ANAMET Report 025
June 1999

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M Rodríguez

ANAMET REPORT

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Comments on this report should be sent to:

Mail: ANAMET
Building 72
National Physical Laboratory
Queens Road
Teddington
Middlesex
TW11 0LW

Fax: 020 8943 6037 (UK)
+44 20 8943 6037 (International)

This report has been approved by the ANAMET Steering Committee.

1520/SEA/2.5K/0699

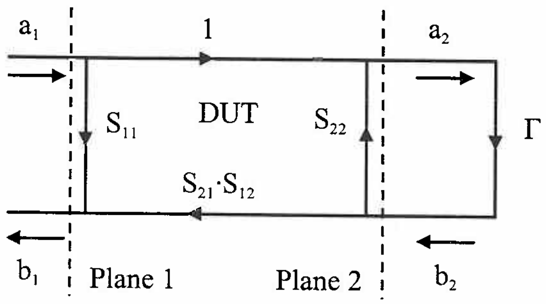
MEASUREMENT OF NON-INSERTABLE DEVICES USING ONE-PORT CALIBRATIONS

Manuel Rodríguez
INTA, Spain

1. INTRODUCTION

During the past, some attention has been devoted to the characterisation – or removal! – of non-insertable devices, such as adapters and cables [1], [2]. Our aim is to present an easy-to-implement procedure for characterisation of such devices, based on two simple One-Port calibrations. This technique can be applied both to the measurement of the non-insertable device (characterisation) or to the measurement of any artefact connected through it (removal). For reasons of clarity, the approach followed here will be that of the characterisation.

2. FLOW GRAPH OF THE SIGNALS INVOLVED



We try to depict here the main signals present. The non-insertable device is represented by its scattering parameters. At this point the key assumption is made of considering the device under test reciprocal, that is S_{21} equals S_{12}. By doing so, we are anticipating one of the limitations of the method: since it is based on two One-Port calibrations, it is the product $S_{21} \cdot S_{12}$ which is measured. In the above figure, this is explicity shown by grouping both parameters into one. However, this is a common assumption when measuring these kinds of devices, [1].

The above flow graph certainly looks like the well-known three-term error model in One-Port calibrations, [3]. This will provide us with some degree of familiarity when dealing with the following expressions. The method described here assumes a calibrated Plane 1 to which the non-insertable DUT is attached, and it is based on a so-called *manual* calibration performed in Plane 2.

Calibration in Plane 1 can be performed with the aid of the firmware on either of the VNA Ports. It can be also verified against known standards, just like any One-Port calibration. However, the calibration in Plane 2 has to be performed manually, for example using a spreadsheet or whichever programming language we are familiar with.

3.- MANUAL CALIBRATION

Any device of known reflection coefficient Γ, when connected to Plane 2 is measured by the VNA in Plane 1 as:

$$\Gamma_m = \frac{b_1}{a_1} = S_{11} + \frac{\Gamma \cdot S_{21}^{\ 2}}{1 - S_{22} \cdot \Gamma}$$

Where the assumption of considering $S_{21}=S_{12}$ reduces the number of unknowns to three. The so-called *manual* calibration consists in putting three known standards in place of Γ and extracting the scattering parameters of the device under test from the resulting three-equation system.

In an OSL calibration the three standards chosen are an Open circuit, a Short circuit and a Load. Let us represent their reflection coefficients as Γ_L (the Load), Γ_{OC} (the Open circuit) and Γ_{SC} (the Short circuit). Measurement of the three terminations provide us with the following set of equations:

$$\Gamma_{m\,(Load)} = S_{11} + \frac{\Gamma_L \cdot S_{21}^{\ 2}}{1 - S_{22} \cdot \Gamma_L} \equiv A \qquad \text{Eqn. (1.1)}$$

$$\Gamma_{m\,(Open)} = S_{11} + \frac{\Gamma_{OC} \cdot S_{21}^{\ 2}}{1 - S_{22} \cdot \Gamma_{OC}} \equiv B \qquad \text{Eqn. (1.2)}$$

$$\Gamma_{m\,(Short)} = S_{11} + \frac{\Gamma_{SC} \cdot S_{21}^{\ 2}}{1 - S_{22} \cdot \Gamma_{SC}} \equiv C \qquad \text{Eqn. (1.3)}$$

