ANAMET REPORT

Three-term error correction
in one-port calibrations

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THREE-TERM ERROR CORRECTION
IN ONE-PORT CALIBRATIONS

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1. THE THREE-TERM ERROR MODEL

In Figure 1 the signal flow graph for a One-Port measurement is represented. Details are given in [1]. The three error terms considered are:

E_{DF}: Directivity
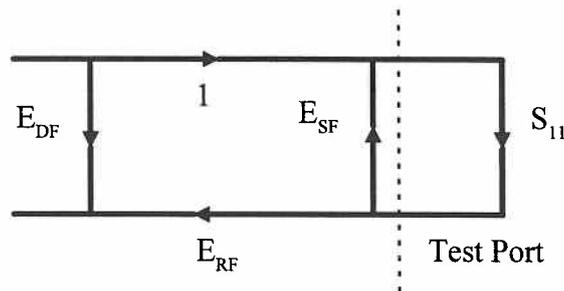E_{SF}: Source Match
E_{RF}: Reflection Tracking

Figure 1.- Signal flow graph

Calibration consists in putting three known standards in place of S_{11} and extracting the error terms from the resulting three-equation system. From the flow graph, the measured reflection coefficient is given by:

$$S_{11,m} = E_{DF} + \frac{S_{11} \cdot E_{RF}}{1 - E_{SF} \cdot S_{11}}$$

In an OSL calibration the three standards chosen are an Open circuit, a Short circuit and a Load. Let us represent their reflection coefficients as Γ_L (the Broadband Load), Γ_{OC} (the Open circuit) and Γ_{SC} (the Short). Measurement of the three terminations provide us with the following set of equations:

$$S_{11,m(Load)} = E_{DF} + \frac{\Gamma_L \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_L} \equiv A$$

$$S_{11,m(Open)} = E_{DF} + \frac{\Gamma_{OC} \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_{OC}} \equiv B$$

$$S_{11,m(Short)} = E_{DF} + \frac{\Gamma_{SC} \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_{SC}} \equiv C$$

For reasons of simplicity we shall assume ideal terminations, so the reflection coefficients considered for the calibration standards are $\Gamma_L = 0$, $\Gamma_{OC} = 1$ and $\Gamma_{SC} = -1$. As explained in [2], these assumptions are not far from reality in everyday calibrations, in particular for the broadband load, and the loss of generality is compensated by the simplicity in the calculations that follow. The above equations thus become:

$$S_{11,m\,(Load)} = E_{DF} \equiv A$$

$$S_{11,m\,(Open)} = E_{DF} + \frac{E_{RF}}{1 - E_{SF}} \equiv B$$

Eqns. (1)

$$S_{11,m\,(Short)} = E_{DF} - \frac{E_{RF}}{1 + E_{SF}} \equiv C$$

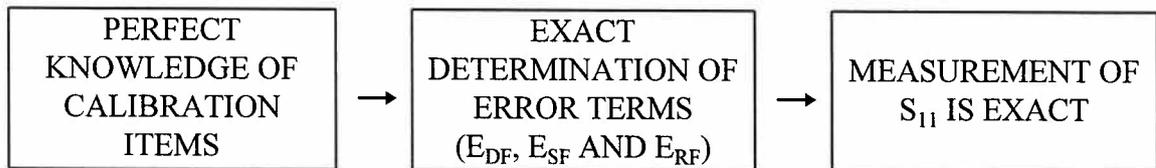Solving the three equations for E_{DF}, E_{SF} and E_{RF} we obtain:

$$E_{DF} = A$$

$$E_{SF} = \frac{2 \cdot A - B - C}{C - B}$$

Eqns. (2)

