Converting between VSWR and VRC formats for reflection measurements

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In ANA tip No. 4, some simple equations were given for converting between the common logarithmic and linear formats used to express reflection and transmission coefficients. This 'tip' uses a similar approach to convert between the Voltage Standing Wave Ratio (VSWR) and Voltage Reflection Coefficient (VRC) formats used to express reflection measurements.

The relationship between VSWR, r, and VRC, Γ, is as follows:

$$r = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{1}$$

Equation (1) can be re-arranged to give the magnitude of the VRC (|VRC|) in terms of VSWR:

$$|\Gamma| = \frac{r - 1}{r + 1} \tag{2}$$

Now, from the law of propagation of uncertainty,

$$u^2(r) \approx \left(\frac{dr}{d|\Gamma|} \right)^2 u^2(|\Gamma|) \tag{3}$$

where $u(r)$ and $u(|\Gamma|)$ are the uncertainties in the VSWR and |VRC|, respectively. So, from equation (1):

$$\frac{dr}{d|\Gamma|} = \frac{2}{(1 - |\Gamma|)^2} \tag{4}$$

therefore:

$$u(r) \approx \frac{2}{(1 - |\Gamma|)^2} u(|\Gamma|) \tag{5}$$

and, using equation (2):

$$u(|\Gamma|) \approx \frac{2}{(r + 1)^2} u(r) \tag{6}$$

It is useful to note that when $|\Gamma| \rightarrow 0$, $r \rightarrow 1$, so equations (5) and (6) further simplify to become, respectively:

$$u(r) \approx 2 \times u(|\Gamma|) \tag{7}$$

$$u(|\Gamma|) \approx \frac{u(r)}{2} \tag{8}$$

Equations (1), (2), (5), (6), and sometimes equations (7) and (8), can be used to convert between VSWR and |VRC| representations of reflection measurements. Two examples are given overleaf.

Example 1

The measured $|\Gamma|$ of a nominal 2.0 VSWR mismatch termination was found to be 0.3288 ± 0.0078. Equation (1) is used to find the equivalent measured VSWR:

$$r = \frac{1 + 0.3288}{1 - 0.3288} = 1.980$$

and equation (5) to find the uncertainty in VSWR:

$$u(r) = \frac{2}{(1 - 0.3288)^2} \times 0.0078 = 0.035$$

Therefore, we have:

$$\boxed{r = 1.980 \pm 0.035}$$

Example 2

The measured VSWR of a nominal near-matched load was found to be 1.012 ± 0.011. Equation (2) is used to find the equivalent VRC:

$$|\Gamma| = \frac{1.012 - 1}{1.012 + 1} = 0.0060$$

and equation (6) to find the uncertainty in |VRC|:

$$u(|\Gamma|) \approx \frac{2}{(1.012 + 1)^2} \times 0.011 = 0.0054$$

Therefore, we have:

$$\boxed{|\Gamma| = 0.0060 \pm 0.0054}$$

It should be noted that, on this occasion, since $r \approx 1$ (i.e. $|\Gamma| \approx 0$), so equation (8) can also be used to find the uncertainty in |VRC|:

$$u(|\Gamma|) \approx \frac{0.011}{2} = 0.0055$$

which shows acceptably close agreement with the value determined using equation (6).

Comments

The above equations can be very useful for providing a quick and easy method of converting measurement results from VSWR to VRC, and vice versa. However, there are circumstances when their application becomes inappropriate. For example, if $|\Gamma|$ is close to unity then one of the uncertainty intervals, $u(|\Gamma|)$ or $u(r)$, will become highly asymmetric. This situation can occur when measuring the reflection coefficient of short- or open-circuit terminations.

Also, strictly speaking, uncertainty statements should be converted from expanded uncertainties (*e.g.* at a 95% level of confidence) to standard uncertainties *before* applying the law of propagation of uncertainty, and then converted back again afterwards. Not doing this may cause problems when the uncertainties being treated are relatively large.

Under the above circumstances it may be more appropriate to use alternative techniques. These will be discussed in another ANA_tips Note, to follow shortly!