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Averaging repeat measurements of low magnitude *S*-parameters

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Averaging repeat measurements of low magnitude S-parameters

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Abstract

This report presents two methods of statistically analysing repeat measurements of complex S-parameters. Both methods summarise the data by computing the arithmetic mean and standard deviation for the repeat measurements. However, the two methods produce different summary values when the magnitude of the complex S-parameter is close to zero (i.e. close to the origin in the complex plane). This effect is verified using both experimental and simulated data. Recommendations are given on the more suitable method for analysing complex S-parameters of any given value.

1. Introduction

When analysing and interpreting results obtained from a series of repeated ANA complex S-parameter measurements, it is usually more helpful to present the data in terms of its magnitude and phase rather than the real and imaginary components. Magnitude and phase are physical concepts and often relate to the physical processes occurring within the ANA and/or device under test. For example, phase can be related to electrical path length, and magnitude to signal loss (or signal absorbed). However, the real and imaginary components of the S-parameters cannot be easily related to such physical mechanisms in such a way. This makes the real and imaginary components more difficult to interpret, in a physical sense.

This report compares two methods of analysing repeat complex S-parameter measurements. For simplicity, only the magnitude of the complex voltage reflection coefficient, Γ, is considered here but both methods (and recommendations concerning their application) are equally applicable to all forms of S-parameter for both reflection and transmission data.

The two methods are based on calculating an arithmetic mean for the measurement data and a measure of dispersion, the standard deviation. The first method (labelled 'Method 1') involves performing the calculations on the recorded magnitude values, whereas the second method ('Method 2') performs calculations on the real and imaginary components of Γ which are then converted to their equivalent mean magnitude value.

Having presented and compared the equations used in both methods, this report describes an experimental investigation undertaken to compare the results produced using the two methods. A data simulator is then used to extend the experimental observations by evaluating the large sample behaviour for such data.

2. Analysing data using 'Method 1'

Method 1 involves calculating the arithmetic mean of the magnitudes of n repeat measurements, as shown in equation 1:

$$\overline{|\Gamma|} = \frac{1}{n}\sum_{i=1}^{n} |\Gamma|_i \,, \tag{1}$$

where $\overline{|\Gamma|}$ is the mean magnitude and $|\Gamma|_i$ represents the magnitude of the ith measurement of Γ.

The associated standard deviation summarising the dispersion in the data about the mean calculated using equation (1) is given by

$$s\left(\overline{|\Gamma|}\right) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(|\Gamma|_i - \overline{|\Gamma|}\right)^2} \,. \tag{2}$$

3. Analysing data using 'Method 2'

Method 2 determines the arithmetic mean magnitude by calculating the individual means of the real and imaginary components. These are then combined to give the mean magnitude, $\left|\overline{\Gamma^*}\right|$, as follows

$$\left|\overline{\Gamma^*}\right| = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\mathrm{Re}(\Gamma_i)\right)^2 + \left(\frac{1}{n}\sum_{i=1}^{n}\mathrm{Im}(\Gamma_i)\right)^2} \,, \tag{3}$$

where $\mathrm{Re}(\Gamma_i)$ and $\mathrm{Im}(\Gamma_i)$ are the real and imaginary components of Γ_i, respectively. To measure the dispersion in the data about this mean value, two standard deviations are used. These represent dispersion in the real and imaginary components separately and are given by

$$\begin{aligned} s\left(\left|\overline{\Gamma^*}\right|:\mathrm{Re}\right) &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\mathrm{Re}(\Gamma_i) - \overline{\mathrm{Re}(\Gamma)}\right)^2} \\ s\left(\left|\overline{\Gamma^*}\right|:\mathrm{Im}\right) &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\mathrm{Im}(\Gamma_i) - \overline{\mathrm{Im}(\Gamma)}\right)^2} \end{aligned} \,, \tag{4}$$

where $\overline{\mathrm{Re}(\Gamma)}$ and $\overline{\mathrm{Im}(\Gamma)}$ are the mean values of the real and imaginary components of Γ respectively.

4. Comparing the two methods

It can be seen that for more than one measurement, equation (1) will always yield a positive non-zero number (except for the unlikely case where the real and imaginary components of all n measurements are zero). It is possible, however, for equation (3) to equal zero. Let us take the hypothetical case where four measurements of Γ have been performed - these are shown in figure 1.