$$E_{RF} = \frac{2 \cdot (A - B) \cdot (A - C)}{C - B}$$

2. ERROR CORRECTION

Having obtained the three error terms, the firmware extracts the corrected value of S_{11} from the measured one, and presents it to the user:
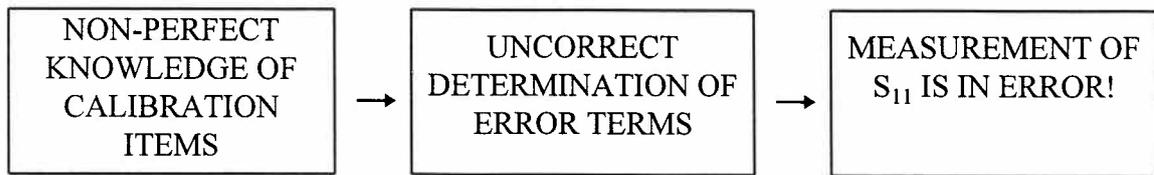
$$S_{11,corr} = \frac{S_{11,m} - E_{DF}}{E_{RF} + E_{SF} \cdot \left(S_{11,m} - E_{DF}\right)}$$

Eqn. (3)

The formulas for determination of the error terms are exact as long as the calibration items are what they are expected to be (ideal terminations with reflection coefficients $\Gamma_L = 0$, $\Gamma_{OC} = 1$ and $\Gamma_{SC} = -1$). Even when Γ_{OC} or Γ_{SC} are more carefully modelled as frequency-dependent, the following statement can be made:

| PERFECT KNOWLEDGE OF CALIBRATION ITEMS	→	EXACT DETERMINATION OF ERROR TERMS (E_{DF}, E_{SF} AND E_{RF})	→	MEASUREMENT OF S_{11} IS EXACT

In other words, the corrected value of S_{11} as per eqn. (3) exactly coincides with S_{11} (*true value*).

Using calibration items that differ from their assumed values gives raise to uncorrect estimation of E_{DF}, E_{SF} and E_{RF}. The overall effect is a measured reflection coefficient $S_{11,corr}$ which is not equal to its *true value*:

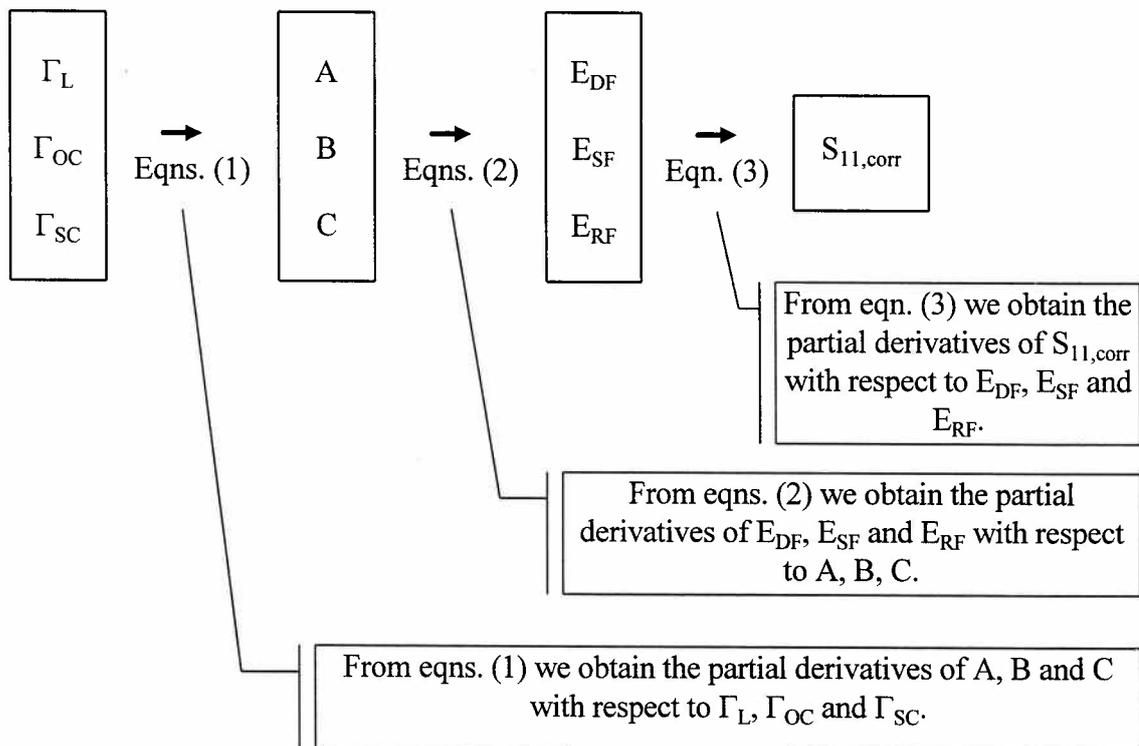| NON-PERFECT KNOWLEDGE OF CALIBRATION ITEMS	→	UNCORRECT DETERMINATION OF ERROR TERMS	→	MEASUREMENT OF S_{11} IS IN ERROR!