<u>Broadband Load</u>: The assumption is usually made to consider the Broadband Load in the Calibration Kit as an ideal termination within a given frequency range. With $\Gamma_L = 0$, eqn. (1.1) provides us with the first scattering parameter:

$$\Gamma_{m\,(Broadband\,Load)} \equiv A = S_{11}$$

<u>Sliding Load</u>: From a given frequency point upwards, use is made of a Sliding Load for determination of S_{11}. In this case, three - or more - measurements are made for different positions of the Load, eqn. (1.1) thus becoming:

$$\Gamma_{m\,(Sliding\,Load)} = S_{11} + \frac{\Gamma_{SL,i} \cdot S_{21}^{\ 2}}{1 - S_{22} \cdot \Gamma_{SL,i}} \equiv A_i$$

Where $\Gamma_{SL,i}$ is the reflection coefficient of the Sliding Load. From the determination of the centre of any three measurements, we finally obtain S_{11} as:

$$Centre\,[A_i]\big|_{i=1,2,3} \equiv A = S_{11}$$

<u>Open and Short circuits</u>: Once S_{11} is obtained, eqns. (1.2) and (1.3) can be reduced to the following linear system:

$$\begin{bmatrix} \Gamma_{OC} & \Gamma_{OC} \cdot (B - A) \\ \Gamma_{SC} & \Gamma_{SC} \cdot (C - A) \end{bmatrix} \cdot \begin{bmatrix} S_{21}^{\ 2} \\ S_{22} \end{bmatrix} = \begin{bmatrix} B - A \\ C - A \end{bmatrix}$$

Which can finally be solved for S_{21} and S_{22}:

$$S_{21} = \sqrt{\frac{(A-B)\cdot(A-C)}{(C-B)}\cdot\left(\frac{1}{\Gamma_{OC}}-\frac{1}{\Gamma_{SC}}\right)} \qquad \text{Eqns. (2)}$$

$$S_{22} = \frac{1}{(C-B)}\cdot\left(\frac{(A-B)}{\Gamma_{OC}}-\frac{(A-C)}{\Gamma_{SC}}\right)$$

4.- VERIFICATION

Having obtained the three scattering parameters, it is possible to determine the value of Γ, attached to Plane 2, from the measured reflection coefficient in Plane 1. In other words, the method allows us to "remove" the effect of the non-insertable device and to measure any reflection coefficient at Plane 2. We can make use of this feature for the purposes of verifying our *manual* calibration. Connecting a known verification item to Plane 2, we can extract the corrected value of Γ as:

$$\Gamma_{corr} = \frac{\Gamma_m - S_{11}}{S_{21}^{\ 2} + S_{22}\cdot(\Gamma_m - S_{11})}$$

And compare it against the reference values, with a previously defined criteria to accept or to reject the calibration.

5.- UNCERTAINTY

In order to deterministically show the effect of the calibration items in Plane 2 on the measurement of the scattering matrix [S], we shall examine the dependence of S_{11}, S_{21} and S_{22} on the individual measurements which we have called A, B and C, tracing it back to Γ_L, Γ_{OC} and Γ_{SC} (see Appendix for details). In terms of uncertainty, and assuming rss combination:

$$u_{11} = |S_{21}|^2 \cdot u_L$$

$$u_{21} = \left|\frac{S_{21}}{2}\right|\cdot\sqrt{\left|\frac{1-S_{22}\cdot\Gamma_{OC}}{\Gamma_{OC}}+\frac{1-S_{22}\cdot\Gamma_{SC}}{\Gamma_{SC}}\right|^2\cdot u_L^{\ 2}+\left|\frac{\Gamma_{SC}/\Gamma_{OC}}{\Gamma_{SC}-\Gamma_{OC}}\right|^2\cdot u_{OC}^{\ 2}+\left|\frac{\Gamma_{OC}/\Gamma_{SC}}{\Gamma_{SC}-\Gamma_{OC}}\right|^2\cdot u_{SC}^{\ 2}}$$

