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One-port Calibration: Nonideal Standards and Residual Error Terms

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Abstract

In this article general relations between actual and assumed parameters of three standards and residual error terms are presented. Then, results for the special case OSL are discussed.

INTRODUCTION

In recent time, several ANAMET publications have been concerned in the problem of the influence of imperfect standards to the measurement errors in one-port calibration. Since residual error terms are commonly used to uncertainty calculation, the aim of this article is to show how the standard imperfections influence the residual error terms.

BRIEF DESCRIPTION OF ONE-PORT CALIBRATION

Systematic errors in one port VNA measurement can be represented by the error model which can be seen as the virtual two-port. This two-port is commonly considered to be linear. Hence, it can be fully described by three complex constants D,T,M at each frequency point. To determine these three constants a measurement of three arbitrary (not identical) perfectly known reflection standards is sufficient.

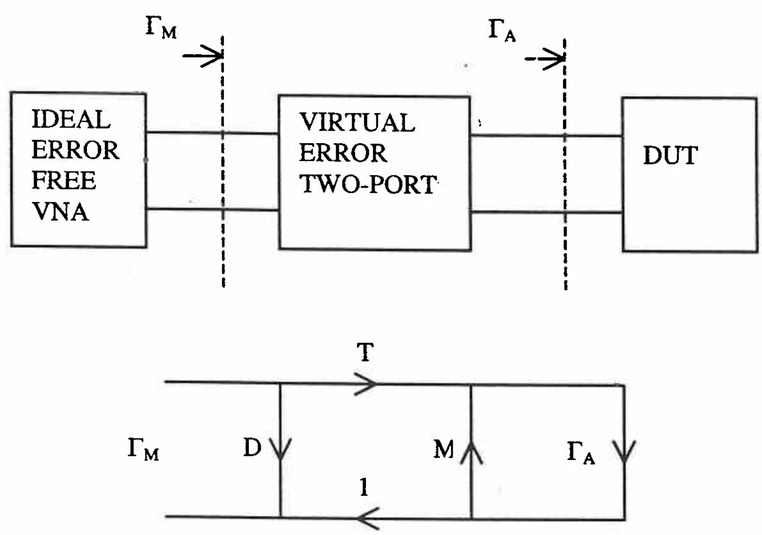

Fig.1 One-port measurement and its signal flowgraph representation

During the one port calibration procedure three known standards are sequentially measured and values D,T,M are calculated solving set of equations (1) for the three standards

$$\Gamma_{Mi} = D + \frac{T\Gamma_{Ai}}{1 - M\Gamma_{Ai}} \tag{1}$$

where

Γ_{Mi} ... measured reflection coefficient of the i-th standard
Γ_{Ai} ... actual reflection coefficient of the i-th standard
D ... error term Directivity

T ... error term Tracking
M ... error term Test Port Match.

The inverse transformation (2) is then used to calculate actual values Γ_A of measured reflection coefficient.

$$\Gamma_A = \frac{\Gamma_M - D}{T + M(\Gamma_M - D)} \tag{2}$$

ONE-PORT CALIBRATION WITH IMPERFECT STANDARDS

Let's consider the situation when the parameters of standards are not perfectly known. The virtual error two-port from fig.1 can be substituted by two cascaded virtual two-ports as can be seen in fig.2.

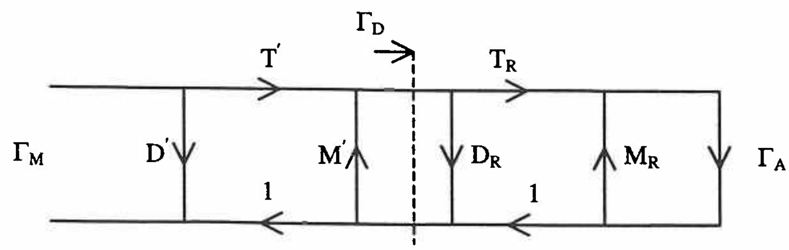

Fig.2 Signal flowgraph representation of the measurement with imperfect standards