Thus we can ask ourselves whether it should be reasonable to introduce the uncertainty of the calibration kit into the overall measurement uncertainty:

CALIBRATION KIT UNCERTAINTY	→	MEASUREMENT UNCERTAINTY u_{S11}
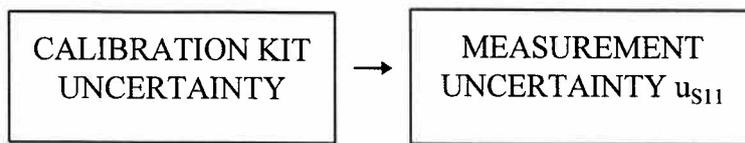
3. PARTIAL DERIVATIVES

Our aim is now to deterministically show the effect of the calibration items on the measurement error. To do this, we will bear in mind the dependance of $S_{11,corr}$ on the error terms E_{DF}, E_{SF} and E_{RF}, tracing it back to Γ_L, Γ_{OC} and Γ_{SC} via the three individual measurements which we have called A, B and C.

Γ_L Γ_{OC} Γ_{SC}	→ Eqns. (1)	A B C	→ Eqns. (2)	E_{DF} E_{SF} E_{RF}	→ Eqn. (3)	$S_{11,corr}$

From eqn. (3) we obtain the partial derivatives of $S_{11,corr}$ with respect to E_{DF}, E_{SF} and E_{RF}.

From eqns. (2) we obtain the partial derivatives of E_{DF}, E_{SF} and E_{RF} with respect to A, B, C.

From eqns. (1) we obtain the partial derivatives of A, B and C with respect to Γ_L, Γ_{OC} and Γ_{SC}.

3.1. Effect of the Load

Partial derivatives of (1) with respect to Γ_L:

$$A \equiv S_{11,m\,(Load)} = E_{DF} + \frac{\Gamma_L \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_L}$$

$$\frac{\partial A}{\partial \Gamma_L}\bigg|_{\Gamma_L=0} = \frac{E_{RF}}{\left(1 - E_{SF} \cdot \Gamma_L\right)^2}\bigg|_{\Gamma_L=0} = E_{RF}$$

$$\frac{\partial B}{\partial \Gamma_L} = 0$$

$$\frac{\partial C}{\partial \Gamma_L} = 0$$

Partial derivatives of (2) with respect to A:

$$\frac{\partial E_{DF}}{\partial A} = 1$$

$$\frac{\partial E_{SF}}{\partial A} = \frac{\left(E_{SF}^2 - 1\right)}{E_{RF}}$$

$$\frac{\partial E_{RF}}{\partial A} = 2 \cdot E_{SF}$$

Partial derivatives of (3) with respect to the error terms E_{DF}, E_{SF} and E_{RF}:

$$\frac{\partial S_{11,corr}}{\partial E_{DF}} = -\frac{\left(1 - E_{SF} \cdot S_{11}\right)^2}{E_{RF}}$$

$$\frac{\partial S_{11,corr}}{\partial E_{SF}} = -S_{11}^2$$

$$\frac{\partial S_{11,corr}}{\partial E_{RF}} = \frac{S_{11}}{E_{RF}}\left(E_{SF} \cdot S_{11} - 1\right)$$

