ANAMET REPORT

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Comparison of Hewlett-Packard and draft EAL methods of assessing network analyser measurement uncertainties

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This report has been approved by the ANAMET Steering Committee.

COMPARISON OF HEWLETT-PACKARD AND DRAFT EAL METHODS OF ASSESSING NETWORK ANALYSER MEASUREMENT UNCERTAINTIES

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INTRODUCTION

This document was produced to demonstrate the differences which exist between the "Draft EAL Procedure for the assessment of Vector Network Analysers" and the method used by Hewlett-Packard at their U.K. Customer Support Centre. Since uncertainty assessment of Vector Network Analysers is still an evolving science it is not possible to make any statement regarding the correctness of either method.

ASSESSMENT OF RESIDUAL ERROR TERMS

Both methods use similar procedures for the assessment of the residual error terms. However, with modern Vector Network Analysers it is not normally possible to find an airline which has significantly better match than the analysers directivity or source match. This problem is somewhat alleviated because it is also not possible to obtain a suitable electrical calibration to show that the airline at least meets it's manufacturers specification.

The following pages demonstrate the procedures used by Hewlett-Packard at their UK Service Centre for assessing the measurement uncertainties and compare the results to those obtained using the draft EAL method.

SIGNAL FLOW GRAPHS:

The method of signal flow graphs enables calculations of the properties of complicated microwave networks to be performed simply and systematically. It is of particular importance when dealing with networks connected in cascade, although this will not be dealt with in this procedure. In essence, it consists of representing complex wave amplitudes by points in a diagram, and scattering parameters of the networks by directed lines connecting these points. By applying certain rules it is then possible to simplify the diagrams of complicated circuits and deduce their overall properties.

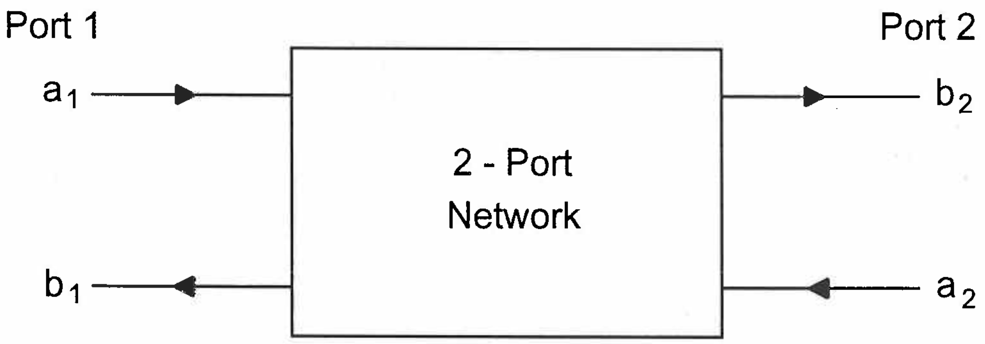


Fig 1: 2 Port Network

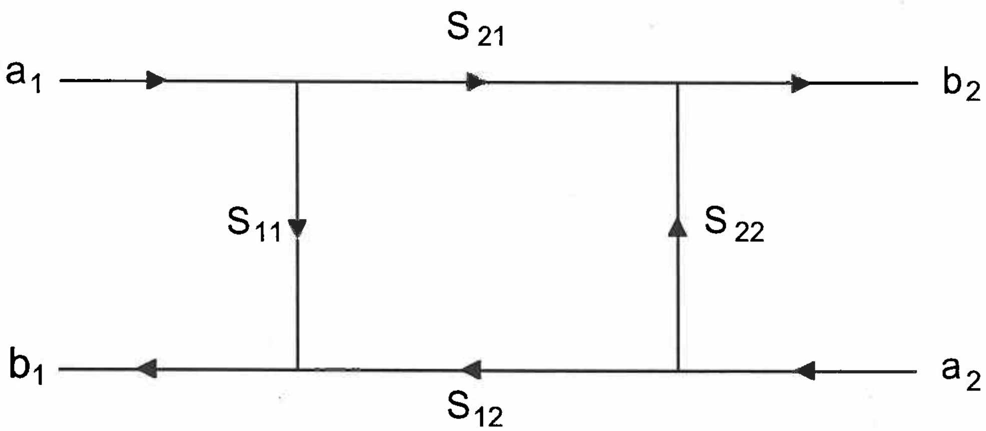


Fig 2: Scattering parameter signal flowgraph

The characteristics of a two port microwave network are now usually specified using S-parameters. In Fig 1, a_1 and a_2 represent complex wave amplitudes entering the system, while b_1 and b_2 represent complex outgoing wave amplitudes. When S-parameters are employed, the relationships between these wave amplitudes are as follows:

$$b_1 = S_{11}.a_1 + S_{12}.a_2 \qquad\qquad b_2 = S_{21}.a_1 + S_{22}.a_2$$

In an ideal situation, when a matched load is connected to port 2, $a_2 = 0$, and when a matched load is connected to port 1, $a_1 = 0$. Thus it follows from the above equations that:

$S_{11} = b_1/a_1$ *when port 2 is perfectly matched*

$S_{12} = b_1/a_2$ *when port 1 is perfectly matched*

$S_{21} = b_1/a_1$ *when port 2 is perfectly matched*

$S_{22} = b_2/a_2$ *when port 1 is perfectly matched*

In a signal flow graph, complex wave amplitudes such as a_1, a_2, b_1 and b_2 are represented by points, or nodes, and the S-parameters are represented by directed lines. Fig 2 illustrates the signal flow graph for the two port network of Fig 1. The arrow directions are from the independent variables (a_1 or a_2) to the dependant variables (b_1 or b_2). The value of a node is the sum of all signals entering it, each signal being the value of the node from which it comes multiplied by the path coefficient.

When several microwave networks are cascaded, the signal flow graphs of the individual networks can be joined together and the ratio between any two complex wave amplitudes can then be written down straight away using Mason's non-touching loop rule.

In a signal flow graph, the following rules apply:

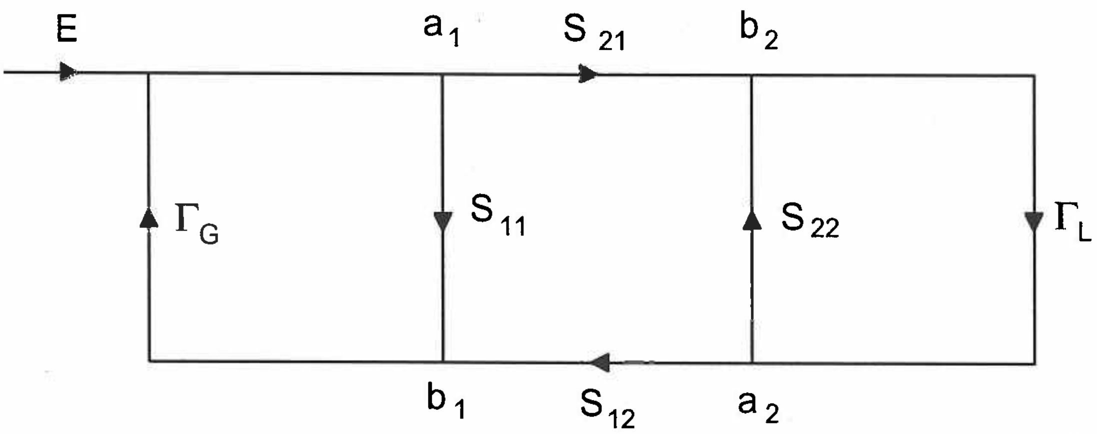


Fig 4: Example signal flowgraph

RULE 1: The value of a path is the product of all path coefficients encountered en route. A path must always follow the directions of the arrows and no node may be passed more than once. In Fig 4, the value of one path from a_1 to b_1 is equal to S_{11}, and the value of the second path from a_1 to b_1 is equal to $S_{21}\,\Gamma_L\,S_{12}$.

RULE 2: A first order loop is a closed path which can be followed, always in the direction of the arrows, without passing any node more than once. Its value is the product of all path coefficients encountered en route. In Fig 4, there are at least three first order loops. Their values are seen to be:

$$\Gamma_G S_{11}, \qquad \Gamma_L S_{22}, \qquad \Gamma_G S_{21} \Gamma_L S_{12}$$

RULE 3: A second order loop is the product of the values of any two first order loops which do not touch at any point. In Fig 4, there is one second order loop whose value is:

$$\Gamma_G S_{11} S_{22} \Gamma_L$$

RULE 4: A third order loop is the product of any three non-touching first order loops etc.

RULE 5: The ratio T of the complex wave amplitude at point Y to that at an independant point X is given by Mason's non-touching rule:

$$T = \frac{P1(1 - \Sigma\alpha L1 + \Sigma\alpha L2 - ...) + P2(1 - \Sigma\beta L1 + \Sigma\beta L2 + ...) + ...}{(1 - \Sigma L1 + \Sigma L2 - ...)}$$

where:

$P1$	is one path from X to Y
$P2$	is a different path from X to Y, etc.
$\Sigma L1$	is the sum of all first order loops
$\Sigma L2$	is the sum of all second order loops, etc.
$\Sigma\alpha L1$	is the sum of all first order loops not touching P1
$\Sigma\alpha L2$	is the sum of all second order loops not touching P1
$\Sigma\beta L1$	is the sum of all first order loops not touching P2
$\Sigma\beta L2$	is the sum of all second order loops not touching P2

etc.