During the calibration procedure values of error terms T', D', M' are calculated by VNA. Thus, the in cal kit definition stored values Γ_{Di} are transformed to the measured data Γ_{Mi} instead of the actual values Γ_{Ai}. We can say, the VNA is "mystified". Hence, the measured data are corrected improperly. The second two-port from fig.2 is to be of interest now. It transforms the actual data of standards to their in cal kit definition stored values. Obviously, s parameters of this two-port are identical with the residual error terms. Their values can be obtained solving set of three equations (3) for three standards

$$\Gamma_{Di} = D_R + \frac{T_R \Gamma_{Ai}}{1 - M_R \Gamma_{Ai}} \tag{3}$$

where

Γ_{Di} ... theoretical, in cal kit definition stored data of the i-th standard
Γ_{Ai} ... actual reflection coefficient of the i-th standard
D_R, M_R, T_R ... residual error terms of the VNA after three error term calibration.

After certain modification, the solution presented in [1] (p. 1105) can be used to compute the residual error terms as follows:

$$D_R = -\frac{\Gamma_{D1}\Gamma_{D2}\Gamma_{A3}(\Gamma_{A1} - \Gamma_{A2}) - \Gamma_{D1}\Gamma_{D3}\Gamma_{A2}(\Gamma_{A1} - \Gamma_{A3}) + \Gamma_{D2}\Gamma_{D3}\Gamma_{A1}(\Gamma_{A2} - \Gamma_{A3})}{(\Gamma_{A1} - \Gamma_{A3})(\Gamma_{D1}\Gamma_{A1} - \Gamma_{D2}\Gamma_{A2}) - (\Gamma_{A1} - \Gamma_{A2})(\Gamma_{D1}\Gamma_{A1} - \Gamma_{D3}\Gamma_{A3})} \tag{4}$$

$$M_R = -\frac{(\Gamma_{D1} - \Gamma_{D3})(\Gamma_{A1} - \Gamma_{A2}) - (\Gamma_{D1} - \Gamma_{D2})(\Gamma_{A1} - \Gamma_{A3})}{(\Gamma_{A1} - \Gamma_{A3})(\Gamma_{D1}\Gamma_{A1} - \Gamma_{D2}\Gamma_{A2}) - (\Gamma_{A1} - \Gamma_{A2})(\Gamma_{D1}\Gamma_{A1} - \Gamma_{D3}\Gamma_{A3})} \tag{5}$$

$$T_R = \frac{(\Gamma_{D1}-\Gamma_{D3})(\Gamma_{D1}\Gamma_{A1}-\Gamma_{D2}\Gamma_{A2})-(\Gamma_{D1}-\Gamma_{D2})(\Gamma_{D1}\Gamma_{A1}-\Gamma_{D3}\Gamma_{A3})}{(\Gamma_{A1}-\Gamma_{A3})-(\Gamma_{D1}\Gamma_{A1}-\Gamma_{D2}\Gamma_{A2})-(\Gamma_{A1}-\Gamma_{A2})(\Gamma_{D1}\Gamma_{A1}-\Gamma_{D3}\Gamma_{A3})} + D_R M_R \tag{6}$$

Let

$$\Gamma_{Ai} = \Gamma_{Di} + \Delta\Gamma_i \tag{7}$$

for i = 1 ... 3.