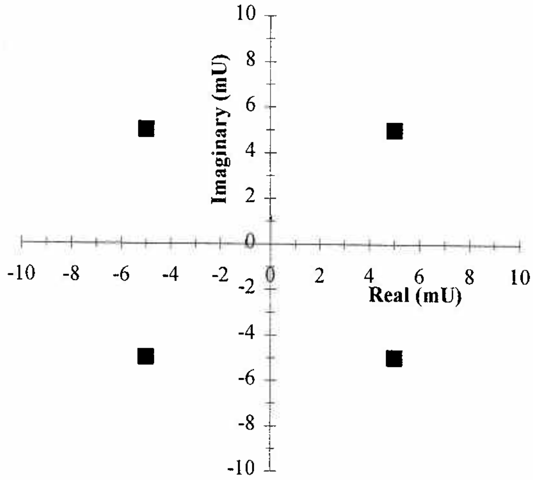


Figure 1 - Four hypothetical Γ measurements shown in the complex plane[†].

Due to the symmetrical scatter about the origin, the mean of the real and imaginary components (Method 2) is zero, hence resulting in a mean magnitude of zero. However, calculating the mean of the four measured magnitudes (Method 1), gives a result of approximately 7 mU (i.e. non-zero!). Intuitively, the result of zero magnitude seems more realistic for this example, which could represent four hypothetical measurements of a well-matched load with connector repeatability errors.

The dispersion in the data (as indicated by the calculated standard deviation) also differs using each method. For the example in figure 1, Method 1 yields a standard deviation of zero (since all four measurements have identical magnitude), whereas Method 2 produces a value of approximately 12 mU (i.e. greater than zero). A standard deviation of zero implies no dispersion in the data which clearly is not the case with this example.

5. Experimental investigation

An experimental investigation was undertaken to explore the differences that can arise when using the above two methods. Data with a relatively high degree of scatter close to the origin of the complex plane was generated using a precision well-matched load connected to a calibrated ANA via a BNC connection. A BNC connection was chosen because of its anticipated poor connection repeatability at RF. It was hoped that this would generate sufficient scatter in the measurements to illustrate the computational discrepancies found above.

[†] The author wishes to acknowledge his colleague, Dr. P. R. Young, NPL, for this example.

After measuring Γ at a range of frequencies from 300 kHz to 3 GHz , the well-matched load was disconnected and re-connected at the BNC measurement reference plane. This process was repeated to give six repeat measurements.

6. Experimental results

Table 1 shows the mean magnitudes and their associated standard deviations calculated from the six repeat measurements of Γ, using Method 1 and Method 2. The last column represents the ratio of Method 1 to Method 2, defined by

$$\Delta = \frac{\overline{|\Gamma|}}{\left|\overline{\Gamma^*}\right|}, \quad (5)$$

and is a measure of the discrepancy between the two mean calculations. This clearly shows that there is negligible difference ($\Delta = 1$) between the two methods at low frequencies, where the measurements are relatively unaffected by connector repeatability and hence exhibit a low degree of scatter. However, at higher frequencies, where scatter due to connector repeatability is more prominent, the discrepancy between the two methods becomes increasingly significant. This is shown in figure 2 which plots the calculated mean Γ using both methods, as a function of frequency. From the values given in Table 1 and plotted in Figure 2, it can be seen that Method 2 consistently yields lower values than Method 1 for the calculated mean when the scatter in the data is more pronounced. This agrees with the prediction made in section 4.

	Method 1		**Method 2**			
Frequency (MHz)	$\overline{\lvert\Gamma\rvert}$ Eqn. (1)	s Eqn. (2)	$\lvert\overline{\Gamma^*}\rvert$ Eqn. (3)	s(Re) Eqn. (4)	s(Im) Eqn. (4)	Δ Eqn. (5)
0.3	0.002052	0.000012	0.002052	0.000012	0.000005	1.00
1	0.002037	0.000012	0.002037	0.000012	0.000003	1.00
3	0.002055	0.000012	0.002055	0.000012	0.000005	1.00
10	0.002056	0.000010	0.002056	0.000010	0.000013	1.00
30	0.002144	0.000020	0.002143	0.000020	0.000049	1.00
100	0.002161	0.000087	0.002154	0.000075	0.000186	1.00
300	0.002580	0.000484	0.002567	0.000434	0.000354	1.00
1000	0.001830	0.000871	0.001445	0.000374	0.001460	1.27
1500	0.004066	0.000784	0.003484	0.002190	0.001044	1.17
2000	0.005429	0.002332	0.004636	0.001853	0.003403	1.17
2500	0.004404	0.004547	0.001002	0.005849	0.002921	4.40
3000	0.009535	0.008607	0.002462	0.005905	0.011876	3.87