Finally, the measurement error ε_{S11} is given by the law of propagation of uncertainties:

$$\frac{\partial S_{11,corr}}{\partial E_{DF}} \cdot \frac{\partial E_{DF}}{\partial A} \cdot \frac{\partial A}{\partial \Gamma_L} \cdot \varepsilon_L + \frac{\partial S_{11,corr}}{\partial E_{SF}} \cdot \frac{\partial E_{SF}}{\partial A} \cdot \frac{\partial A}{\partial \Gamma_L} \cdot \varepsilon_L + \frac{\partial S_{11,corr}}{\partial E_{RF}} \cdot \frac{\partial E_{RF}}{\partial A} \cdot \frac{\partial A}{\partial \Gamma_L} \cdot \varepsilon_L = \dots$$

$$\dots = -\left(1 - E_{SF} \cdot S_{11}\right)^2 \cdot \varepsilon_L - S_{11}^2 \cdot \left(E_{SF}^2 - 1\right) \cdot \varepsilon_L + 2 \cdot E_{SF} \cdot S_{11} \cdot \left(E_{SF} \cdot S_{11} - 1\right) \cdot \varepsilon_L$$

Where ε_L is the complex error with which the reflection coefficient Γ_L of the termination is known. Rearranging terms:

$$\boxed{\varepsilon_{S11} = \left(S_{11}^2 - 1\right) \cdot \varepsilon_L}$$

Eqn. (4)

3.2. Effect of the Open Circuit

Partial derivatives of (1) with respect to Γ_{OC}:

$$\frac{\partial A}{\partial \Gamma_{OC}} = 0$$

$$B \equiv S_{11,m(Open)} = E_{DF} + \frac{\Gamma_{OC} \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_{OC}}$$

$$\frac{\partial B}{\partial \Gamma_{OC}}\bigg|_{\Gamma_{OC}=1} = \frac{E_{RF}}{\left(1 - E_{SF} \cdot \Gamma_{OC}\right)^2}\bigg|_{\Gamma_{OC}=1} = \frac{E_{RF}}{\left(1 - E_{SF}\right)^2}$$

$$\frac{\partial C}{\partial \Gamma_{OC}} = 0$$

Partial derivatives of (2) with respect to B:

$$\frac{\partial E_{DF}}{\partial B} = 0$$

$$\frac{\partial E_{SF}}{\partial B} = \frac{\left(1 - E_{SF}^2\right) \cdot \left(1 - E_{SF}\right)}{2 \cdot E_{RF}}$$

$$\frac{\partial E_{RF}}{\partial B} = \frac{\left(1 - E_{SF}\right)^2}{2}$$

The partial derivatives of (3) with respect to E_{DF}, E_{SF} and E_{RF} are as in the previous section. Let us finally denote as ε_{OC} the error with which the reflection coefficient of the Open Circuit is known. The measurement error ε_{S11} is given by:

$$\frac{\partial S_{11,corr}}{\partial E_{SF}} \cdot \frac{\partial E_{SF}}{\partial B} \cdot \frac{\partial B}{\partial \Gamma_{OC}} \cdot \varepsilon_{OC} + \frac{\partial S_{11,corr}}{\partial E_{RF}} \cdot \frac{\partial E_{RF}}{\partial B} \cdot \frac{\partial B}{\partial \Gamma_{OC}} \cdot \varepsilon_{OC} = \dots$$

$$\dots = -S_{11}^2 \cdot \frac{\left(1 + E_{SF}\right)}{2} \cdot \varepsilon_{OC} + \frac{S_{11}}{2} \cdot \left(E_{SF} \cdot S_{11} - 1\right) \cdot \varepsilon_{OC}$$

And rearranging terms:

$$\boxed{\varepsilon_{S11} = \frac{-S_{11} \cdot \left(1 + S_{11}\right)}{2} \cdot \varepsilon_{OC}}$$

Eqn. (5)

