

# ANAMET REPORT

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The ANAMET Data Analysis Game

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# THE ANAMET DATA ANALYSIS GAME

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## Abstract

*This report describes a data analysis "game" involving the visual interpretation of several sets of two-dimensional (complex) data, each exhibiting different characteristics. The data sets are typical of those produced by ANAMET measurement comparisons. The purpose of the game is to raise the level of awareness and appreciation of some of the difficulties which can be experienced by data analysts in such a measurement comparison and by microwave metrologists in general when dealing with multiple measurements of complex (vector) quantities.*

*To date, the game has been played twice in public. The results of the game on each of these occasions are presented in this report, together with a short discussion. The results are used to assess the correspondence between averaging by eye and analytical statistical averaging techniques.*

## INTRODUCTION

This report describes a data analysis "game" in which participants assess visually which of a number of options provide satisfactory average values for each of three data sets. Each option is derived using a different statistical technique and each data set exhibits different characteristics. The term average is used here to denote a summary value which is representative of the data set as a whole.

The game has been played twice in public: firstly during the 6th ANAMET meeting, which took place at SESC, DRA Aquila, Bromley, on the 23rd May 1996; and secondly at the 25th Automated RF & Microwave Measurements Society (ARMMS) conference, which took place at the Limpley Stoke Hotel, Bath, on the 31st October and 1st November 1996. The results produced by the game on these two occasions are given, and discussed, in this report. Finally, these results are used to assess any likely correspondence between averaging by eye and analytical statistical techniques.

## THE GAME - HOW IT WAS PLAYED

The game was played in open forum with members raising their hands to register their votes. The idea of the game was for the players to act as data analysts by visually inspecting three sets of data, presented in two-dimensional form as points in the complex plane (as in the polar

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<sup>1</sup> Assessment Services, working under contract to NPL, % DRA Malvern, UK.

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presentations available on vector network analysers). Each participant was given a set of data plots, as found in Appendices I - IV, and a sheet with the following instructions:

*In this "game" there are three sets of measurement comparison data points.*

*Each data point represents the measurement of a complex (two-dimensional) parameter.*

*For each data set, a summary value is required which is representative of the data set and against which individual data points can be compared.*

*Which of the three options provide suitable summary values for each data set?*

For each of the three data sets, the participants voted as to the suitability of each option as a summary value by raising their hands. Participants were allowed to vote for more than one option if they considered more than one option to be a suitable summary value. The number of votes for each option were recorded in a table.

### THE MEASUREMENT DATA SETS

Appendix I contains plots of the three data sets used for the game. Each data point is represented by a black dot. Appendix II shows data set 1 with the three alternative average values (options 1A, 1B and 1C), Appendix III shows data set 2 with the three alternative average values (options 2A, 2B and 2C), and Appendix IV shows data set 3 with the three alternative average values (options 3A, 3B and 3C). The averages are represented on the plots by a ×.

**Note:-** *The reader wishing to play this game for themselves should at this point turn to the Appendices and record their votes by ticking the appropriate boxes in table 1, below. This having been done, the reader can compare their votes with those obtained by the players at the ANAMET and ARMMS meetings which are given in the next section of this report. This is followed by a description of the statistical techniques used to provide the averaging candidates.*

	Tick (✓) box to indicate an acceptable average value		
	Option A	Option B	Option C
Data set 1			
Data set 2			
Data set 3			

*Table 1: Table to record the reader's choices when playing the game.*

## THE RESULTS OF THE GAMES

Table 2 shows the results produced by the game when it was played for the first time, at the 6th ANAMET meeting. Table 3 shows the results produced when the game was played for a second time, at the 25th ARMMS meeting.

	Number of votes		
	Option A	Option B	Option C
Data set 1	12	16	12
Data set 2	4	0	22
Data set 3	5	7	15

*Table 2: Game results produced at the ANAMET meeting. (22 players.)*

	Number of votes		
	Option A	Option B	Option C
Data set 1	10	12	13
Data set 2	4	0	19
Data set 3	3	2	16

*Table 3: Game results produced at the ARMMS meeting. (22 players.)*

## THE IDENTITY OF THE STATISTICAL AVERAGING TECHNIQUES

With the voting completed, the identities of the three different statistical averaging techniques used were revealed to the participants. We do likewise, below.