$$u_{22} = \sqrt{\left|\frac{(1-S_{22}\cdot\Gamma_{OC})\cdot(1-S_{22}\cdot\Gamma_{SC})}{\Gamma_{OC}\cdot\Gamma_{SC}}\right|^2\cdot u_L^{\ 2}+\left|\frac{1-S_{22}\cdot\Gamma_{SC}}{\Gamma_{OC}\cdot(\Gamma_{OC}-\Gamma_{SC})}\right|^2\cdot u_{OC}^{\ 2}+\left|\frac{1-S_{22}\cdot\Gamma_{OC}}{\Gamma_{SC}\cdot(\Gamma_{SC}-\Gamma_{OC})}\right|^2\cdot u_{SC}^{\ 2}}$$

The values of u_L, u_{OC} and u_{SC} are obtained from our knowledge of the calibration items. If the Load, the Open and the Short circuit are part of a Calibration Kit, the uncertainty could be directly obtained from the manufacturer specifications.

As for the Sliding Load, it must be said that its effect on the measurement uncertainty tends to be neglected. The reason is that, by definition of its use, the centre of a series of positions is taken as the equivalent Directivity E_{DF}, regardless of the reflection coefficient presented by the Load. This is only true as long as the reflection coefficient of the input connector is small enough. Otherwise, it should be included into u_L.

Regarding the calibration in Plane 1, we can make use of the common procedure for assessment of VNAs, [4]. Alternatively, one could follow a similar approach to that described here for the calibration in Plane 2.

6.- MEASUREMENT RESULTS

We include here the results obtained for an adapter 3.5mm(f) / N-Type(m), internally labelled as model 35N1. The 3.5 mm(f) end is taken as Port 1. In Table 1 and Figure 1 the verification against a calibrated 50 dB attenuator is shown. Its reflection coefficient as measured by NIST is taken as the reference value.

For reason of simplicity, only the effect of the *manual* calibration in Plane 2 has been included, so the verification, as regards the uncertainty, is not complete. The uncertainty terms considered for the calibration items are typical values: u_L=0.006, u_{OC}=u_{SC}=0.01. Nevertheless, the agreement between the values measured by INTA and by NIST is fairly good.

Freq (GHz)	Γ_{INTA}	u Γ_{INTA}	Γ_{NIST}	u Γ_{NIST}
.05	0.012	0.006	0.0121	0.0040
.1	0.012	0.006	0.0120	0.0037
.5	0.012	0.006	0.0130	0.0035
1	0.014	0.006	0.0139	0.0035
2	0.018	0.006	0.0194	0.0036
3	0.024	0.006	0.0234	0.0037
4	0.026	0.006	0.0233	0.0038
5	0.025	0.006	0.0230	0.0039
6	0.022	0.006	0.0222	0.0040
7	0.016	0.006	0.0180	0.0042
8	0.011	0.006	0.0125	0.0044
9	0.011	0.006	0.0106	0.0044
10	0.025	0.006	0.0203	0.0046
11	0.041	0.006	0.0385	0.0048
12	0.055	0.006	0.0563	0.0050
13	0.065	0.006	0.0659	0.0053
14	0.067	0.006	0.0716	0.0057
15	0.060	0.006	0.0646	0.0060
16	0.046	0.006	0.0500	0.0064
17	0.036	0.006	0.0419	0.0069
18	0.048	0.006	0.0516	0.0075