3.3. Effect of the Short Circuit

Partial derivatives of (1) with respect to Γ_{SC}:

$$\frac{\partial A}{\partial \Gamma_{SC}} = 0$$

$$\frac{\partial B}{\partial \Gamma_{SC}} = 0$$

$$C \equiv S_{11,m(Short)} = E_{DF} + \frac{\Gamma_{SC} \cdot E_{RF}}{1 - E_{SF} \cdot \Gamma_{SC}}$$

$$\frac{\partial C}{\partial \Gamma_{SC}}\bigg|_{\Gamma_{SC}=-1} = \frac{E_{RF}}{\left(1 - E_{SF} \cdot \Gamma_{SC}\right)^2}\bigg|_{\Gamma_{SC}=-1} = \frac{E_{RF}}{\left(1 + E_{SF}\right)^2}$$

Partial derivatives of (2) with respect to C:

$$\frac{\partial E_{DF}}{\partial C} = 0$$

$$\frac{\partial E_{SF}}{\partial C} = \frac{\left(1 - E_{SF}^2\right) \cdot \left(1 + E_{SF}\right)}{2 \cdot E_{RF}}$$

$$\frac{\partial E_{RF}}{\partial C} = -\frac{\left(1 + E_{SF}\right)^2}{2}$$

The partial derivatives of (3) with respect to E_{DF}, E_{SF} and E_{RF} are known. Finally, the measurement error ε_{S11} as a function of ε_{SC}:

$$\frac{\partial S_{11,corr}}{\partial E_{SF}} \cdot \frac{\partial E_{SF}}{\partial C} \cdot \frac{\partial C}{\partial \Gamma_{SC}} \cdot \varepsilon_{SC} + \frac{\partial S_{11,corr}}{\partial E_{RF}} \cdot \frac{\partial E_{RF}}{\partial C} \cdot \frac{\partial C}{\partial \Gamma_{SC}} \cdot \varepsilon_{SC} = \ldots$$

$$\ldots = -S_{11}^2 \cdot \frac{\left(1 - E_{SF}\right)}{2} \cdot \varepsilon_{SC} - \frac{S_{11}}{2} \cdot \left(E_{SF} \cdot S_{11} - 1\right) \cdot \varepsilon_{SC}$$

And rearranging terms:

$$\boxed{\varepsilon_{S11} = \frac{S_{11} \cdot \left(1 - S_{11}\right)}{2} \cdot \varepsilon_{SC}} \qquad \text{Eqn. (6)}$$

Happily enough, equations (4), (5) and (6) do not depend on the error terms E_{DF}, E_{SF} and E_{RF}. This means that we need not know them when determining the measurement uncertainty (they can therefore remain *internal* within the firmware!).

4. OVERALL EFFECT ON THE MEASUREMENT ERROR

Suming up eqns. (4), (5) and (6) we obtain (7), which gives us the measurement error as a function of the errors with which the three calibration items are known, as well as the *true value* of the reflection coefficient S_{11}.

$$\varepsilon_{S11} = \left(S_{11}^{2} - 1\right) \cdot \varepsilon_{L} - \frac{S_{11} \cdot \left(1 + S_{11}\right)}{2} \cdot \varepsilon_{OC} + \frac{S_{11} \cdot \left(1 - S_{11}\right)}{2} \cdot \varepsilon_{SC}$$

Eqn. (7)

Where ε_{S11}, ε_{L}, ε_{OC} and ε_{SC} are, in general, complex vectors defined with respect to the theoretical values as follows:

$$\varepsilon_{S11} = S_{11,corr} - S_{11}$$
$$\varepsilon_{L} = \Gamma_{L} - 0$$
$$\varepsilon_{OC} = \Gamma_{OC} - 1$$
$$\varepsilon_{SC} = \Gamma_{SC} - (-1)$$

The differential approach followed here gives good results as long as the deviations from the ideal values of the calibration kit are small enough. But, of course, in a practical case, we will be lacking any information about complex errors, and, more important, we will be ignoring the *true value* of S_{11}. Thus an approximate approach is needed in order to set bounds to the measurement uncertainty.

