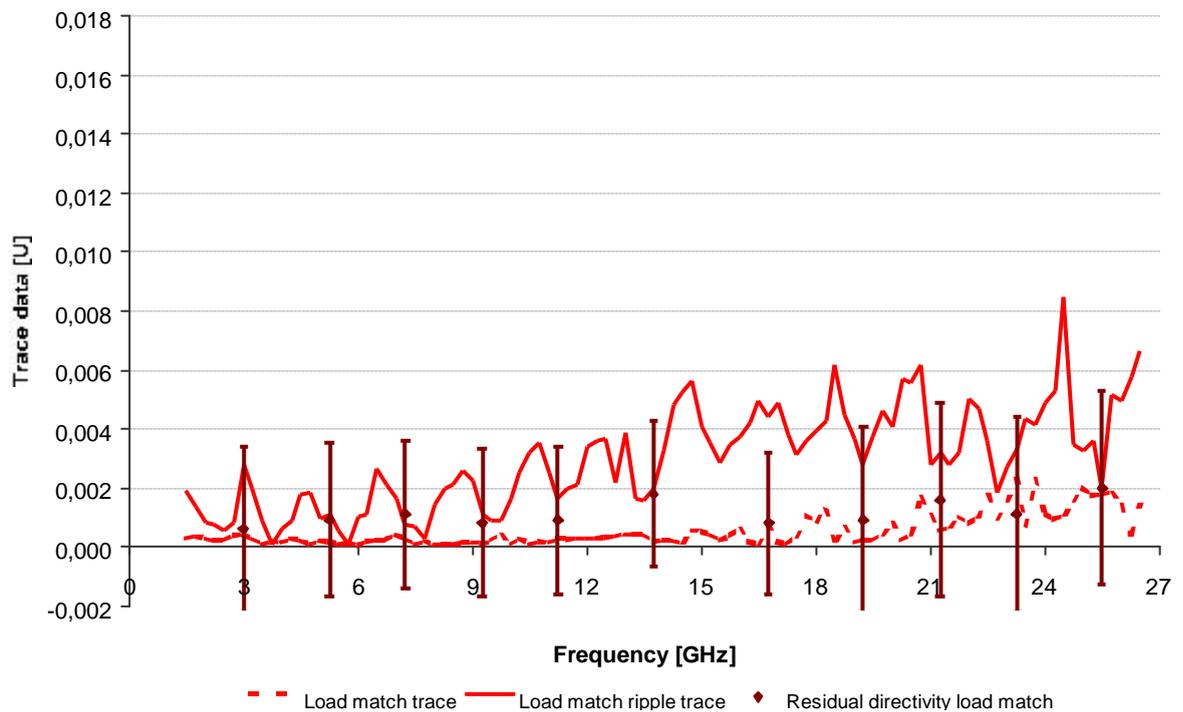


The fundamental assumption for the "ripple" technique was that the reflection coefficient vectors should be constant and the best way to obtain a constant load is probably to use as perfect load as it is possible. One way to get a "perfect" load is to use the "artificial" load match of a Full 2-Port calibration. So in the next measurement of the residual directivity of the sliding load calibration the reference airline is terminated with the "perfect" load match of the Full 2-Port calibration.

Figure 13 Set-up for the load match residual directivity measurement.



Figure 14 Load match measured data of the Full 2-Port sliding load calibration.