Let's assume the difference $\Delta\Gamma i$ between actual and assumed reflection coefficient of standards is small. Substituting (7) into (4)-(6) we can obtain after some algebraic manipulations and neglecting the second order terms

$$D_R = \frac{\Delta\Gamma_3\Gamma_{D1}\Gamma_{D2}(\Gamma_{D2}-\Gamma_{D1})+\Delta\Gamma_2\Gamma_{D3}\Gamma_{D1}(\Gamma_{D1}-\Gamma_{D3})+\Delta\Gamma_1\Gamma_{D2}\Gamma_{D3}(\Gamma_{D3}-\Gamma_{D2})}{\Gamma_{D2}^2(\Gamma_{D3}-\Gamma_{D1})+\Gamma_{D1}^2(\Gamma_{D2}-\Gamma_{D3})+\Gamma_{D3}^2(\Gamma_{D1}-\Gamma_{D2})} \tag{8}$$

$$M_R = \frac{\Delta\Gamma_3(\Gamma_{D2}-\Gamma_{D1})+\Delta\Gamma_2(\Gamma_{D1}-\Gamma_{D3})+\Delta\Gamma_1(\Gamma_{D3}-\Gamma_{D2})}{\Gamma_{D2}^2(\Gamma_{D3}-\Gamma_{D1})+\Gamma_{D1}^2(\Gamma_{D2}-\Gamma_{D3})+\Gamma_{D3}^2(\Gamma_{D1}-\Gamma_{D2})} \tag{9}$$

$$T_R = 1 + \frac{\Delta\Gamma_2(\Gamma_{D3}^2-\Gamma_{D1}^2)+\Delta\Gamma_3(\Gamma_{D1}^2-\Gamma_{D2}^2)+\Delta\Gamma_1(\Gamma_{D2}^2-\Gamma_{D3}^2)}{\Gamma_{D2}^2(\Gamma_{D3}-\Gamma_{D1})+\Gamma_{D1}^2(\Gamma_{D2}-\Gamma_{D3})+\Gamma_{D3}^2(\Gamma_{D1}-\Gamma_{D2})} \tag{10}$$

Data indicated by the VNA after calibration with imperfect standards can be expressed

$$\Gamma_D \cong D_R + T_R\Gamma_A + T_R M_R \Gamma_A^2 \tag{11}$$

where Γ_A is actual value of the measured reflection coefficient.

Let

$$\Delta\Gamma = \Gamma_A - \Gamma_D \tag{12}$$

Substituting (8)-(11) into (12) we can express how the errors of standards and value of measured reflection coefficient influence the measurement error. After some manipulations and neglecting errors of the second order we obtain

$$\Delta\Gamma = \Delta\Gamma_1 \frac{(\Gamma_D-\Gamma_{D2})(\Gamma_D-\Gamma_{D3})}{(\Gamma_{D1}-\Gamma_{D2})(\Gamma_{D1}-\Gamma_{D3})} + \Delta\Gamma_2 \frac{(\Gamma_D-\Gamma_{D1})(\Gamma_D-\Gamma_{D3})}{(\Gamma_{D2}-\Gamma_{D1})(\Gamma_{D2}-\Gamma_{D3})} + \Delta\Gamma_3 \frac{(\Gamma_D-\Gamma_{D1})(\Gamma_D-\Gamma_{D2})}{(\Gamma_{D3}-\Gamma_{D1})(\Gamma_{D3}-\Gamma_{D2})} \tag{13}$$

It can be seen that the result is the same as that obtained in a slightly different way in [2]. Let's pay attention to individual fractions in expression (13). We will assume the value Γ_D of measured reflection coefficient approaches to that of the standard Γ_{Di}. It can be seen that the value of the fraction in product with $\Delta\Gamma_i$ approaches to 1 while the other fractions approaches to zero. Hence, if the value of reflection coefficient approaches to that of the standard, the error (or uncertainty) is of the same value as that of the standard.

OPEN-SHORT-LOAD (OSL) CALIBRATION

Now we will try to simplify expressions (8)-(10) for the case OSL.

