ANA *Tips:* No. **9**
May 2003

Converting between VSWR and VRC formats for reflection measurements
Part 2: techniques for high reflecting devices

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Introduction

In ANA_Tip No 8, some simple equations were given for converting between the Voltage Standing Wave Ratio (VSWR) and Voltage Reflection Coefficient (VRC) formats used to express reflection measurements. However, it was pointed out that there are instances when the use of these equations becomes inappropriate. Specifically, when measuring high reflecting devices such as short- and open-circuits. Under these circumstances, the equivalent uncertainty interval in either VSWR or VRC becomes highly asymmetric. This 'tip' shows how to calculate the actual uncertainty interval and gives examples showing when it is, and when it is not, necessary to evaluate the uncertainty interval in this way.

The method

From ANA_tips No 8, the relationship between VSWR, r, and VRC, Γ, can be given as either:

$$r = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (1)$$

or, re-arranged, as:

$$|\Gamma| = \frac{r-1}{r+1} \qquad (2)$$

The method presented here to convert between VSWR and VRC, and vice versa, is a simple one and follows the rules given previously in ANA_tips No 5. For a given value, x, with uncertainty, $u(x)$, (where x can be either VSWR or VRC):

1. Calculate the minimum value, $x_{\min} = x - u(x)$, and the maximum value, $x_{\max} = x + u(x)$.
2. Convert x, $x_{\min}$ and $x_{\max}$ into the equivalent parameter format, by using either equation (1) or (2), as appropriate. We'll call these converted values y, $y_{\min}$ and $y_{\max}$.
3. Calculate the upper and lower uncertainty intervals in y, as $y^{+} = y_{\max} - y$ and $y^{-} = y - y_{\min}$.
4. Express the result as $y \left\{ \pm \begin{matrix} y^{+} \\ y^{-} \end{matrix} \right\}$ or $y \pm u(y)$, if $y^{+} = y^{-}$, which we then call $u(y)$.

To illustrate this approach, we apply the method to two simple examples, and compare the resulting uncertainties with those produced using the techniques given in ANA_tips No 8. Firstly, we apply the method to example 1 from ANA_tips No 8 then we apply both methods to a measurement of a high reflecting device.

Example 1

The measured $|\Gamma|$ of a nominal 2.0 VSWR mismatch termination was found to be 0.3288 ± 0.0078.

1. $|\Gamma|_{\min} = 0.3288 - 0.0078 = 0.3210$, and $|\Gamma|_{\max} = 0.3288 + 0.0078 = 0.3366$.

2. The equivalent VSWR values for $|\Gamma|$, $|\Gamma|_{\min}$ and $|\Gamma|_{\max}$ are found using equation (1): $r = 1.980$; $r_{\min} = 1.946$; $r_{\max} = 2.015$.

3. The upper and lower uncertainty intervals for the VSWR are: $2.015 - 1.980 = 0.035$, and $1.980 - 1.946 = 0.034$.
4. Therefore, neglecting the very small amount of asymmetry in this uncertainty interval (*i.e.* 0.001, being the absolute difference between 0.034 and 0.035), we can say, pragmatically, that:

$$\boxed{r = 1.980 \pm 0.035}$$

This result agrees with the result obtained for this example in ANA_tips No 8.

Example 2

The measured $|\Gamma|$ of a offset short-circuit was found to be 0.9912 ± 0.0072.

1. $|\Gamma|_{\min} = 0.9912 - 0.0072 = 0.9840$, and $|\Gamma|_{\max} = 0.9912 + 0.0072 = 0.9984$.
2. The equivalent VSWR values for $|\Gamma|$, $|\Gamma|_{\min}$ and $|\Gamma|_{\max}$ are found using equation (1): $r = 226$; $r_{\min} = 124$; $r_{\max} = 1249$.
3. The upper and lower uncertainty intervals for the VSWR are: $1249 - 226 = 1023$, and $226 - 124 = 102$.
4. Here, we are unable to neglect the very large amount of asymmetry in the uncertainty interval (*i.e.* 921, being the absolute difference between 1023 and 102), so we need to say:

$$\boxed{r = 226\left\{\pm\begin{matrix}1023\\102\end{matrix}\right\}}$$

However, if we use the formula given in ANA_tips No 8 (equation 5) to find the uncertainty in VSWR, we obtain the following result:

$$\boxed{r = 226 \pm 186}$$

Comments

The above result clearly disagrees with the result obtained using the method presented in this ANA_tips Note. The reason is that the simple formulae presented in ANA_tips No 8 cannot take account of the asymmetric uncertainties which result when the non-linearity in equations (1) and (2) becomes significant (*i.e.* when the measured VRC is close to unity). It is therefore advisable to use the method presented in this ANA_tips Note when converting results of high reflecting loads.

Also, strictly speaking (as mentioned in ANA_tips No 8), uncertainty statements should be converted from expanded to standard uncertainties *before* propagating uncertainties, and then converted back again afterwards. Again, this can cause problems especially where the non-linearity in the equations becomes significant. To acquire a deeper understanding of what happens under these circumstances, it may be more appropriate to examine what happens to the *distributions* of the measured quantities when they are transformed from one format to another, rather than the uncertainty intervals themselves. Such an approach was proposed in ANAMET Report 033 and this can be extended to deal with other parameter transformations, including the VSWR to VRC transformation described in this Note.