Table 1 - Summary statistics for experimental data

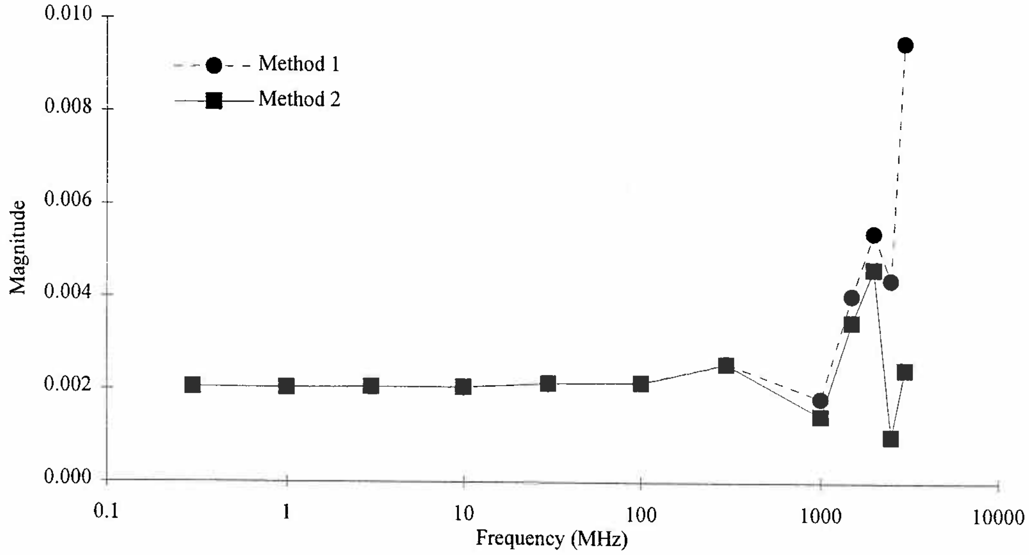


Figure 2 - Comparison of the methods of calculating mean magnitudes as a function of frequency.

7. Data simulation

Statistical estimates based on a sample size of only six, as used above, are likely to be unreliable compared with large sample estimates. To test more rigorously the behaviour observed in the preceding sections, a data simulator was used to generate 10 000 pseudo repeat measurements.

The simulator generates a random set of values for the real and imaginary components of Γ. These are sampled from a parent two-component Gaussian distribution which is defined by the user in terms of a mean and a standard deviation for the real and imaginary components. In practice, the sample drawn from the parent distribution will also be Gaussian in nature, but with a slightly different mean and standard deviation to the parent distribution. This is due to the finite sample size.

The simulator was used to generate simulated repeat measurement data under the following three conditions:

Simulation 1. A set of 10 000 repeat measurements for a perfectly matched load, i.e. $|\Gamma| = 0$, for each of a series of different values for the standard deviation, ranging from 0.00001 to 0.01 (identical in both real and imaginary parts). The increasing standard deviation for each set of 10 000 repeat measurements represents an increase in the measurement scatter due to worsening connector repeatability and/or electrical noise.

Simulation 2. The above series of simulations were repeated except using a hypothetical near-matched load, with $|\Gamma|$ set to 0.005, in place of the perfectly matched load used above.

Simulation 3. The series of simulations was again repeated except using a hypothetical high-reflect device, $|\Gamma|$ set to 0.95.