For a series of  $n$  complex data points,  $z_i$  (where  $i = 1, \dots, n$ ), the three options were calculated as follows:

**Option A:** Calculate the arithmetic mean of the real components of the complex data points and the arithmetic mean of the imaginary components of the complex data points. *I.e.*,

(i) calculate the mean of the real components,  $\overline{Re(z)}$ ;

$$\overline{Re(z)} = \frac{1}{n} \sum_{i=1}^n Re(z_i)$$

(ii) calculate the mean of the imaginary components,  $\overline{Im(z)}$ ;

$$\overline{Im(z)} = \frac{1}{n} \sum_{i=1}^n Im(z_i)$$

Option A is the point,  $\bar{z}$ , given by;

$$\bar{z} = \overline{Re(z)} + j \overline{Im(z)}$$

where  $j = \sqrt{-1}$ .

**Option B:** Calculate the arithmetic mean of the magnitude values of the complex data points and the arithmetic mean of the phase values of the complex data points. *I.e.*,

(i) calculate the mean of the magnitude values;

$$\overline{|z|} = \frac{1}{n} \sum_{i=1}^n |z_i|$$

(ii) calculate the mean of the phase values;

$$\overline{\phi(z)} = \frac{1}{n} \sum_{i=1}^n \phi(z_i)$$

Option B is the point described by these two values, having a magnitude of  $\overline{|z|}$  and a phase of  $\overline{\phi(z)}$ .

**Option C:** Option C is the spatial median [1] of the complex data points. The spatial median of the data set is that point in the complex plane for which the sum of the distances between it and the points of the data set is a minimum. *I.e.*, it minimises:

$$\sum_{i=1}^n |z_i - \mu|$$

over all possible choices of  $\mu$ .

## OBSERVATIONS AND DISCUSSION

The results obtained (*i.e.*, the votes cast) both times the game was played show very similar trends. Although a small number of participants played in both events, the majority of players were first-timers. This adds credence to the results obtained by the game demonstrating a degree of independence and reproducibility.

### Data set 1

For the first data set, all three statistical averages were found to be acceptable to a high proportion of the players - the number of votes cast for each option being similar. The data set was relatively well-behaved, exhibiting no really unusual features, and so the three estimators provided very similar average values.

### Data set 2

For the second data set, Option C (the spatial median) was by far the most popular average. Option B (mean calculations performed on magnitude and phase components) received *no* votes and Option A (mean calculations performed on real and imaginary components) only four votes.

The data set displayed two interesting features. Firstly, the set contained a value far-removed from the majority of values (a statistical outlier) and, secondly, the data was scattered around the origin of the complex plane (although the axes were not drawn on the data plot). The presence of the outlier adversely affected the average produced by Option A. This is consistent with earlier investigations into the effects of outliers on mean-based calculations [2]. The scatter of the data around the origin of the complex plane adversely affected the average produced by Option B. This effect has been reported elsewhere [3].

The voting for this data set indicates that the spatial median is more likely (compared to the other two options) to provide an acceptable average for data scattered around the origin containing an unusual value. (It should be remembered that the players were not told the identities of the statistical averaging techniques before voting. This helped to prevent any personal preference for a given statistical method biasing the votes.)

### Data set 3

For the third data set, Option C again received the majority of votes from the participants, with a much smaller number voting for Options A and B.

This data set contained *two* unusual values which were in different places in the complex plane. These two values moved both mean-based averages away from a more acceptable position for the average.

Again, the voting for this data set indicates that the spatial median is more likely (compared to the other two options) to provide an acceptable average for data sets containing two unusual values.

## CONCLUSIONS

The ANAMET Data Analysis Game has enabled participants to visually assess the plausibility of proposed averaging techniques for vector data exhibiting different characteristics.