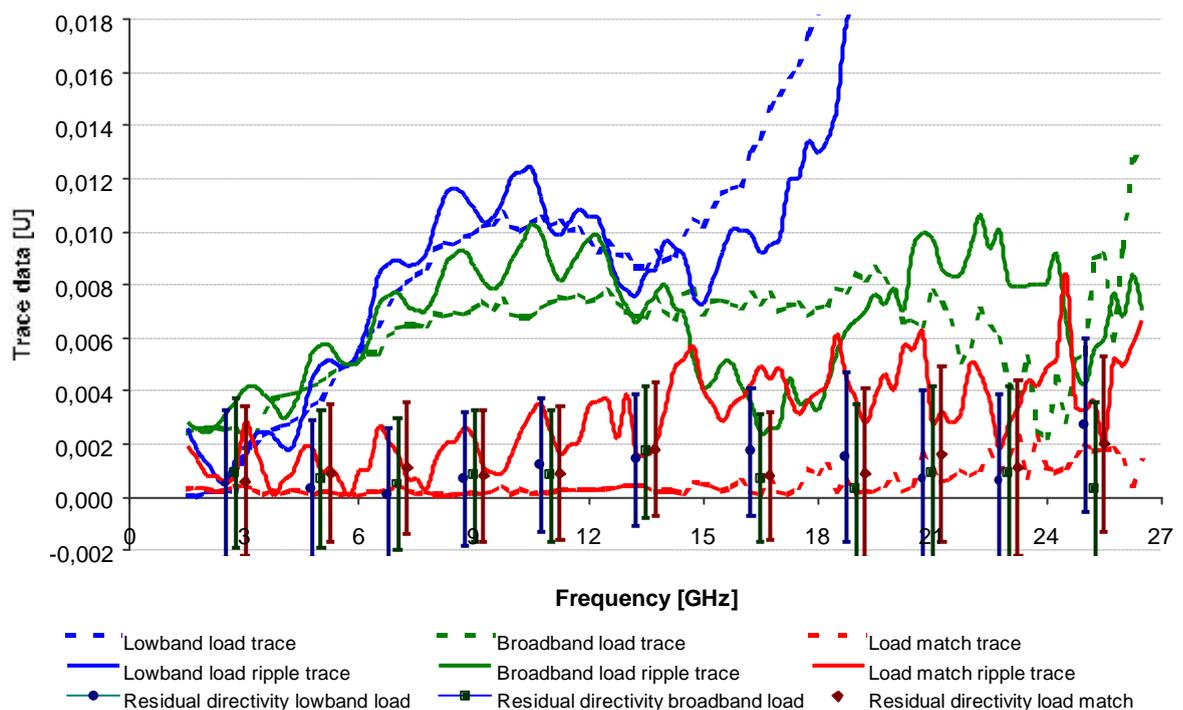


To test the technique a “poor” lowband load is also used to measure the residual directivity of the sliding load calibration.

Figure 15 Set-up for the lowband load residual directivity measurement.



Figure 16 Comparison of the measured data obtained with three different loads.



The residual directivity measurement results obtained with three different loads to terminate the reference airline seem to be consistent.

Other comparisons between “manually” evaluated “ripple” measurements and this “vector ripple” technique also seem to agree but it is very difficult to make measurements with the high degree of accuracy needed to clearly see differences between the “manual” technique and this “vector ripple” technique. One always have the questions about the repeatability of the connector interface and the difference between the theoretical value (calculated from dimensions and material constants) of the reflection coefficient of a reference airline and the electrically measured value.

4 Conclusions

A least squares fit of a circle in the complex plane can be used as an algorithm for the evaluation of the residual directivity "ripple" trace obtained with a vector network analyser. A big advantage with this technique is that all measurement points in the trace will be used to evaluate the trace.

The ripple trace can be divided into frequency intervals that cover about a period of the ripple each. These relatively short frequency intervals will help us to fulfil the fundamental assumption that both the load reflection coefficient and the directivity is constant, both in magnitude and phase, in the frequency interval used to evaluate the measurement.

The technique can distinguish between the residual directivity vector and the load vector so a "perfectly" matched load can be used to terminate the airline. One way to get a "perfect" load is to use the "artificial" load match of a Full 2-Port calibration.

5 References

1. Why Tolerate Unnecessary Measurement Errors, Wiltron Technical Review No. 5, June 1979.
2. An algorithm for least squares fit of a circle, Jan Rudander.
3. EA Guidelines on the Evaluation of Vector Network Analysers (VNA), EA-10/12, European co-operation for Accreditation, May 2000.