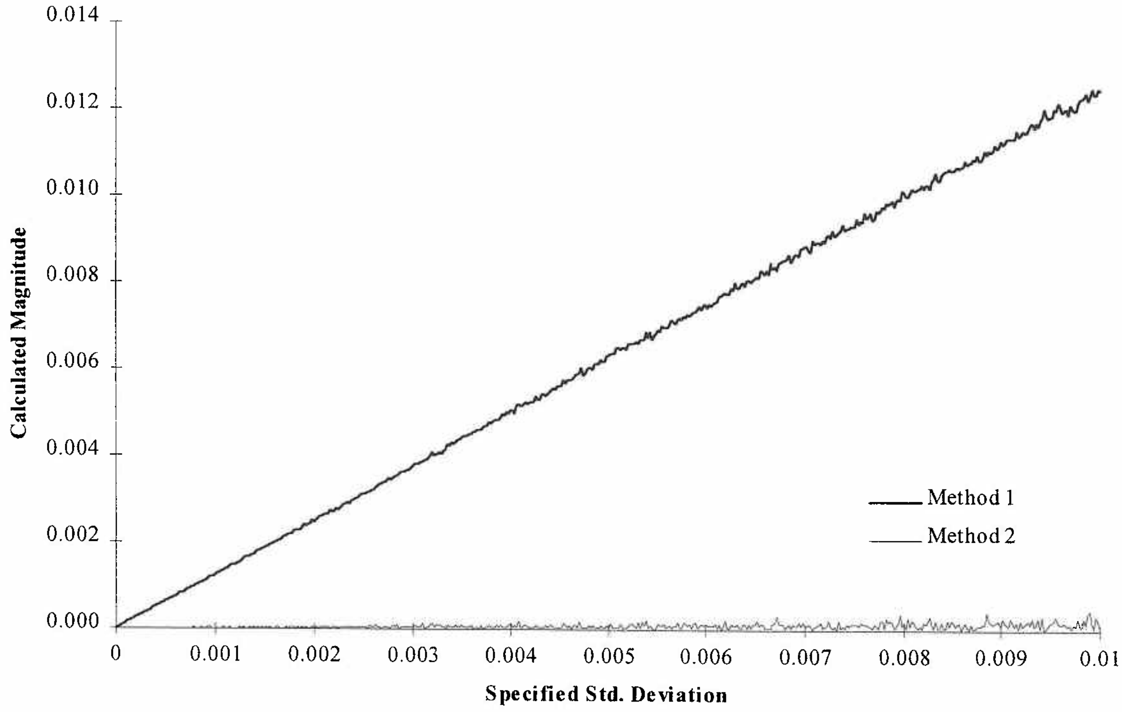
8. Simulation results

Simulation 1. Figure 3(a) shows the mean magnitudes, calculated using Method 1 and Method 2, as a function of specified standard deviation. It can be seen that Method 2 calculates a value for the mean of $|\Gamma|$ close to zero and remains independent of the size of the standard deviation. This value therefore shows good agreement with the value specified in the simulator, i.e. zero. However, Method 1 calculates values for the mean of $|\Gamma|$ which are *non-zero* and vary as a function of the standard deviation.

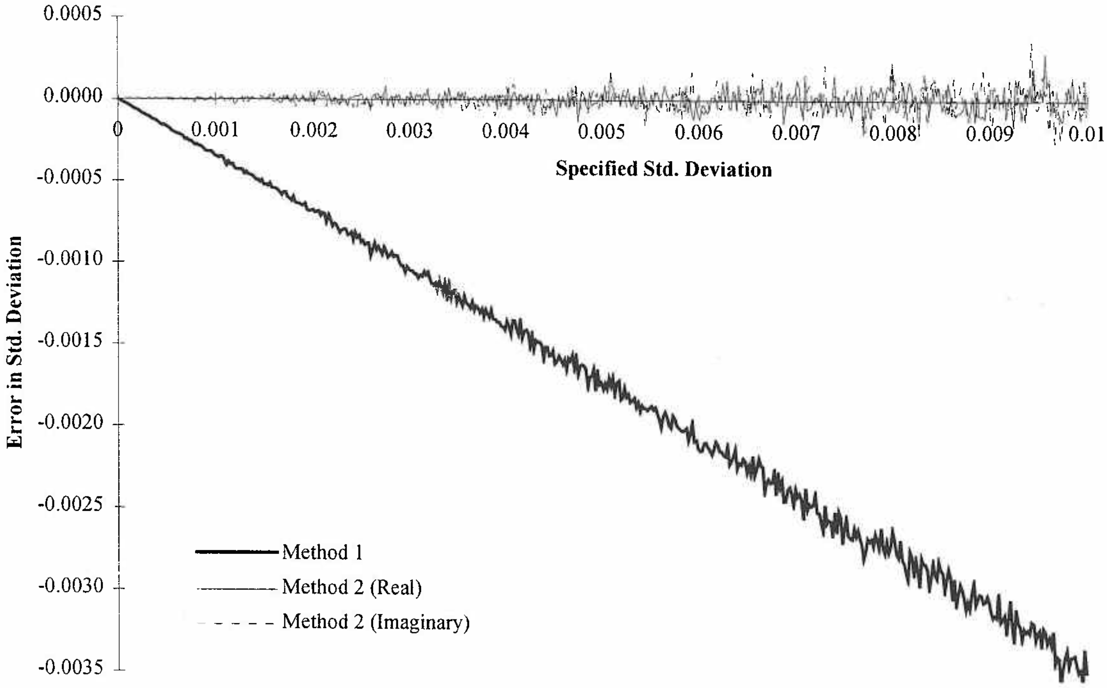
Figure 3(b) shows the difference (error) between the standard deviation calculated by the two methods under investigation and the standard deviation specified in the simulator, as a function of the specified standard deviation. Method 2 calculates values for the standard deviation (in both real and imaginary components) which are acceptably close to the value specified in the simulator. (The small fluctuations observed in the calculated standard deviations are due to the finite sample size.) Method 1 calculates values for the standard deviation which are systematically *smaller* than the value set in the simulator. This difference between the calculated standard deviation and that set in the simulator increases with standard deviation. Method 1 therefore under-estimates the actual degree of scatter in the measurement data.