If the participants in these two games can be considered as representative samples drawn from the total population of possible participants (*i.e.*, all living humans)<sup>3</sup>, then for the type of data sets considered, we can say that, when judged by eye, a spatial median average is preferable to the more conventional mean-based averages given in this report. This indicates that the spatial median is closer to the sort of average our instinct tells us is right!

## ACKNOWLEDGEMENTS

The authors would like to thank all those who participated in the game on the two occasions it has been played. In particular, for providing the game with "intelligent eyes" for critical visual data analysis.

## REFERENCES

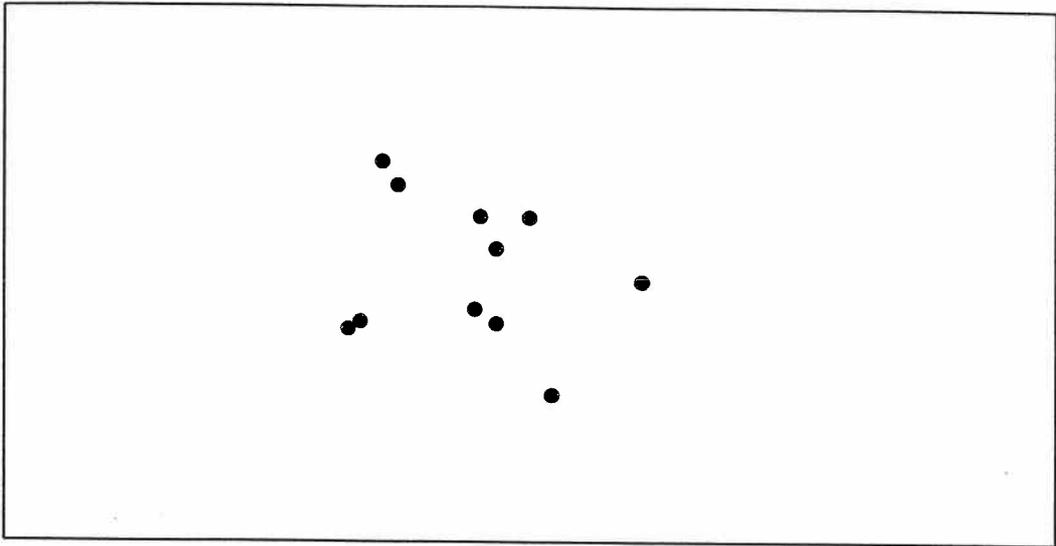
- [1] N M Ridler and J C Medley. Making a good estimate of a vector quantity. *ANALyse* Note Number 15, March 1996.
- [2] N M Ridler. Making a good estimate of a measurand. *ANALyse* Note Number 7, January 1995.
- [3] J P Ide. A comedy of errors *or* That's not what I mean by 'mean'. *ANALyse* Note Number 6, November 1994.

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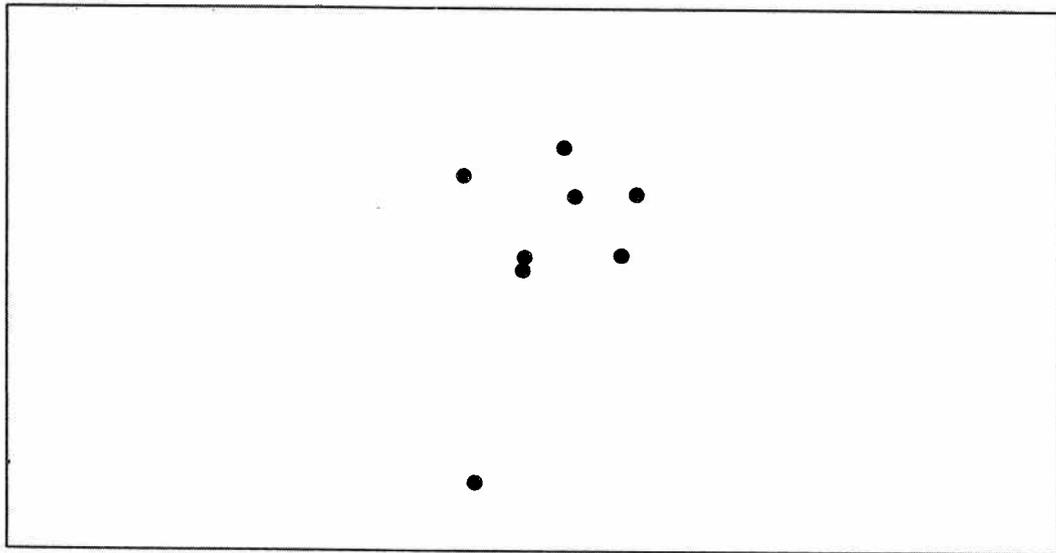
<sup>3</sup> Whether a gathering of microwave engineers/scientists can be considered a representative sample of the human race is an interesting hypothesis which probably would not be endorsed by the rest of our fellow humans!!