Table 1.- Verification against a known 50 dB attenuator, female connector

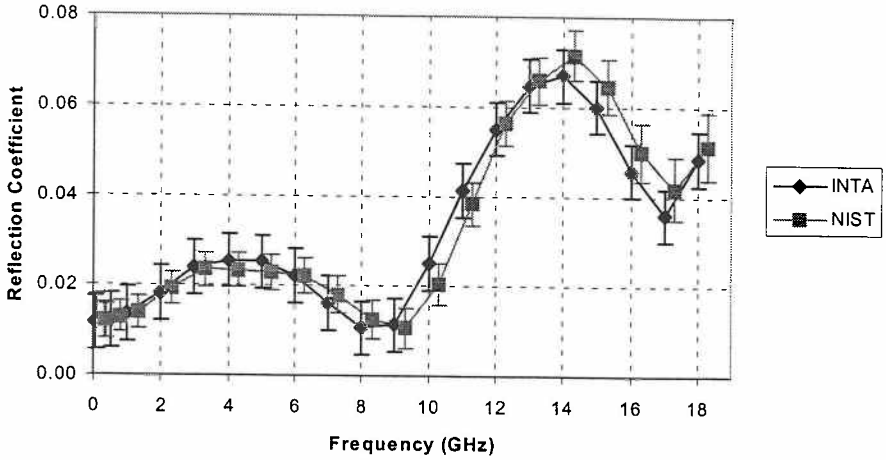


Figure 1.- Verification results. A frequency shift of 0.3 GHz has been applied to the NIST values.

In Table 2, the measured [S] parameters of the adapter are shown, together with the associated uncertainty. Again, it does not include the effect of calibration in Plane 1, nor Type A contributions (repeatability).

Freq (GHz)	S_{11}	u S_{11}	S_{21} (dB)	U S_{21} (dB)	S_{21} (deg)	S_{22}	u S_{22}
.05	0.001	0.006	0.006	0.031	-2.54	0.002	0.009
.1	0.002	0.006	0.009	0.031	-5.04	0.004	0.009
.5	0.005	0.006	0.020	0.031	-25.10	0.007	0.009
1	0.008	0.006	0.032	0.031	-50.09	0.010	0.009
2	0.011	0.006	0.047	0.031	79.94	0.008	0.009
3	0.013	0.006	0.061	0.031	30.00	0.015	0.009
4	0.018	0.006	0.073	0.031	-19.93	0.025	0.009
5	0.027	0.006	0.084	0.031	-69.86	0.026	0.009
6	0.033	0.006	0.096	0.031	60.22	0.023	0.009
7	0.036	0.006	0.104	0.031	10.31	0.033	0.009
8	0.033	0.006	0.112	0.031	-39.61	0.038	0.009
9	0.026	0.006	0.117	0.031	-89.57	0.032	0.009
10	0.018	0.006	0.123	0.031	40.46	0.010	0.009
11	0.014	0.006	0.133	0.031	-9.54	0.015	0.009
12	0.020	0.006	0.141	0.031	-59.68	0.028	0.009
13	0.023	0.006	0.146	0.031	70.14	0.020	0.009
14	0.022	0.006	0.152	0.031	19.95	0.019	0.009
15	0.025	0.006	0.160	0.031	-30.29	0.029	0.009
16	0.031	0.006	0.166	0.031	-80.65	0.044	0.009
17	0.037	0.006	0.169	0.031	48.80	0.032	0.009
18	0.047	0.006	0.179	0.031	-1.73	0.026	0.009

Table 2.- Measured [S] parameters of a 3.5mm(f) / N-Type(m) adapter

In the measurement of S_{21}, an ambiguity of ±180° exists, due to the fact that it is the product $S_{21} \cdot S_{12}$ which is measured. In the above table, the result of extracting the square root is represented, for which the phase values are distributed between -180° and +180°.

7.- CONCLUSIONS

1) A simple, easy-to-implement method for characterisation of non-insertable devices has been presented. It is based on two One-Port calibrations, one of which has to be performed *manually*. The popular OSL calibration, including a Sliding Load for frequencies above 3 GHz, can be used.

2) The [S] parameters of the device under test are obtained from the individual measurements of the calibration items. The formulas that relate them are straight-forward. Since complex quantities are involved, the method allows us to obtain the parameters in magnitude and phase.