Simulation 2. The results for the simulated near-matched load (with $|\Gamma|$ set to 0.005) are shown in figures 4(a) and 4(b). Similar trends to those observed for the perfectly matched load in Figure 3 are exhibited except in this case, the two methods of calculation show better agreement at low values of specified standard deviation.

Simulation 3. Moving further away from the origin of the complex plane, figures 5(a) and 5(b) show the effects of the two calculation methods on averaging simulated measurements of an item with $|\Gamma| = 0.95$. It can be seen that at this (relatively high) magnitude value, there is no observable difference between the two methods, both in terms of calculated mean and standard deviation.

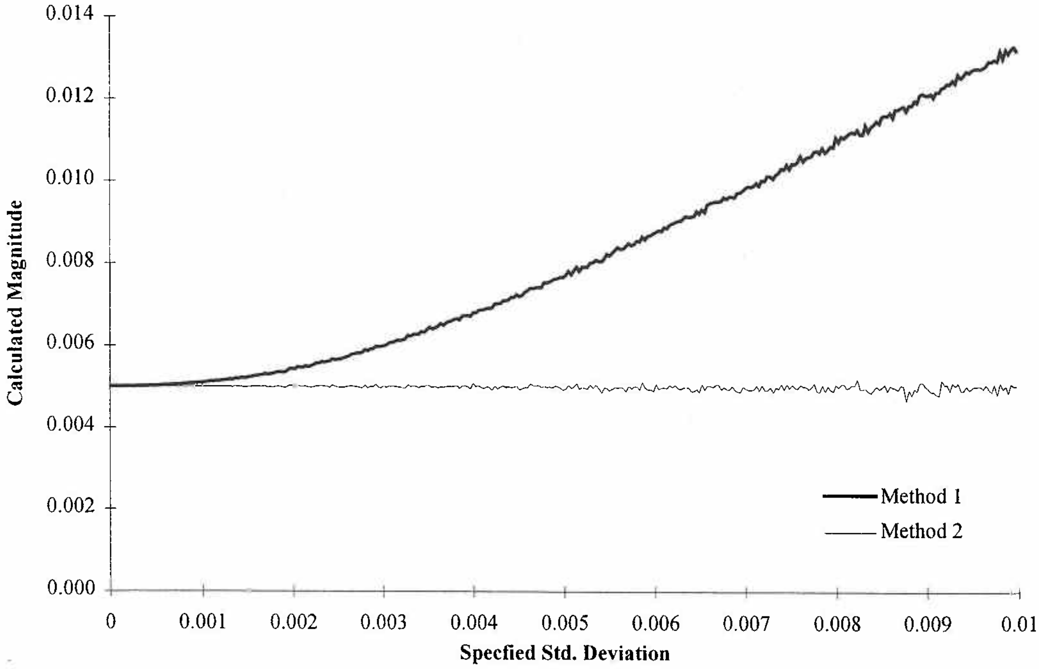


(a)