## Appendix I:

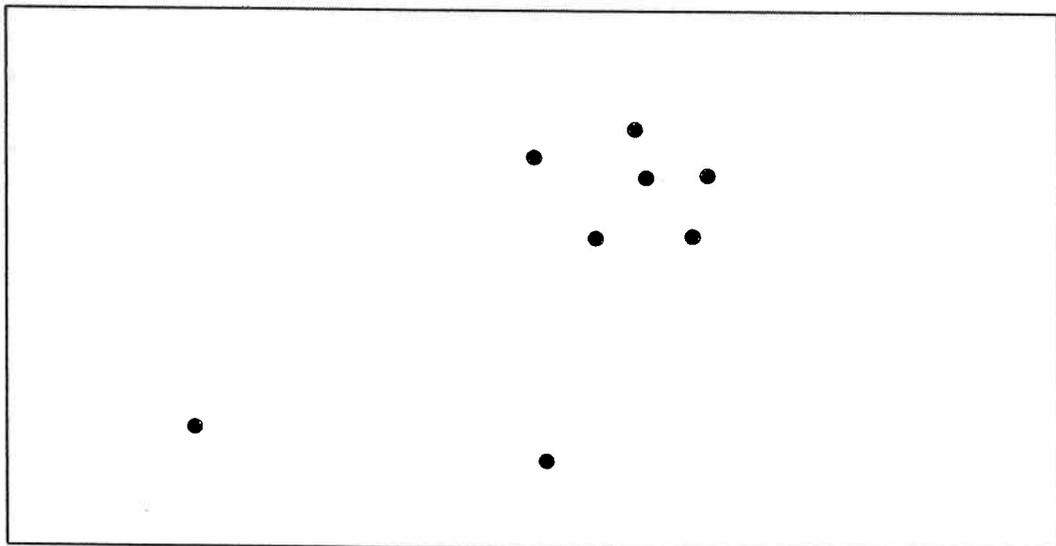
### The three data sets



DATA SET 1



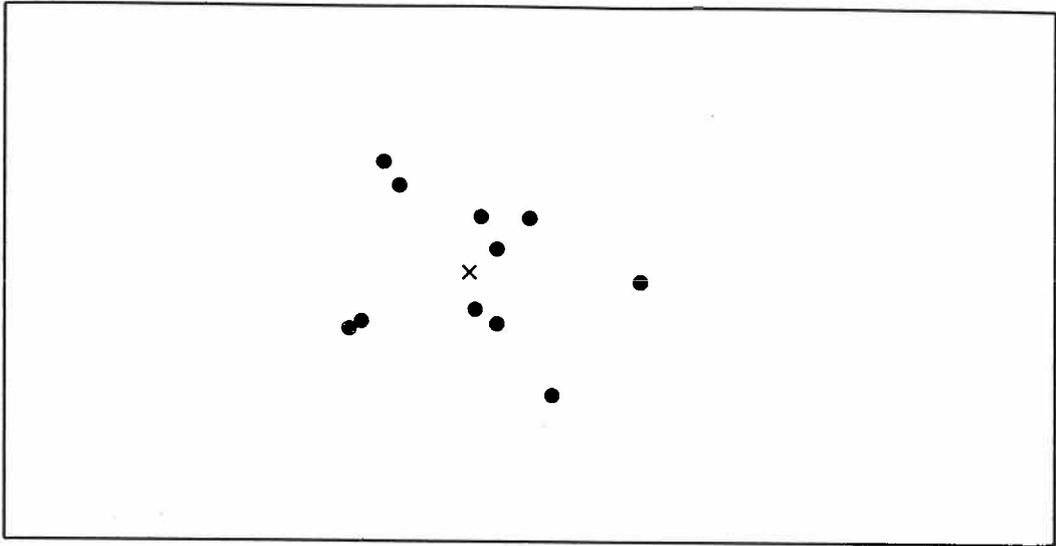
DATA SET 2