5. SCALAR APPROACH

In the previous section, we have arrived at this vectorial expression relating the *true value* of S_{11} and its measured value, as obtained from the ANA: $S_{11} = S_{11,corr} - \varepsilon_{S11}$. If we are just interested in the magnitude of the measurand, that is if we are making scalar measurements, then the following statement can be made:

$$\left|S_{11}\right| \le \left|S_{11,corr}\right| + \left|\varepsilon_{S11}\right|$$

In terms of uncertainty, we can express the result of our measurement as:

$$\left|S_{11}\right| = \left|S_{11,corr}\right| \pm u_{S11}$$

Where u_{S11} can be obtained as the magnitude of ε_{S11}, with $S_{11,corr}$ as an estimate of S_{11}. Assuming worst-case vectorial combination:

$$u_{S11} = \left|S_{11,corr}^{2} - 1\right| \cdot u_{L} + \left|\frac{S_{11,corr} \cdot \left(1 + S_{11,corr}\right)}{2}\right| \cdot u_{OC} + \left|\frac{S_{11,corr} \cdot \left(1 - S_{11,corr}\right)}{2}\right| \cdot u_{SC}$$

Whereas assuming root-sum-of-squares:

$$u_{S11} = \sqrt{\left| S_{11,corr}^2 - 1 \right|^2 \cdot u_L^2 + \left| \frac{S_{11,corr} \cdot \left(1 + S_{11,corr}\right)}{2} \right|^2 \cdot u_{OC}^2 + \left| \frac{S_{11,corr} \cdot \left(1 - S_{11,corr}\right)}{2} \right|^2 \cdot u_{SC}^2}$$

6. EXAMPLE

The internal error terms are as follows:

$$E_{DF} = 0.003 \,|\, 135°$$
$$E_{SF} = 0.005$$
$$E_{RF} = 0.99$$

The calibration items are:

$$\Gamma_L = 0.005 \,|\, 45° \text{ (46 dB return loss)}$$
$$\Gamma_{OC} = 1.01 - j\,0.01$$
$$\Gamma_{SC} = -0.98$$

Let us assume that $S_{11} = 0.5 + j\cdot0.5$ is the *true* reflection coefficient, measured after correction as:

$$S_{11,corr} = 0.49242 + j\,0.49565$$

Equation (7) can be used if we take the following complex vectors for the calibration items:

$$\varepsilon_L = 0.005 \,|\, 45°$$
$$\varepsilon_{OC} = 0.01 - j\,0.01$$
$$\varepsilon_{SC} = 0.02$$

In this case, eqn. (7) yields:

$$\varepsilon_{S11} = -0.00780 - j\,0.00427$$

Which agrees very well with the complex amount by which the measurement of S_{11} is in error.

Let us now follow an approximate approach, assuming that we are just involved in scalar measurements. The corrected value of S_{11} - as obtained from the Analyser:

$$\left| S_{11,corr} \right| = \left| 0.49242 + j\,0.49565 \right| = 0.699$$

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If we compute the scalar uncertainty with $u_L = 0.005$, $u_{OC} = 0.014$ and $u_{SC} = 0.02$, we finally obtain:

$$u_{S11} = 0.018 \text{ (worst case)}$$
$$u_{S11} = 0.011 \text{ (rss)}$$

In both cases:

$$|S_{11}| = 0.699 \pm 0.018 \text{ (worst case)}$$
$$|S_{11}| = 0.699 \pm 0.011 \text{ (rss)}$$

the measured value includes the *true value* 0.707 within the uncertainty interval.

7. CONCLUSIONS

We have seen that every error in the calibration kit affects the measurement of S_{11}. Partial differentiation can help us very accurately estimate the resultant complex error (valid only for small deviations!). Formulas have been derived that show the relationship among the errors involved.

However, complex errors are difficult to deal with, and also the true value S_{11} is unknown. Use can be made of approximate formulas, in terms of scalar uncertainty and of measured S_{11}, which also produce good results.

A final question remains open to debate: should we include this into the ANA overall uncertainty as an additional contribution?

8. REFERENCES

[1]   A Knowlson and I Instone. "Comparison of HP and draft EAL methods of assessing network analyser measurement uncertainties". ANAMET Report 005, February 1997.

[2]   M Rodríguez. "One-Port Calibration: Sliding Load vs. Broadband Load". ANAlyse Number 26, March 1998.