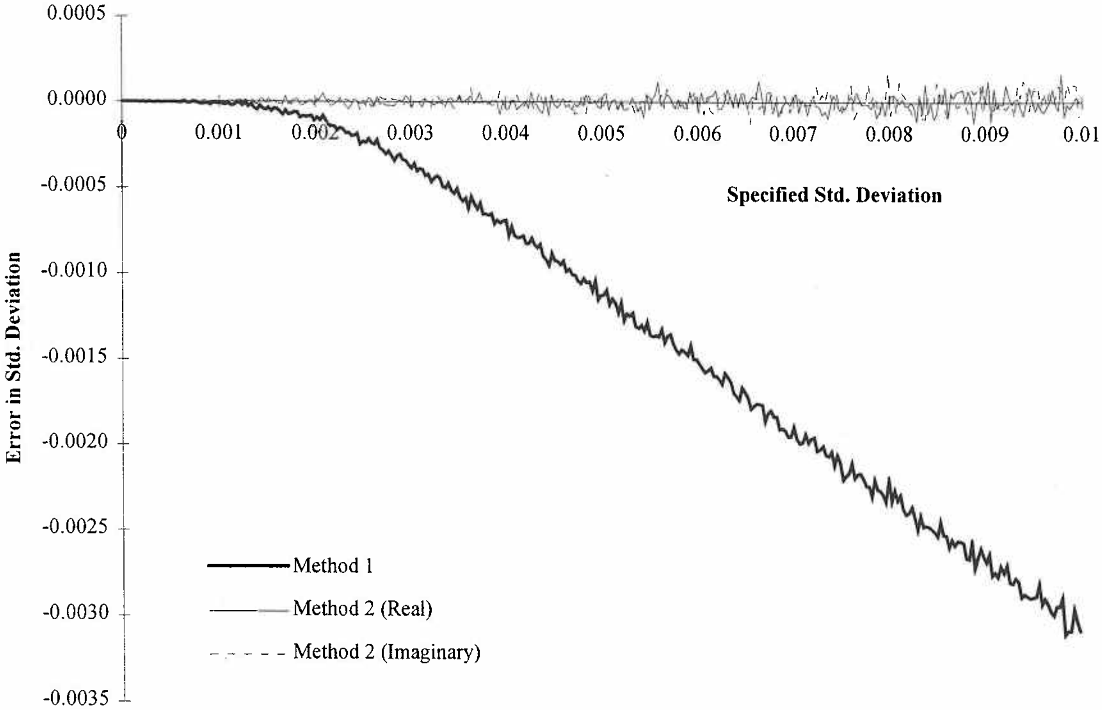


(b)

Figure 3 - Results for simulation of multiple connections of a perfectly matched load. (a) Magnitudes as calculated by Methods 1 and 2, (b) the difference between calculated and specified standard deviations.