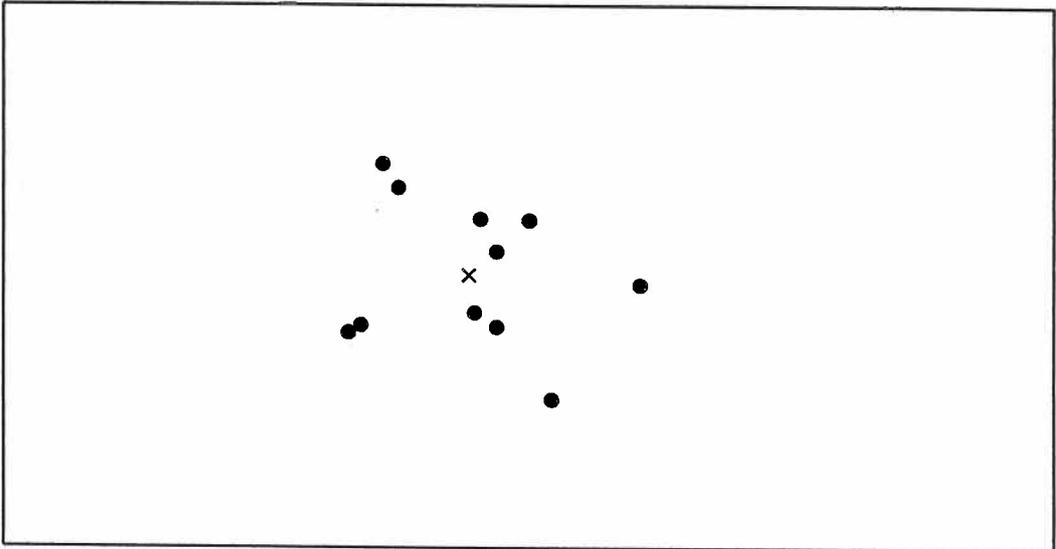
DATA SET 3

## Appendix II:

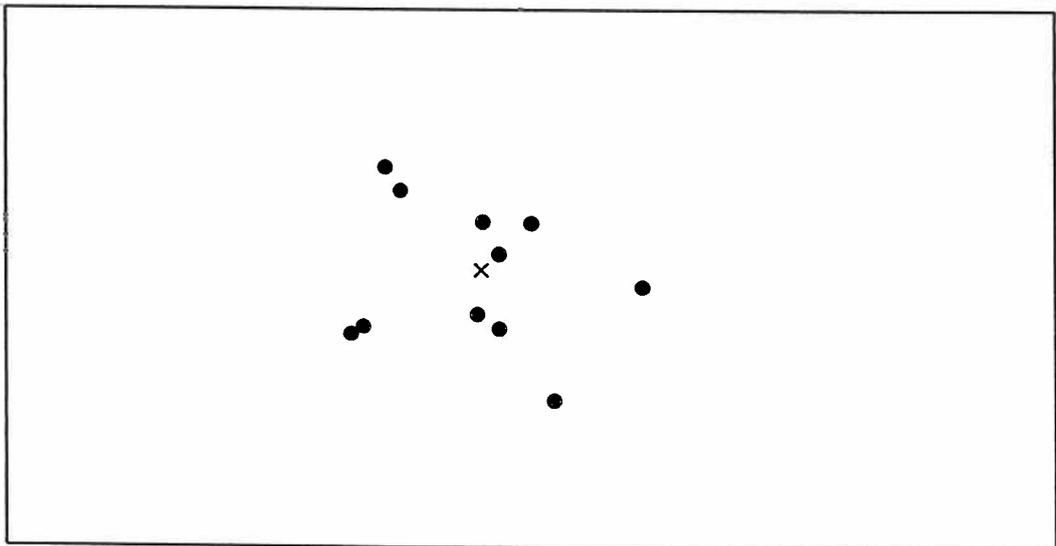
Data set 1 - options A, B and C