Let $\Delta\Gamma_1 = \Delta\Gamma_{SHORT}$, $\Gamma_{D1} = -1$
$\Delta\Gamma_2 = \Delta\Gamma_{OPEN}$, $\Gamma_{D2} = 1$
$\Delta\Gamma_3 = \Delta\Gamma_{LOAD}$, $\Gamma_{D3} = 0$

Then we obtain

$$D_R = -\Delta\Gamma_{LOAD} \tag{14}$$

$$M_R = \Delta\Gamma_{LOAD} - \frac{\Delta\Gamma_{SHORT} + \Delta\Gamma_{OPEN}}{2} \tag{15}$$

$$T_R = 1 + \frac{\Delta\Gamma_{SHORT} - \Delta\Gamma_{OPEN}}{2} \tag{16}$$

As can be seen, the residual directivity depends only on the load, residual port match depends on all three standards and reflection tracking is dependent only on the standards short and open. Further important fact is that the terms $\Delta\Gamma_{OPEN}$ and $\Delta\Gamma_{SHORT}$ appears in expression (15) in sum while in (16) are subtracted. It means that residual reflection tracking is not affected by errors of open and short when both errors are of the same value. Similarly, the residual test port match is not affected when these errors are of the same value but of the opposite sign. The most typical situation is that the reflection coefficient of the standards open and short (or offset open and offset short) is well defined in magnitude but uncertainty in phase due to mechanical tolerances is significantly greater. Assuming both the phase and magnitude differences between actual and assumed standards small we can rewrite equation (16) as follows

$$T_R \cong \left(1 - \frac{\Delta M_{SHORT} + \Delta M_{OPEN}}{2}\right) \cdot e^{j\left(-\frac{\Delta\phi_{SHORT} + \Delta\phi_{OPEN}}{2}\right)} \tag{17}$$

where $\Delta\phi_{SHORT}$, $\Delta\phi_{OPEN}$ are the phase differences and ΔM_{SHORT} , ΔM_{OPEN} are the magnitude differences between the actual and assumed reflection coefficients of the standards. The phase differences are specified by the manufacturer of the calibration kit. For example, the maximum specified values are 0.6° (HP 7mm cal kits) up to 2° (HP 3.5mm cal kits). Note that the phase uncertainty for opens is typically slightly greater than that for shorts. The magnitude uncertainty of individual standards is not specified but magnitude of residual reflection tracking is specified. As can be seen in [3], the magnitude of residual reflection tracking after OSL calibration differs from ideal value only by several thousandths up to several hundredths of dB. Thus, the dominant uncertainty contribution due to residual reflection tracking is usually the uncertainty in phase.

Now we will try to determine how much can be the overall error affected by the errors of individual standards. The expression (13) can be simplified for OSL calibration as follows

$$\Delta\Gamma = \Delta\Gamma_{SHORT}\frac{\Gamma^2 - \Gamma}{2} + \Delta\Gamma_{OPEN}\frac{\Gamma^2 + \Gamma}{2} + \Delta\Gamma_{LOAD}\left(1 - \Gamma^2\right) \tag{18}$$

where Γ is the value of the measured reflection coefficient. Let's examine the terms being in product with differences $\Delta\Gamma$ of the individual standards in expression (18). We expect the magnitude of measured reflection coefficient Γ to be less or equal 1. Obviously, the maximum value of the term being in product with $\Delta\Gamma_{SHORT}$ is 1 when the measured reflection coefficient value is –1. Thus, the maximum possible error (or uncertainty) caused by imperfect definition of the short is of the same value as the error of the short. This maximum appears when the value of reflection coefficient is the same as that of the short. The maximum value of the term being in product with $\Delta\Gamma_{OPEN}$ is 1 when the measured reflection coefficient value is 1. Thus, the maximum possible error caused by imperfect definition of the open is of the same value as the error of the open. This maximum appears when the value of reflection coefficient is the same as that of the open. The maximum value of the term in product with $\Delta\Gamma_{LOAD}$ is 2 when the value of the measured reflection coefficient is ±j. Thus, the maximum possible error caused by imperfect definition of the load is two times greater than the error of the load. This maximum appears when the magnitude of measured reflection coefficient is 1 and the phase is ±90° (exactly said, if the phase difference between the measured reflection coefficient and short or open is ±90°). We can see, the load is the most critical calibration item. In practice, calibration kits are often designed with respect to this fact. For example, in HP 7mm and 3.5mm calibration kits is the uncertainty of the reflection coefficient of the

