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Assessment of VNAs: Determination of Test Port Match

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Assessment of VNAs: Determination of Test Port Match

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Abstract

This article deals with the problem of effective test port match determination. A procedure is proposed that utilizes both magnitude and phase of the measured reflection coefficient of a short circuit terminated air line.

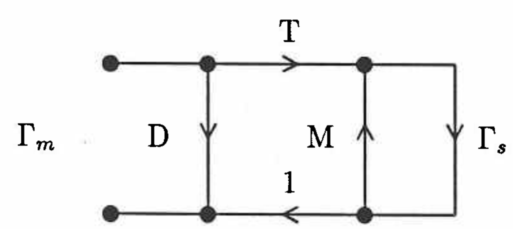
1 Introduction

A modification of the procedure for the effective test port match evaluation presented in [1] is proposed. The aim of this work is to reduce the effective test port match uncertainty caused by the unknown phases of D and M. The measurement procedure is the same as described in [1] but, in addition, the information about measured phase is utilized.

2 Description

The flowgraph for the one port error model is shown in fig. 1. Using Mason's rule, the reflection coefficient Γ_m can be expressed

$$\Gamma_m = D + \frac{T\Gamma_s}{(1 - \Gamma_s M)} \tag{1}$$



Γ_s ..... offset short reflection coefficient
T ..... residual error term Tracking
D ..... residual error term Directivity
M ..... residual error term Test Port Match
Γ_m ..... measured reflection coefficient

Figure 1: One port error model

For $|\Gamma_s M| \ll 1$, $T = 1$ we can get

$$\Gamma_m = D + T\Gamma_s(1 + \Gamma_s M) = D + \Gamma_s + \Gamma_s^2 M \tag{2}$$

$$\frac{\Gamma_m - \Gamma_s}{\Gamma_s} = \frac{D}{\Gamma_s} + \Gamma_s M \tag{3}$$

The reflection coefficient of an ideal offset short is

$$\Gamma_s = -1e^{-j2\alpha l} \tag{4}$$

where $\alpha = 2\pi/\lambda$,
l ... length of the air line (including offset length of the short)

Substituting eq (4) into eq (3) we can obtain

$$\frac{\Gamma_m - \Gamma_s}{\Gamma_s} = \frac{\Gamma_m}{\Gamma_s} - 1 = -De^{j2\alpha l} - Me^{-j2\alpha l} \tag{5}$$

Let

$$\Gamma_m = |\Gamma_m|e^{j\phi_m} \tag{6}$$

$$\Gamma_s = |\Gamma_s|e^{j\phi_s} \tag{7}$$

Then

$$\frac{\Gamma_m}{\Gamma_s} - 1 = \frac{|\Gamma_m|}{|\Gamma_s|}(cos(\phi_m - \phi_s) + jsin(\phi_m - \phi_s)) - 1 \tag{8}$$

For $|\Gamma_m - \Gamma_s| \ll 1$ we can write[1]

$$\frac{\Gamma_m}{\Gamma_s} - 1 = \frac{|\Gamma_m|}{|\Gamma_s|} - 1 + jsin(\phi_m - \phi_s) \tag{9}$$

We can see that the real part depends on the magnitude and the imaginary part on the phase of Γ_m/Γ_s only. Hence, we will separate the real and imaginary parts of (5).

Let

$$D = |D|e^{j\phi_D} \tag{10}$$

$$M = |M|e^{j\phi_M} \tag{11}$$

$$Re\left(\frac{\Gamma_m}{\Gamma_s} - 1\right) = \frac{|\Gamma_m|}{|\Gamma_s|} - 1 = -|D|cos(2\alpha l + \phi_D) - |M|cos(2\alpha l - \phi_M) \tag{12}$$

$$Im\left(\frac{\Gamma_m}{\Gamma_s} - 1\right) = sin(\phi_m - \phi_s) = -|D|sin(2\alpha l + \phi_D) + |M|sin(2\alpha l - \phi_M) \tag{13}$$

[1] A maximum error of both real and imaginary parts is less than 2.5 percent when $|\Gamma_m - \Gamma_s| < 0.05$

We can see that the maximum possible ripple of both real and imaginary parts is $2(|M| + |D|)$.