OPTION 1A



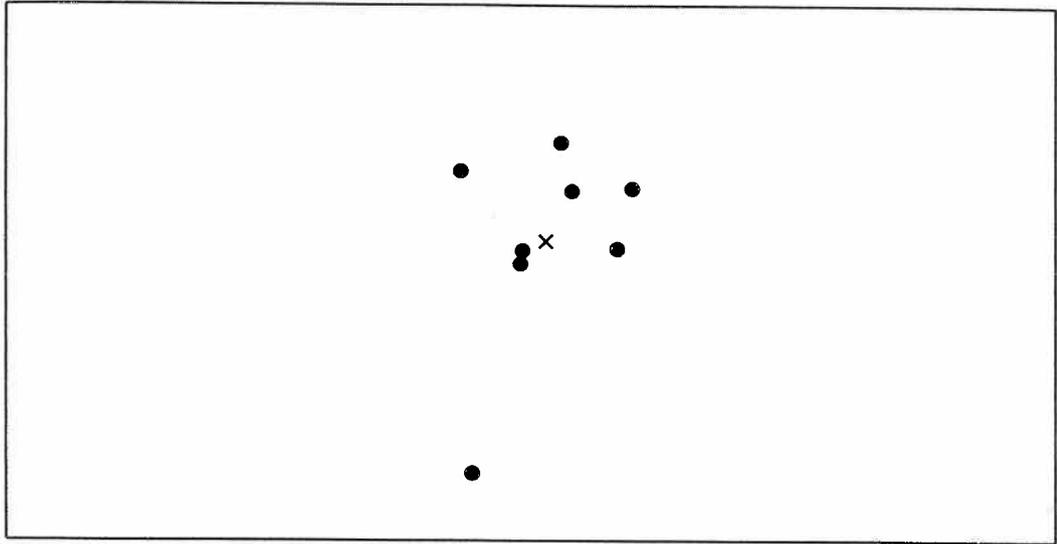
OPTION 1B



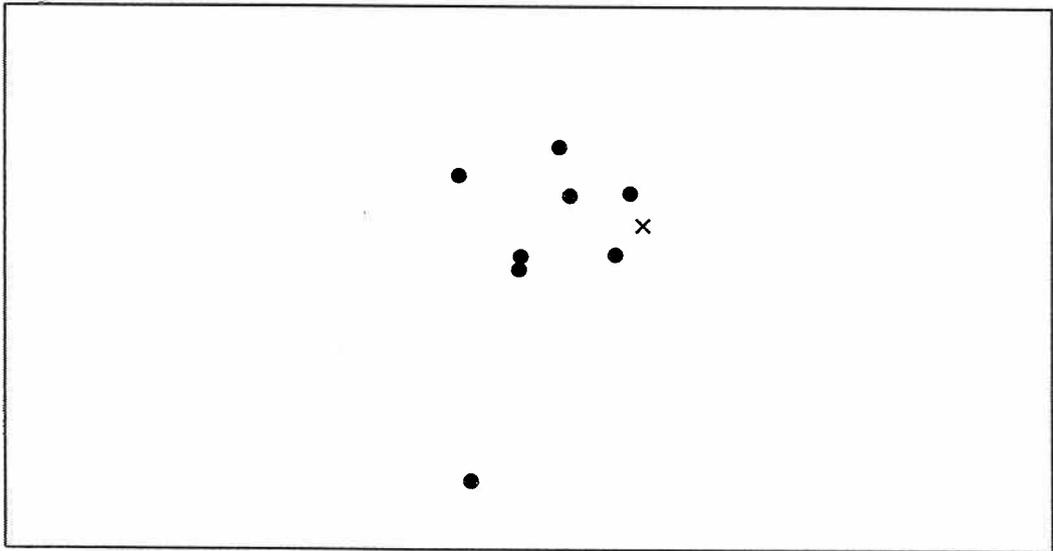
OPTION 1C

## Appendix III:

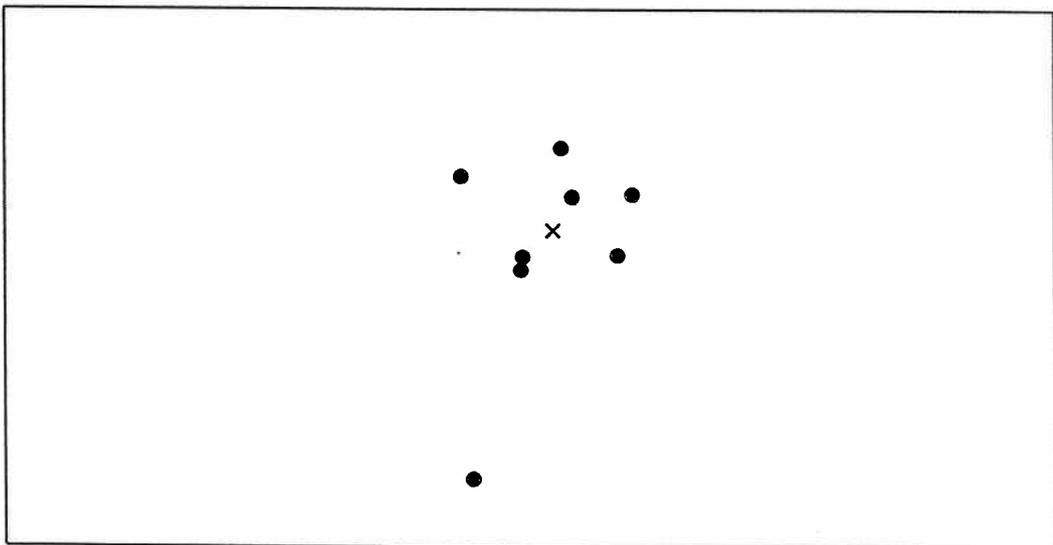
Data set 2 - options A, B and C



OPTION 2A



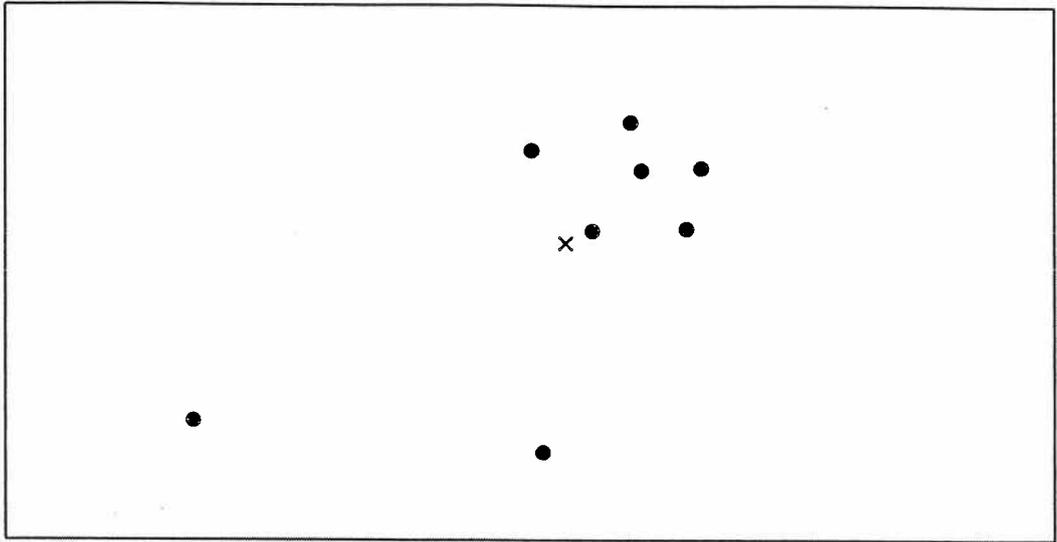
OPTION 2B



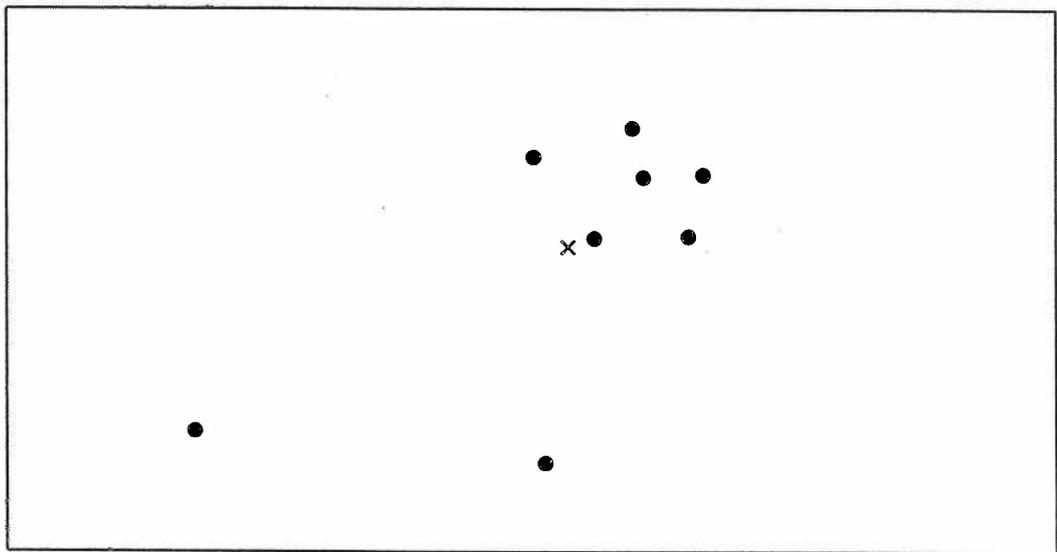