SIGNAL FLOW GRAPHS FOR ANA (TWO PORT) MEASUREMENTS

It can be shown that the residual errors in the test are mathematically related to the actual S-parameter and measured S-parameter using the following terms:

Parameter	Forward Path	Reverse Path
Directivity	E_{DF}	E_{DR}
Isolation	E_{XF}	E_{XR}
Source Match	E_{SF}	E_{SR}
Load Match	E_{LF}	E_{LR}
Transmission Tracking	E_{TF}	E_{TR}
Reflection Tracking	E_{RF}	E_{RR}

Note: The Source Match of one Path is the Load Match of the other and vice versa.

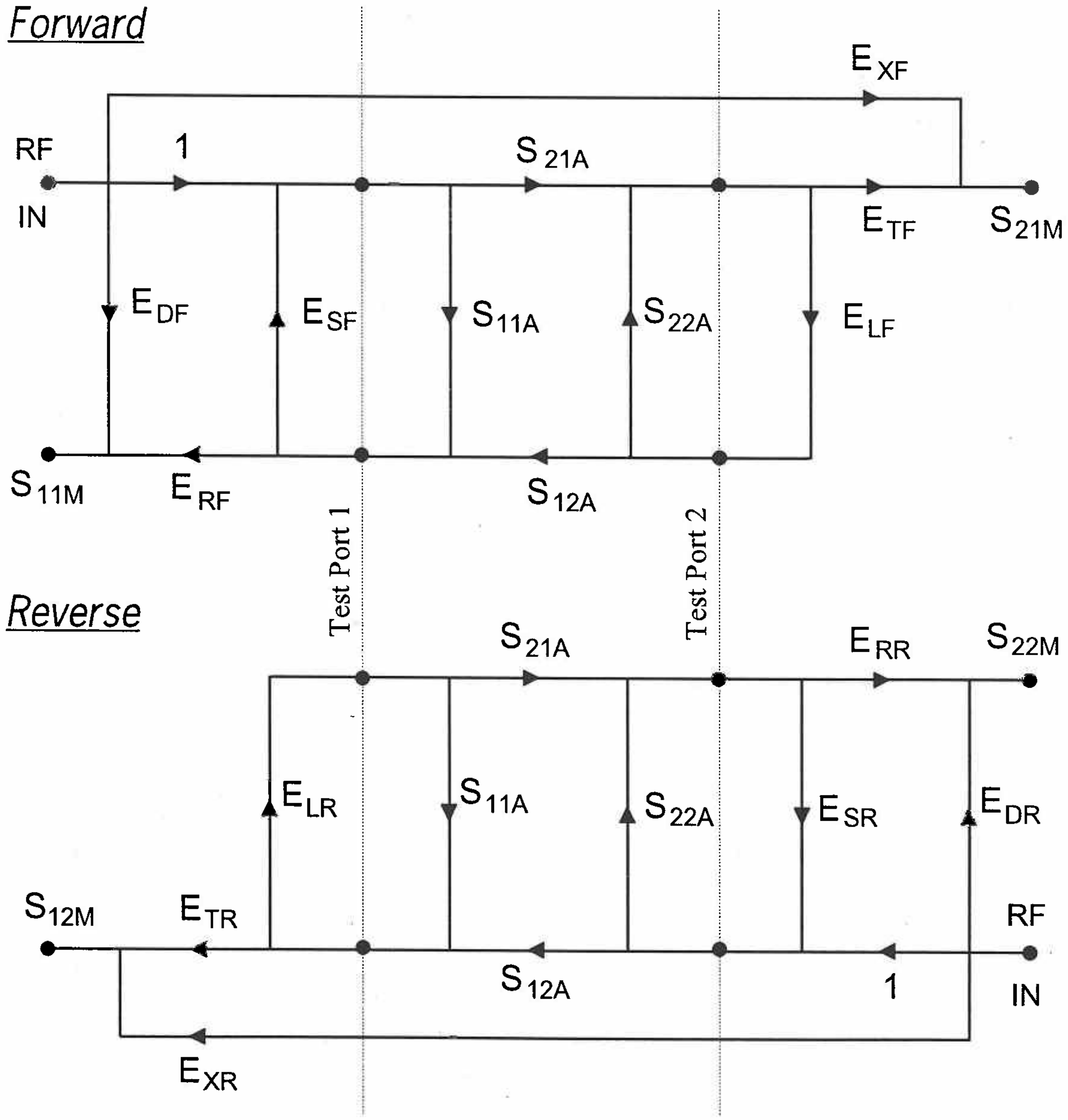


Fig 4: Signal flow graphs, illustrating the forward and reverse paths

TRANSMISSION MEASUREMENTS

Consider an S_{21} measurement, and apply Mason's non-touching rule to the *FORWARD* signal flow graph:

Available paths:	E_{TF}, $E_{TF}\, S_{21A}$
First Order Loops:	$S_{22A}\, E_{LF}$, $E_{SF}\, S_{11A}$, $S_{21A}\, E_{LF}\, S_{12A}\, E_{SF}$
Second Order Loops:	$E_{SF}\, S_{11A}\, S_{22A}\, E_{LF}$

$$S_{21M} = E_{XF} + \frac{E_{TF}\, S_{21A}}{1 - E_{SF}\, S_{11A} - E_{LF}\, S_{22A} - E_{LF}\, E_{SF}\, S_{21A}\, S_{12A} + E_{SF}\, E_{LF}\, S_{11A}\, S_{22A}}$$

The above equation is the general expression for two port measurements relating the measured S-parameter (S_{21M}) to the actual S-parameter (S_{21A}).

REFLECTION MEASUREMENTS

Consider an S_{11} measurement, and apply Mason's non-touching rule to the *FORWARD* signal flow graph:

Available Paths:	E_{DF}, $E_{RF}\, S_{11A}$, $S_{21A}\, S_{12A}\, E_{LF}\, E_{RF}$
First Order Loops:	$E_{SF}\, S_{11A}$, $E_{SF}\, E_{LF}\, S_{21A}\, S_{12A}$, $S_{22A}\, E_{LF}$
Second Order Loops:	$E_{LF}\, E_{SF}\, S_{11A}\, S_{22A}$

$$S_{11M} = E_{DF} + \frac{S_{11A}\, E_{RF} + E_{LF}\, E_{RF}\, S_{11A}\, S_{22A} + S_{21A}\, E_{LF}\, S_{12A}\, E_{RF}}{1 - E_{SF}\, S_{11A} + E_{SF}\, E_{LF}\, S_{21A}\, S_{12A} - S_{22A}\, E_{LF} - E_{LF}\, E_{SF}\, S_{11A}\, S_{22A}}$$

The above equation is the general expression for two port measurements relating the measured S-parameter (S_{11M}) to the actual S-parameter (S_{11A}).

Similar equations can be developed for the remaining S-parameters, S_{12M} and S_{22M}.

FORMULÆ USED IN ASSESSING MEASUREMENT UNCERTAINTIES

(HEWLETT - PACKARD METHOD)

TRANSMISSION MEASUREMENTS

The systematic contribution of the measurement uncertainty is determined by evaluating the following expression, where it has been assumed that all phase relationships are possible (since all quantities are complex).

$$U_{S21M} = [(S_{21A} - S_{21M})^2 + U_R^2 + U_C^2 + U_N^2]^{½}$$

The above equation is the modified expression for two port measurements relating the measured S-parameter (S_{21M}) to the actual S-parameter (S_{21A}), assuming that the worst case phase relationships will occur.

REFLECTION MEASUREMENTS

The systematic contribution of the measurement uncertainty is determined by evaluating the following expression, where it has been assumed that all phase relationships are possible (since all quantities are complex).

$$U_{S11M} = [(S_{11A} - S_{11M})^2 + U_R^2 + U_C^2 + U_N^2]^{½}$$

The above equation is the modified expression for two port measurements relating the measured S-parameter (S_{11M}) to the actual S-parameter (S_{11A}), assuming that the worst case phase relationships will occur.

The following variables are employed in the uncertainty expressions above:

U_{S11M}	=	Reflection Uncertainty
U_{S21M}	=	Transmission Loss Uncertainty
U_R	=	System Repeatability
U_C	=	Connector Repeatability
U_N	=	Noise (assumed negligable in reflection measurements)

Note that the formulæ are not necessarily derived in accordance with "*The ISO Guide to the Expression of Uncertainty in Measurement*" or "*NIS 3003 Edition 8*". Values of uncertainty may change slightly if these new techniques are employed.