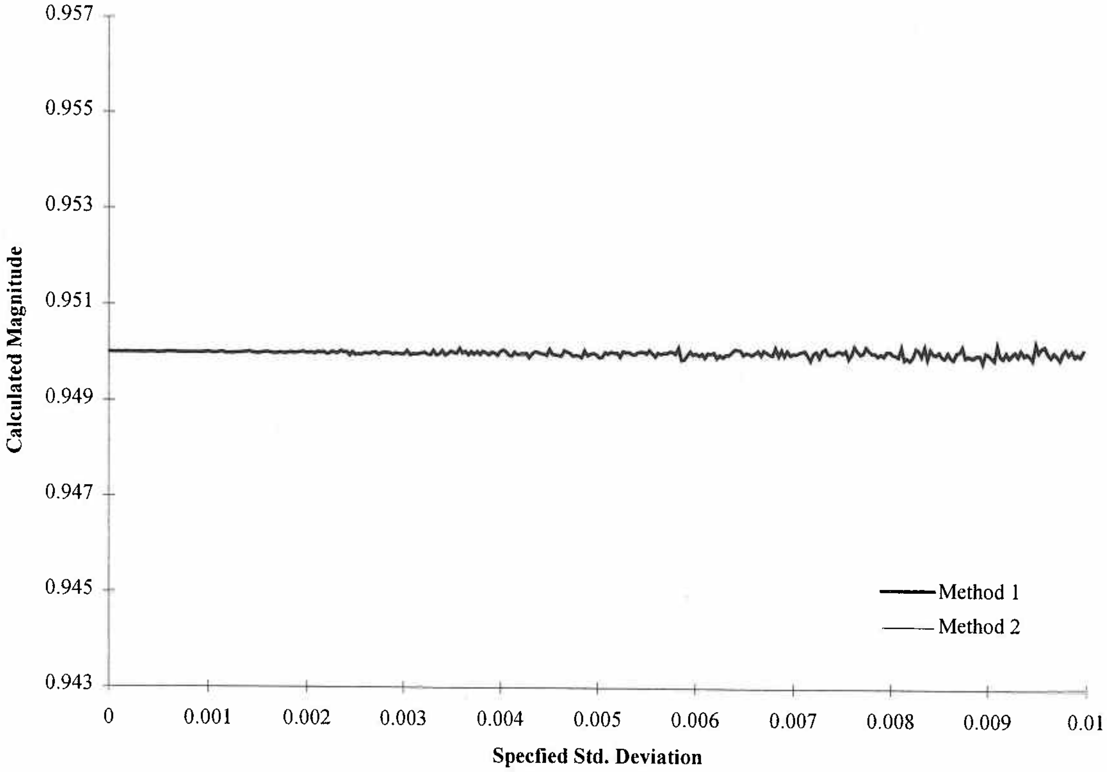


(a)

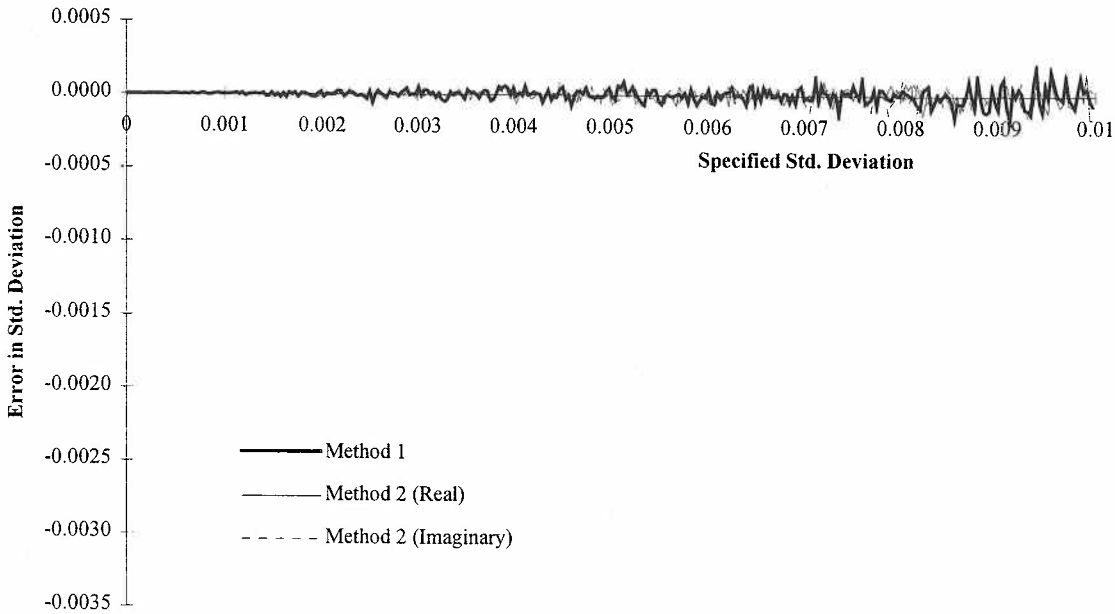


(b)

Figure 4 - Results for simulation of multiple connections of a load with $|\Gamma| = 0.005$. (a) Magnitudes as calculated by Methods 1 and 2, (b) the difference between calculated and specified standard deviations.



(a)



(b)

Figure 5 - Results for simulation of multiple connections of a load with $|\Gamma| = 0.95$.
(a) Magnitudes as calculated by Methods 1 and 2, (b) the difference between calculated and specified standard deviations.

9. Data distributions

The causes of the anomalies observed when using Method 1 can be investigated by examining the distribution of magnitudes about the calculated mean value. Figure 6 shows the distribution of magnitude data obtained from simulation 2 (specified $|\Gamma|$ = 0.005) and simulation 3 ($|\Gamma|$ set to 0.95). In both cases, the effect of a specified standard deviation of 0.01 was investigated.