3) It is possible to verify the *manual* calibration with the aid of any set of known standards. Some criteria can be defined in order to accept or to reject the calibration, which provides us with some additional confidence in our measurements.

4) The whole procedure can be implemented on a spreadsheet or with the aid of any programming language we are familiar with.

5) The measurement uncertainty can also be obtained from the equations that relate the measured [S] parameters, the individual measurements and the calibration items. Although it has not been included in this report, the method can be further improved with the inclusion of uncertainty in Plane 1, as well as repeatability effects.

6) A scalar simplification of the method can be used, only for S_{21} measurement of low-loss devices. In this case, a simple Open / Short calibration in Plane 2 is needed. The insertion loss - in dB - of the device under test would be obtained as half the semi-sum of the return losses measured with the Open and the Short terminations attached.

8.- REFERENCES

[1] J C Medley. "Towards traceability for Adaptor Measurement". 25th Automated RF and Microwave Measurement Society. Bath, UK, November 1996.

[2] I Instone. "Calibrating an ANA – Techniques where Calibration Kits are not available". ANAMET Report 016, March 1998.

[3] M Rodríguez. "Three-term Error Correction in One-Port Calibrations". ANAMET Report 020, November 1998.

[4] J P M de Vreede. "Draft Procedure for the Assessment of Vector Network Analysers (VNA)". ANAMET Report 019, October 1998.

APPENDIX - PARTIAL DERIVATIVES

Partial derivative of A (equation 1.1) with respect to Γ_L:

$$\frac{\partial A}{\partial \Gamma_L}\bigg|_{\Gamma_L=0} = \frac{S_{21}^{\;2}}{(1 - S_{22} \cdot \Gamma_L)^2}\bigg|_{\Gamma_L=0} = S_{21}^{\;2}$$

Partial derivatives of B (equation 1.2) with respect to Γ_{OC}:

$$\frac{\partial B}{\partial \Gamma_{OC}} = \frac{S_{21}^{\;2}}{(1 - S_{22} \cdot \Gamma_{OC})^2}$$

Partial derivatives of C (equation 1.3) with respect to Γ_{SC}:

$$\frac{\partial C}{\partial \Gamma_{SC}} = \frac{S_{21}^{\;2}}{(1 - S_{22} \cdot \Gamma_{SC})^2}$$

<u>Measurement of S_{11}</u>

Partial derivatives of S_{11} with respect to A, B and C:

$$\frac{\partial S_{11}}{\partial A} = 1$$

$$\frac{\partial S_{11}}{\partial B} = \frac{\partial S_{11}}{\partial C} = 0$$

The measurement error is given by the law of propagation of uncertainties:

$$\varepsilon_{11} = \frac{\partial S_{11}}{\partial A} \cdot \frac{\partial A}{\partial \Gamma_L} \cdot \varepsilon_L = S_{21}^{\;2} \cdot \varepsilon_L$$

Where ε_L is the complex error with which the reflection coefficient Γ_L of the termination is known.

<u>Measurement of S_{21}</u>

Partial derivatives of S_{21} with respect to A, B and C:

$$\frac{\partial S_{21}}{\partial A} = \frac{-1}{2 \cdot S_{21}} \cdot \left(\frac{1 - S_{22} \cdot \Gamma_{OC}}{\Gamma_{OC}} + \frac{1 - S_{22} \cdot \Gamma_{SC}}{\Gamma_{SC}} \right)$$

$$\frac{\partial S_{21}}{\partial B} = \frac{1}{2 \cdot S_{21}} \cdot \frac{\Gamma_{OC} \cdot \Gamma_{SC}}{\Gamma_{SC} - \Gamma_{OC}} \cdot \left(\frac{1 - S_{22} \cdot \Gamma_{OC}}{\Gamma_{OC}} \right)^2$$

$$\frac{\partial S_{21}}{\partial C} = \frac{-1}{2 \cdot S_{21}} \cdot \frac{\Gamma_{OC} \cdot \Gamma_{SC}}{\Gamma_{SC} - \Gamma_{OC}} \cdot \left(\frac{1 - S_{22} \cdot \Gamma_{SC}}{\Gamma_{SC}} \right)^2$$