FORMULÆ USED IN ASSESSING MEASUREMENT UNCERTAINTIES

(DRAFT E.A.L. METHOD)

TRANSMISSION MEASUREMENTS

Uncertainty formulæ from the "*Draft EAL Procedure for the Assessment of Vector Network Analysers (VNA)*" are in accordance with "*The ISO Guide to the Expression of Uncertainty in Measurement*" and "*NIS 3003 Edition 8*". Residual error terms are assessed using similar techniques to Hewlett-Packard. The uncertainty due to mismatch loss (M_{TM}) is shown as:

$$M_{TM} = 20 \times \log_{10} \frac{1 \pm (|MS_{11}| + |\Gamma_L S_{22}| + |M\Gamma_L S_{11} S_{22}| + |M\Gamma_L S_{12} S_{21}|)}{1 \pm |M| \times |\Gamma_L|}$$

The standard uncertainty (S_{US21}) for transmission measurements $(S_{21}$ or $S_{12})$ is shown as:

$$S_{US21}^{\,2} = \left[\frac{L}{2}\right]^2 + \left[\frac{M_{TM}}{\sqrt{3}}\right]^2 + \left[\frac{I}{\sqrt{3}}\right]^2 + \left[\frac{R_{dB}}{1}\right]^2$$

The uncertainty (U_{S21}) for transmission measurements $(S_{21}$ or $S_{12})$ is therefore shown as:

$$U_{S21} = 2 \times S_{US21}$$

REFLECTION MEASUREMENTS

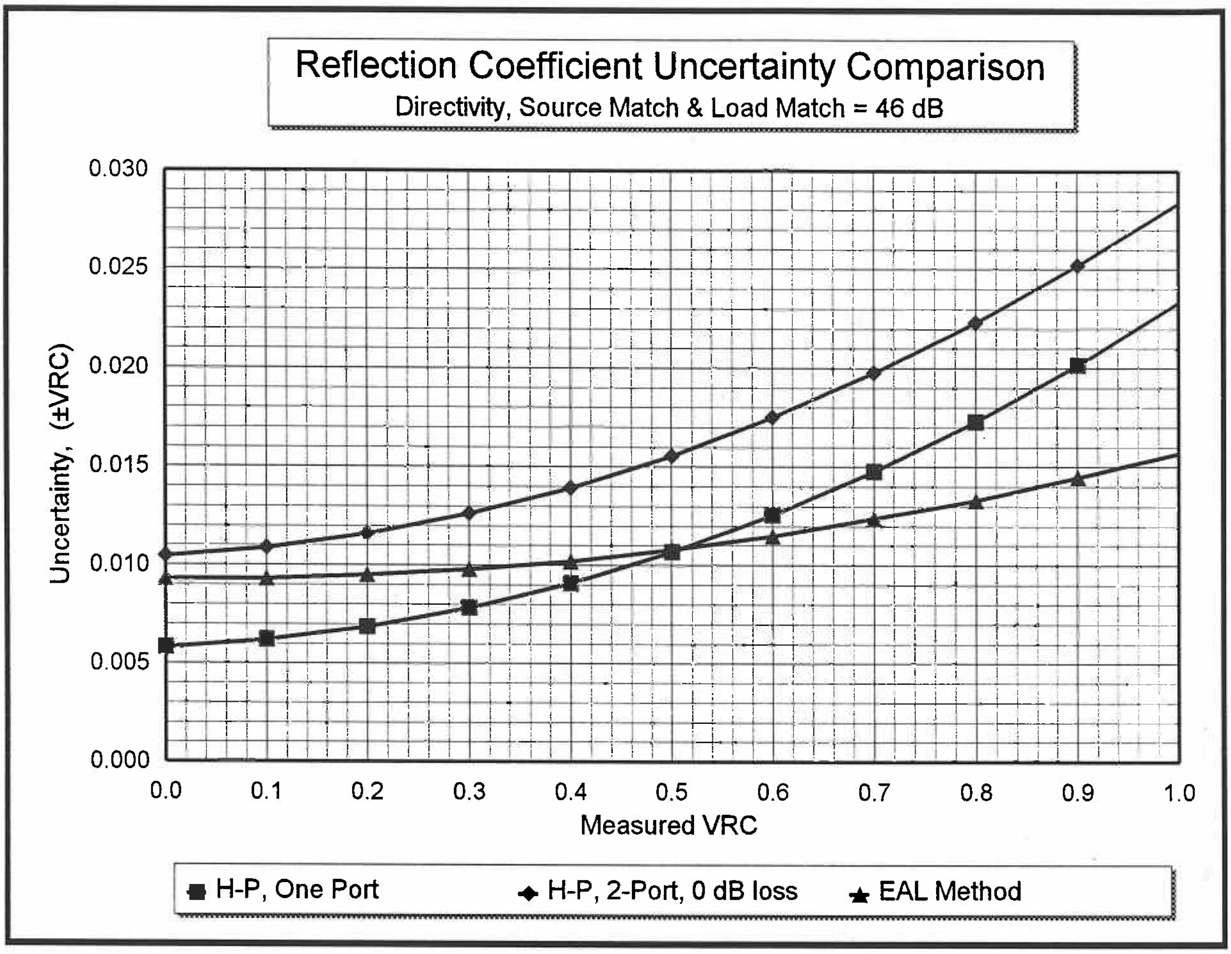
For measurements of S_{11} and S_{22} the error model for a VNA can be represented using only the major error terms as follows:

$$S_{US11}^{\,2} = \left[\frac{D}{\sqrt{2}} + \frac{M \times \Gamma^2}{\sqrt{2}}\right]^2 + \left[\frac{T \times \Gamma}{\sqrt{3}}\right]^2 + \left[\frac{R_{VRC}}{1}\right]^2$$

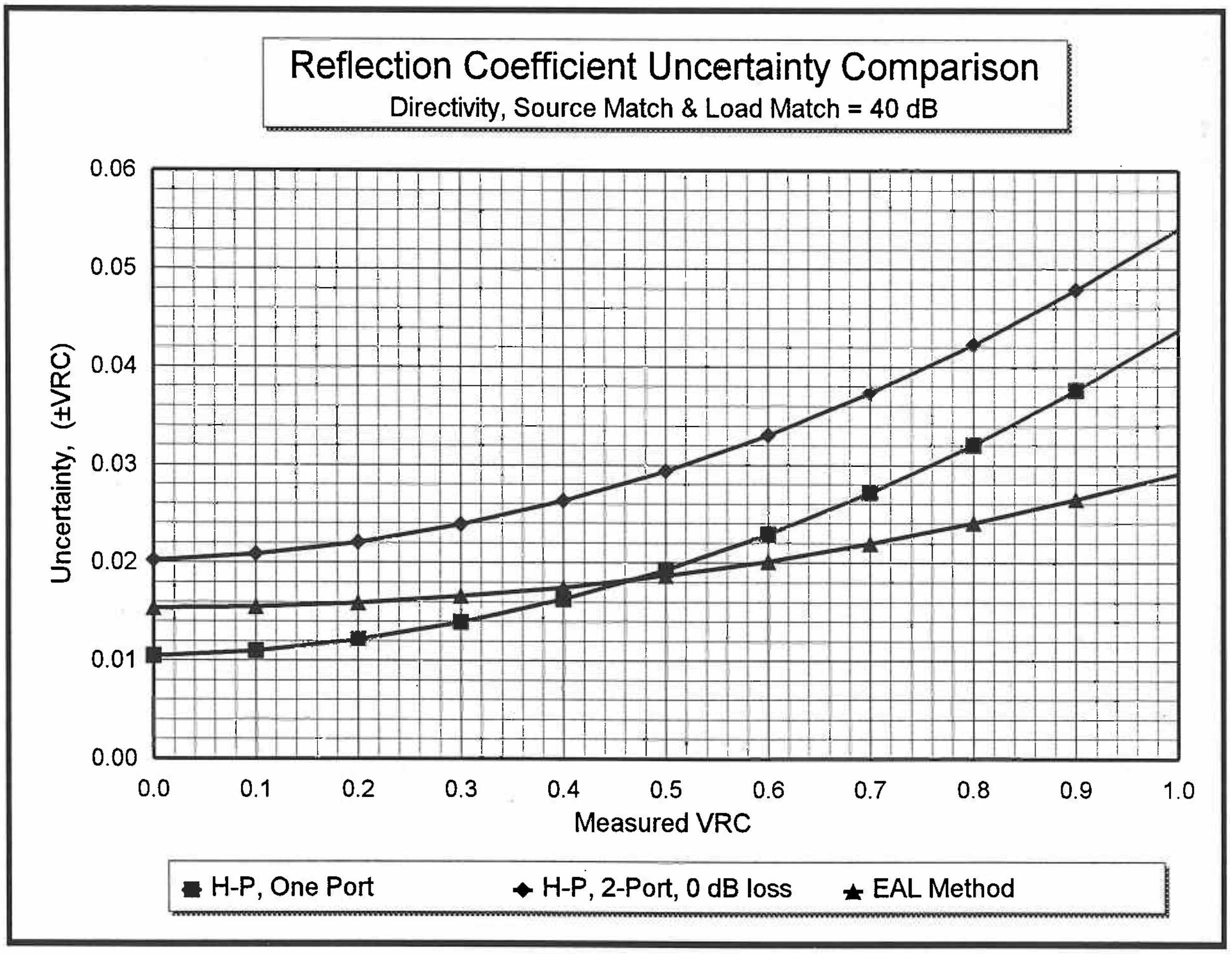
The uncertainty (U_{S11}) for transmission measurements $(S_{11}$ or $S_{22})$ is therefore shown as:

$$U_{S11} = 2 \times S_{US11}$$

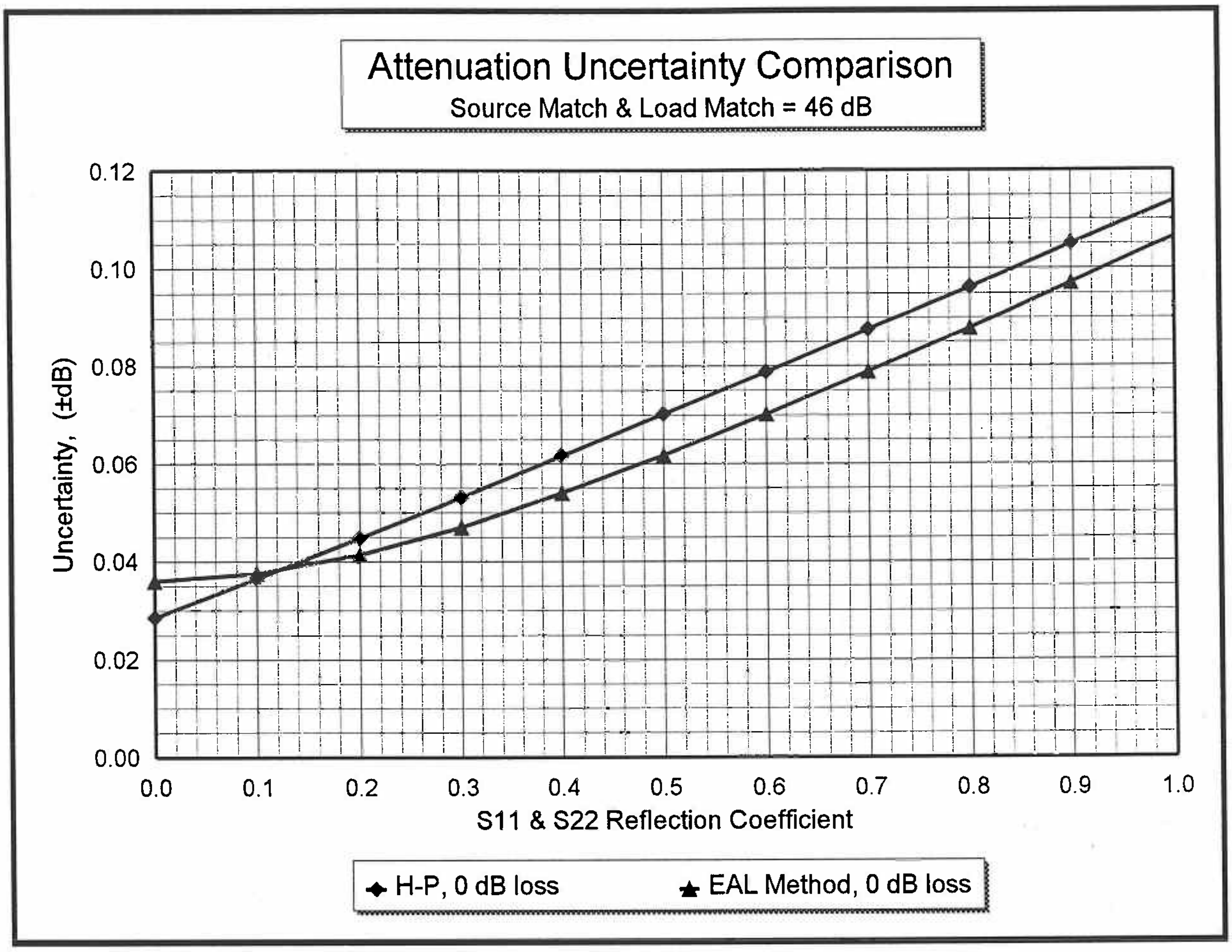
Where:	L	=	Measured System Linearity
	D	=	Effective Directivity
	Γ	=	Reflection Coefficient of the Device Under Test
	M_{TM}	=	the calculated Mismatch
	I	=	the estimated or measured Isolation
	R_{VRC}, R_{dB}	=	represents all of the Random Effects
	M	=	Effective Test Port Match
	Γ_L	=	Effective Load Reflection Coefficient
	S_{11}, S_{22}, S_{21}, S_{12}	=	Scattering Coefficients of the device being measured