open and short due to specified phase tolerances three times or more greater than the specified reflection coefficient of the sliding load.
Till now, we assumed standards open and short without offset. In most of calibration kits the offset lengths in the opens and shorts are designed so that the difference in phase of their reflection coefficients is approximately 180° at all frequencies. Hence, the conclusions described above can be applied analogically for the calibration kits with offset shorts and opens.

INFLUENCE OF ANOTHER ERRORS

It is evident that the errors in measurement of standards due to random errors and linearity affect the residual error terms in the similar way as the deviations between actual and assumed values of the standards. An important question is which contribution is dominant. For illustration, residual error terms for system HP8510C calibrated with several calibration kits [3] are shown in tab.1. It can be seen that the effective directivity and test port match of the system HP 8510 can be improved to as much as 60 dB when TRL calibration is used. Advantage of TRL method is that it doesn't need the exact knowledge of parameters of the standards except the line impedance. As can be seen, the residual error terms for OSL calibration are approximately 10 to 30 dB worse (depending on the cal kit type) than the best capability of the system. Obviously, the residual error terms are primarily affected by the differences between actual and assumed standards for typical OSL calibration.

Cal Kit Type	Connector Type	Return Loss, Fixed Load	Return Loss, Sliding Load	Return Loss Airline @ f_{max}	Residual Directivity (dB) @ f_{max}	Residual Source Match (dB) @ f_{max}
HP 85050B (OSLT)	7mm	≥52dB,DC-2GHz	≥52dB,2-18GHz	—	52	41
HP85052B (OSLT)	3.5mm	≥44dB,DC-3GHz	≥44dB,3-26.5GHz	—	44	31
HP 85054B (OSLT)	Type N	≥48dB,DC-2GHz	≥42dB,2-18GHz	—	42	32
HP 85050C (TRL)	7mm	≥38dB,DC-18GHz	—	>60dB	60	60
HP 85052C (TRL)	3.5mm	≥46dB,DC-2GHz	—	50	50	50
HP 85050D (OSLT)	7mm	≥38dB,DC-18GHz	—	—	40	35
HP 85052D (OSLT)	3.5mm	≥30dB@26.5GHz	—	—	30	25
HP 85054D (OSLT)	Type N	≥34dB@18GHz	—	—	34	28

Tab.1 Cal Kit specifications

CONCLUSIONS

The important fact was shown that the residual error terms after three term one-port calibration are, in an ideal case, determined only by the differences between actual and assumed parameters of standards. These differences can be usually considered to be dominant source of residual errors when typical OSL calibration is used. Further important conclusion is that the residual error terms are independent on the actual error terms of the VNA (in contrast to "response" type of calibration). It is obvious, the uncertainty calculated using residual error term approach could not be underestimated if the residual error terms are well evaluated. Even, this uncertainty can be overestimated, namely for regions in complex plane near to reflective standards, where the uncertainty approaches to that of the standard. The cause of it is, that we calculate only the worst case uncertainty as function of the magnitude of measured reflection coefficient. We can conclude, the uncertainty calculation from uncertainties of individual standards can be considered as the alternative method to the traditional residual error term approach which can yield pessimistic values of uncertainty in some regions of the complex plane.
Having in mind the mechanism of origin of residual error terms we can try to change the cal kit definition to minimize their values. It is based on the fact, that two of three residual error terms can be easily evaluated using

well known ripple method. It seems not to be easy task but this approach was in some special cases succesfully applied. The detailed report on it is to follow in the future.

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