The measurement error is given by the law of propagation of uncertainties:

$$\varepsilon_{21} = \frac{\partial S_{21}}{\partial A}\cdot\frac{\partial A}{\partial \Gamma_L}\cdot\varepsilon_L + \frac{\partial S_{21}}{\partial B}\cdot\frac{\partial B}{\partial \Gamma_{OC}}\cdot\varepsilon_{OC} + \frac{\partial S_{21}}{\partial C}\cdot\frac{\partial C}{\partial \Gamma_{SC}}\cdot\varepsilon_{SC} = \ldots$$

$$\ldots = \frac{-S_{21}}{2}\cdot\left[\left(\frac{1-S_{22}\cdot\Gamma_{OC}}{\Gamma_{OC}} + \frac{1-S_{22}\cdot\Gamma_{SC}}{\Gamma_{SC}}\right)\cdot\varepsilon_L - \frac{\Gamma_{SC}/\Gamma_{OC}}{\Gamma_{SC}-\Gamma_{OC}}\cdot\varepsilon_{OC} + \frac{\Gamma_{OC}/\Gamma_{SC}}{\Gamma_{SC}-\Gamma_{OC}}\cdot\varepsilon_{SC}\right]$$

Where ε_L, ε_{OC} and ε_{SC} are the errors with which the reflection coefficients of the terminations are known.

Measurement of S_{22}

Partial derivatives of S_{22} with respect to A, B and C:

$$\frac{\partial S_{22}}{\partial A} = \frac{(1-S_{22}\cdot\Gamma_{OC})\cdot(1-S_{22}\cdot\Gamma_{SC})}{\Gamma_{OC}\cdot\Gamma_{SC}\cdot S_{21}^{2}}$$

$$\frac{\partial S_{22}}{\partial B} = \frac{(1-S_{22}\cdot\Gamma_{OC})^2\cdot(1-S_{22}\cdot\Gamma_{SC})}{\Gamma_{OC}\cdot S_{21}^{2}\cdot(\Gamma_{OC}-\Gamma_{SC})}$$

$$\frac{\partial S_{22}}{\partial C} = \frac{(1-S_{22}\cdot\Gamma_{SC})^2\cdot(1-S_{22}\cdot\Gamma_{OC})}{\Gamma_{SC}\cdot S_{21}^{2}\cdot(\Gamma_{SC}-\Gamma_{OC})}$$

The measurement error is given by the law of propagation of uncertainties:

$$\varepsilon_{22} = \frac{\partial S_{22}}{\partial A}\cdot\frac{\partial A}{\partial \Gamma_L}\cdot\varepsilon_L + \frac{\partial S_{22}}{\partial B}\cdot\frac{\partial B}{\partial \Gamma_{OC}}\cdot\varepsilon_{OC} + \frac{\partial S_{22}}{\partial C}\cdot\frac{\partial C}{\partial \Gamma_{SC}}\cdot\varepsilon_{SC} = \ldots$$

$$\ldots = \frac{(1-S_{22}\cdot\Gamma_{OC})\cdot(1-S_{22}\cdot\Gamma_{SC})}{\Gamma_{OC}\cdot\Gamma_{SC}}\cdot\varepsilon_L + \frac{(1-S_{22}\cdot\Gamma_{SC})}{\Gamma_{OC}\cdot(\Gamma_{OC}-\Gamma_{SC})}\cdot\varepsilon_{OC} + \frac{(1-S_{22}\cdot\Gamma_{OC})}{\Gamma_{SC}\cdot(\Gamma_{SC}-\Gamma_{OC})}\cdot\varepsilon_{SC}$$

ε_L, ε_{OC} and ε_{SC} are, in general, complex quantities. The differential approach followed here gives good results as long as the deviations from the ideal values of the calibration kit are small enough.