Let's consider the special case $|M| = |D|$:

If $\phi_D + \phi_M = 180°$
ripple in magnitude disappears
ripple $sin(\phi_m - \phi_s) = 2(|M| + |D|)$

If $\phi_D + \phi_M = 0°$
ripple in phase disappears
ripple in magnitude $= 2(|M| + |D|)$

The electrical delay and phase offset functions on a VNA are convenient tools for phase ripple or imaginary part of $\Gamma_m/\Gamma_s - 1$ evaluation. Using the electrical delay function we obtain the measurement result

$$\Gamma_{mdelay} = \frac{\Gamma_m}{e^{-j2\alpha l_{delay}}} \tag{14}$$

Electrical delay should be set so that the measured reflection coefficient of the offset short is close to the point -1 in polar diagram. After setting the phase offset 180° the VNA displays

$$\frac{\Gamma_m}{-e^{-j2\alpha l}} \equiv \frac{\Gamma_m}{\Gamma_s}.$$

The ripple in phase or imaginary part of reflection coefficient can be easily read.

Equations (12), (13) can be rewritten as follows:

$$\begin{aligned} Re(\Gamma_m/\Gamma_s - 1) &= -|D|(cos2\alpha l cos\phi_D - sin2\alpha l sin\phi_D) - \\ &\quad -|M|(cos2\alpha l cos\phi_M + sin2\alpha l sin\phi_M) = \\ &= -(|D|cos\phi_D + |M|cos\phi_M)cos2\alpha l + \\ &\quad +(|D|sin\phi_D - |M|sin\phi_M)sin2\alpha l \end{aligned} \tag{15}$$

$$\begin{aligned} Im(\Gamma_m/\Gamma_s - 1) &= -|D|(sin2\alpha l cos\phi_D - sin2\alpha l sin\phi_D) + \\ &\quad +|M|(sin2\alpha l cos\phi_M + cos2\alpha l sin\phi_M) = \\ &= -(|D|sin\phi_D + |M|sin\phi_M)cos2\alpha l - \\ &\quad -(|D|cos\phi_D - |M|cos\phi_M)sin2\alpha l \end{aligned} \tag{16}$$

For amplitudes of real and imaginary parts we can write:

$$\begin{aligned} A_{Re}^2 &= |D|^2cos^2\phi_D + |M|^2cos^2\phi_M + 2|D||M|cos\phi_D cos\phi_M + \\ &\quad +|D|^2sin^2\phi_D + |M|^2sin^2\phi_M - 2|D||M|sin\phi_D sin\phi_M = \\ &= |D|^2 + |M|^2 + 2|D||M|cos\phi_D cos\phi_M - 2|D||M|sin\phi_D sin\phi_M \end{aligned} \tag{17}$$

$$
\begin{aligned}
A_{Im}^2 &= |D|^2 sin^2\phi_D + |M|^2 sin^2\phi_M + 2|D||M| sin\phi_D sin\phi_M + \\
&\quad + |D|^2 cos^2\phi_D + |M|^2 cos^2\phi_M - 2|D||M| cos\phi_D cos\phi_M = \\
&= |D|^2 + |M|^2 + 2|D||M| sin\phi_D sin\phi_M - 2|D||M| cos\phi_D cos\phi_M
\end{aligned}
\tag{18}
$$

Adding equations (17), (18) we obtain

$$|D|^2 + |M|^2 = \frac{A_{Re}^2 + A_{Im}^2}{2} \tag{19}$$

where

$$A_{Re} = \frac{real\ part\ ripple}{2} \approx \frac{magnitude\ ripple}{2} \tag{20}$$

$$A_{Im} = \frac{imag.\ part\ ripple}{2} \approx \frac{sin(phase\ ripple)}{2} \tag{21}$$

Then, we can exactly calculate $|M|$ when $|D|$ is known:

$$|M| = \sqrt{\frac{A_{Re}^2 + A_{Im}^2}{2} - |D|^2} \tag{22}$$

Let's now consider the influence of the air line and short circuit losses. Till now, we assumed $|\Gamma_s|$ to be equal 1. If $|\Gamma_s| \neq 1$ we can replace in the previous equations $|D|$ and $|M|$ with $|D|/|\Gamma_s|$ and $|M||\Gamma_s|$, respectively.

Then

$$\frac{|D|^2}{|\Gamma_s|^2} + |M|^2|\Gamma_s|^2 = \frac{A_{Re}^2 + A_{Im}^2}{2} \tag{23}$$

$$|M| = \frac{1}{|\Gamma_s|}\sqrt{\frac{A_{Re}^2 + A_{Im}^2}{2} - \frac{|D|^2}{|\Gamma_s|^2}} \tag{24}$$

3 Numerical Check

A numerical check of the above procedure has been carried out. Results are presented in tables 1, 2. The ripple of $|\Gamma_m|$ and $arg(\Gamma_m/\Gamma_s)$ has been calculated for phase steps Γ_s of 10 degree when calculating Γ_m by eq (1). Test port match values have been computed using formulas (25), (26). The second formula takes into account losses of the airline and short circuit.