OPTION 2C

Appendix IV:

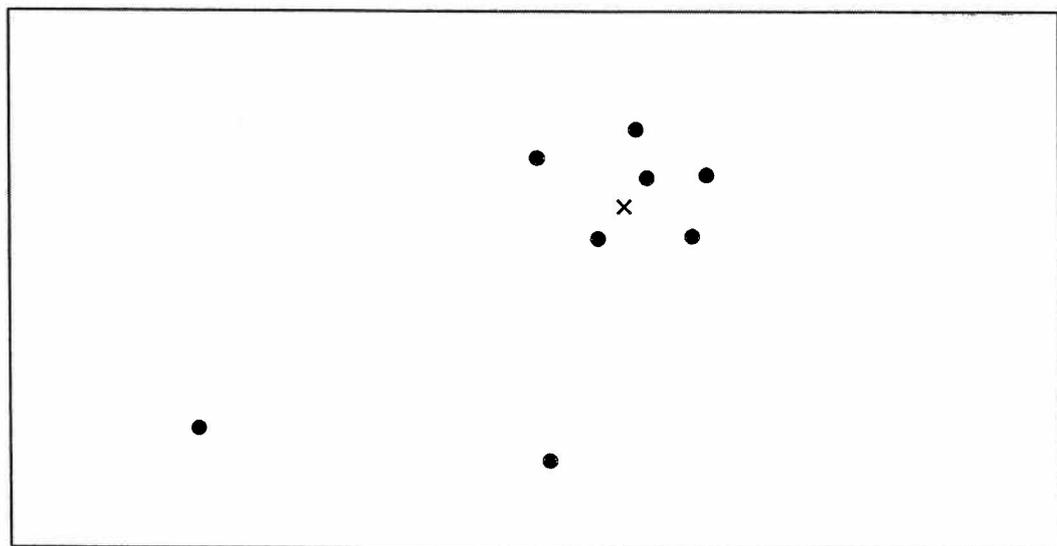
Data set 3 - options A, B and C



OPTION 3A



OPTION 3B



OPTION 3C