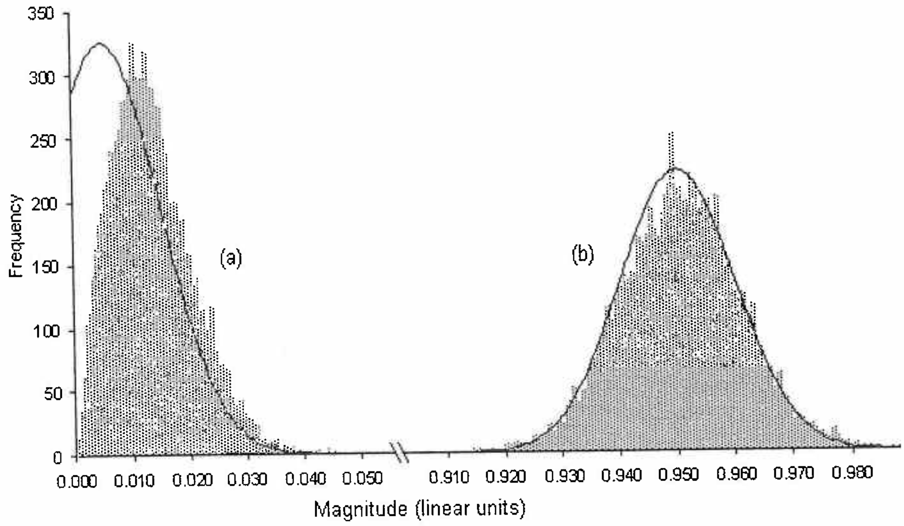


Figure 6 - Histogram illustrating the distribution of simulated magnitudes for a standard deviation of 0.01. Grey areas represent the sampled distribution and the solid curves show the expected Gaussian distribution about each specified mean. Curve (a): Specified mean $|\Gamma|$ = 0.005; Curve (b): Specified mean $|\Gamma|$ = 0.950.

The summary values as calculated using Method 1 are based on the distributions shown in figure 6. For the low magnitude case (simulation 2) where the standard deviation is comparable to the specified magnitude, there is a clear deviation from the expected Gaussian distribution (shown in Figure 6 by the black curve). Conversely, there is excellent agreement with the expected distribution when the specified mean $|\Gamma|$ is much greater than zero and subsequently larger than the standard deviation of 0.01.

This change of shape in the distribution at low magnitudes causes Method 1 to produce a mean value of $|\Gamma|$ which is *greater* than the 'true' value of $|\Gamma|$ (corresponding to the peak of curve (*a*) in Figure 6). This means that by statistically averaging a series of repeat measurements and producing an average using Method 1, has actually *induced a systematic error* into the estimate of $|\Gamma|$ (i.e. the average value). Similarly, the change of shape causes Method 1 to calculate a standard deviation which is *less than* the 'true' standard deviation. This leads to uncertainty intervals based on the standard deviation calculated using Method 1 to be narrower than is appropriate and are therefore *over-optimistic.*

10. Summary and recommendations

A comparison has been made between two different methods (labelled here as 'Method 1' and 'Method 2') of statistically analysing repeat measurements of $|\Gamma|$, to obtain summary values for a 'best estimate' and a 'measure of dispersion' for the data. The comparison included an experimental investigation and the use of a data simulator. The experimental data consisted of six repeat measurements of Γ and indicated a likely discrepancy between the two methods. The data simulator was then used to extend the experimental investigations by simulating large numbers of repeat measurements.

The simulated data confirmed the observations from the experimental investigation, where Method 2 yielded summary values for $|\Gamma|$ which were close to the expected values. These summary values were also independent of the nominal value of $|\Gamma|$ and the dispersion in the $|\Gamma|$ data. Method 1 induces a systematic error in both the summary values. In the case of the mean, it becomes artificially higher than expected with the discrepancy being a function of the scatter in the measurements. For the standard deviation, it becomes artificially lower than expected and is also a function of the scatter in the measurements.

By comparing the results obtained for a matched and near-matched load, it was observed that moving the specified Γ away from the origin causes the two methods to agree at low standard deviations. It is only when the standard deviation becomes comparable to the specified mean magnitude that discrepancies begin to occur. Only in cases where the scatter of data is negligible or the expected value of $|\Gamma|$ is away from the origin in the complex plane will Method 1 produce results that are free from these systematic effects. Method 2, however, invariably gives meaningful results independent of the measurement conditions.

From the findings presented in this report it is recommended that, to be sure of meaningful summary values under *any* measurement conditions, Method 2 be used wherever possible. For convenience however, Method 1 can be used in circumstances where it is confidently known that the measurement scatter is negligible with respect to the magnitude of the *S*-parameter being measured. This is likely to be the case for highly reflective or low loss devices.

11. Acknowledgements

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