$$|M| = \sqrt{\frac{\left(\frac{ripple|\Gamma_m|}{2}\right)^2 + \left(\frac{sin(ripple\ arg(\Gamma_m/\Gamma_s))}{2}\right)^2}{2} - |D|^2} \quad (25)$$

$$|M| = \frac{1}{|\Gamma_s|}\sqrt{\frac{\left(\frac{ripple|\Gamma_m|}{2|\Gamma_s|}\right)^2 + \left(\frac{sin(ripple\ arg(\Gamma_m/\Gamma_s))}{2}\right)^2}{2} - \frac{|D|^2}{|\Gamma_s|^2}} \quad (26)$$

Note that the values of ripple $|\Gamma_m|$ and ripple $arg(\Gamma_m/\Gamma_s)$ can be directly read on the screen of the VNA.

D actual		M actual		|M| calculated by eq (25)	magnitude ripple	sin (phase ripple)
mag.	phase	mag.	phase			
0.01	0	0.01	0	0.0100	0.0400	0.0002
0.01	90	0.01	0	0.0099	0.0282	0.0282
0.01	180	0.01	0	0.0100	0.0000	0.0400
0.01	270	0.01	0	0.0099	0.0282	0.0282
0.01	0	0.01	90	0.0099	0.0282	0.0282
0.01	90	0.01	90	0.0100	0.0000	0.0400
0.01	180	0.01	90	0.0099	0.0282	0.0282
0.01	270	0.01	90	0.0100	0.0400	0.0002
0.01	0	0.01	180	0.0100	0.0000	0.0400
0.01	90	0.01	180	0.0099	0.0282	0.0282
0.01	180	0.01	180	0.0100	0.0400	0.0002
0.01	270	0.01	180	0.0099	0.0282	0.0282
0.01	0	0.01	270	0.0099	0.0282	0.0282
0.01	90	0.01	270	0.0100	0.0400	0.0002
0.01	180	0.01	270	0.0099	0.0282	0.0282
0.01	270	0.01	270	0.0100	0.0000	0.0400
0.01	0	0.03	0	0.0300	0.0801	0.0400
0.01	90	0.03	0	0.0300	0.0633	0.0632
0.01	180	0.03	0	0.0300	0.0401	0.0799
0.01	270	0.03	0	0.0300	0.0633	0.0632
0.03	0	0.01	0	0.0100	0.0800	0.0400
0.03	90	0.01	0	0.0099	0.0632	0.0632
0.03	180	0.01	0	0.0099	0.0400	0.0799
0.03	270	0.01	0	0.0099	0.0632	0.0632

Table 1: Results of numerical check, $|\Gamma_s| = 1$

D actual		M actual		|M| calculated by eq (26)	|M| calculated by eq (25)	magnitude ripple	sin (phase ripple)
mag.	phase	mag.	phase				
0.01	0	0.01	0	0.0100	0.0080	0.0359	0.0046
0.01	90	0.01	0	0.0100	0.0092	0.0255	0.0286
0.01	180	0.01	0	0.0100	0.0102	0.0041	0.0403
0.01	270	0.01	0	0.0100	0.0092	0.0255	0.0286
0.01	0	0.03	0	0.0300	0.0244	0.0677	0.0310
0.01	90	0.03	0	0.0300	0.0255	0.0516	0.0579
0.01	180	0.03	0	0.0300	0.0267	0.0277	0.0758
0.01	270	0.03	0	0.0300	0.0255	0.0516	0.0579
0.03	0	0.01	0	0.0100	0.0112	0.0759	0.0495
0.03	90	0.01	0	0.0096	0.0135	0.0619	0.0695
0.03	180	0.01	0	0.0098	0.0157	0.0441	0.0850
0.03	270	0.01	0	0.0096	0.0135	0.0619	0.0695

Table 2: Results of numerical check, $|\Gamma_s| = 0.89$ (return loss 1 dB)

4 Conclusion

The possibility of eliminating the test port match uncertainty caused by unknown phases of M and D using additional phase ripple measurement has been demonstrated. It is possible to decrease the error of $|M|$ when taking into account losses of air line and short circuit also, but the formulas become more complicated. In our opinion, the less complicated formula (25) would yield acceptable results for the majority of cases.
Note that we suppose there is a possibility of calculating both parameters M and D when only the short circuit terminated precision air line is measured but we haven't found a simple solution to this problem yet.

References

[1] Clarke, R. N.; Ide, J. P.; Orford, G. R.; Ridler, N. M.: Draft EAL Procedure for the Assessment of Vector Network Analysers (VNA), ANAMET Report 002, September 1996