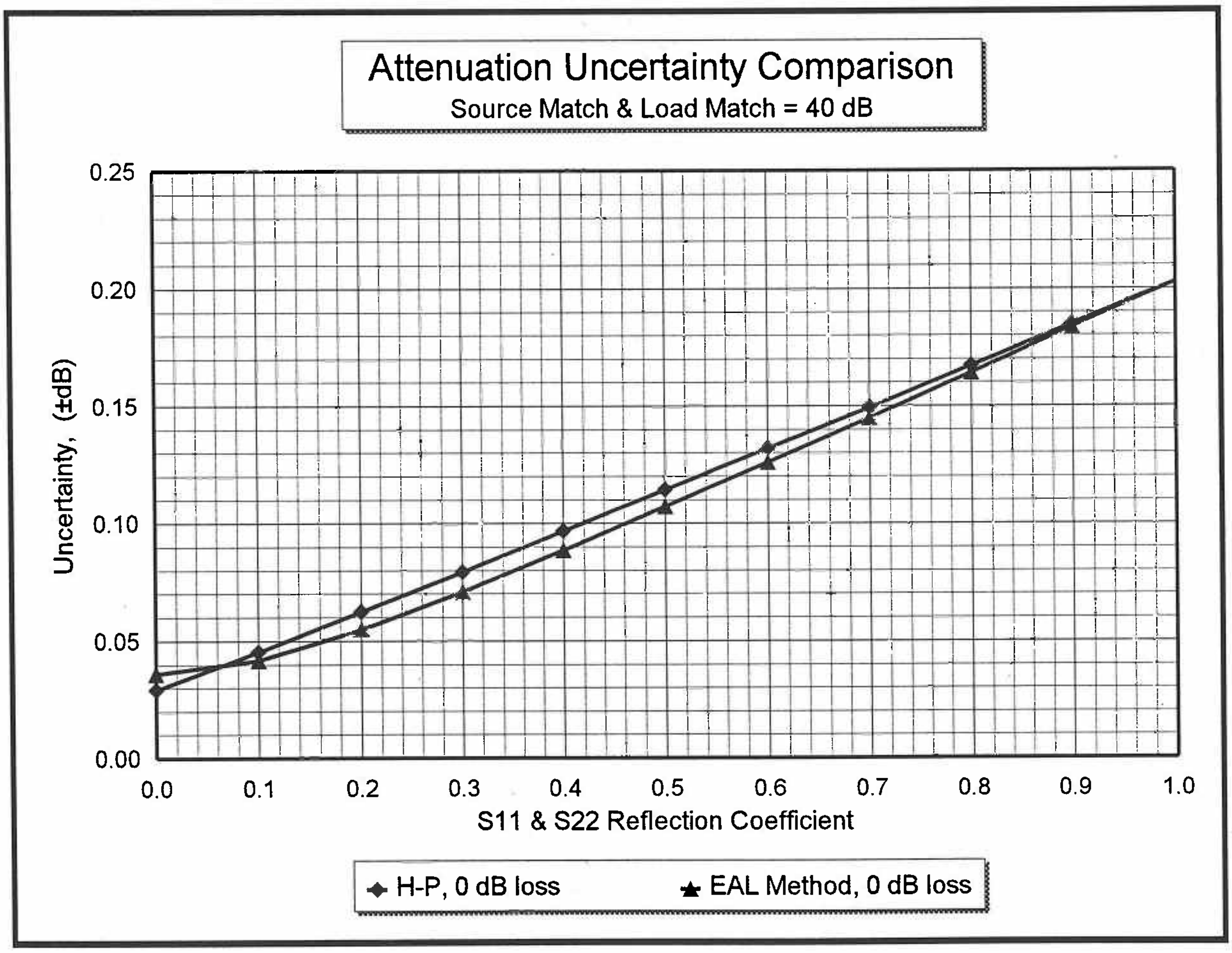
Reflection Coefficient Uncertainty Comparison
Directivity, Source Match & Load Match = 46 dB
Uncertainty, (±VRC)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Measured VRC
H-P, One Port
H-P, 2-Port, 0 dB loss
EAL Method



Reflection Coefficient Uncertainty Comparison
Directivity, Source Match & Load Match = 40 dB
Uncertainty, (±VRC)
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Measured VRC
H-P, One Port
H-P, 2-Port, 0 dB loss
EAL Method



Attenuation Uncertainty Comparison
Source Match & Load Match = 46 dB
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Uncertainty, (±dB)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
S11 & S22 Reflection Coefficient
H-P, 0 dB loss
EAL Method, 0 dB loss



Attenuation Uncertainty Comparison
Source Match & Load Match = 40 dB
0.25
0.20
0.15
0.10
0.05
0.00
Uncertainty, (±dB)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
S11 & S22 Reflection Coefficient
H-P, 0 dB loss
EAL Method, 0 dB loss