Converting between logarithmic and linear formats for reflection and transmission coefficients. Part 2: when the uncertainty of measurement is relatively large

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Introduction

In ANA_tips No 4, some simple equations were given for converting between the common logarithmic and linear formats used to express reflection and transmission coefficients (*i.e.* *S*-parameters). However, it was pointed out that there are instances when the use of these equations becomes inappropriate. Specifically, when the size of the uncertainty in the measurement is significant with respect to the size of the magnitude of the *S*-parameter. Under these circumstances, the equivalent uncertainty interval in either the linear or logarithmic quantity becomes asymmetric. This 'tip' shows how to calculate the actual uncertainty interval and gives examples showing when it is, and when it is not, necessary to evaluate the uncertainty interval in this way.

The method

From ANA_tips No 4, we define, for an *n*-port device, the relationship between logarithmic and linear units, as follows:

$$a = -20\log_{10}\left|S_{ij}\right| \qquad (1)$$

or

$$\left|S_{ij}\right| = 10^{-a/20} \qquad (2)$$

where S_{ij} are scattering parameters, with $i = 1, \ldots, n$, and $j = 1, \ldots, n$, and á is the equivalent logarithmic quantity (*e.g.* return loss, attenuation, etc), measured in dB.

The method we present here to convert between logarithmic and linear quantities, and vice versa, is a simple one. For a given value, x, with uncertainty, $u(x)$, (where x can be either the linear or logarithmic quantity):

1. Calculate the minimum value, $x_{min} = x - u(x)$, and the maximum value, $x_{max} = x + u(x)$.
2. Convert x, x_{min} and x_{max} into the equivalent logarithmic or linear format, by using either equation (1) or (2), as appropriate. We'll call these converted values y, y_{min} and y_{max}.
3. Calculate the upper and lower uncertainty intervals in y, as $y^+ = y_{max} - y$ and $y^- = y - y_{min}$.
4. Express the result as $y \left\{\pm \begin{matrix} y^+ \\ y^- \end{matrix}\right\}$ or $y \pm u(y)$, if $y^+ = y^-$, which we then call $u(y)$.

To illustrate this approach, we apply the method to two simple examples, and compare the resulting uncertainties with those produced using the techniques given in ANA_tips No 4. Firstly, we apply the method to example 1 from ANA_tips No 4, then we apply both methods to a situation where the uncertainty of measurement is relatively large compared to the value being measured.

Example 1

The measured $|S_{11}|$ of a nominal 2.0 VSWR mismatch termination was found to be 0.3288 ± 0.0078.

1. $|S_{11}|_{\mathrm{min}}$ = 0.3288 – 0.0078 = 0.3210, and $|S_{11}|_{\mathrm{max}}$ = 0.3288 + 0.0078 = 0.3366.
2. The equivalent logarithmic (*i.e.* return loss) values for $|S_{11}|$, $|S_{11}|_{min}$ and $|S_{11}|_{max}$ are found using equation (1): return loss = 9.66 dB; minimum return loss = 9.46 dB; maximum return loss = 9.87 dB.

3. The upper and lower uncertainty intervals for the return loss are: 9.87 – 9.66 = 0.21dB, and 9.66 - 9.46 = 0.20 dB
4. Therefore, neglecting the very small amount of asymmetry in the uncertainty interval (*i.e.* 0.01 dB, being the absolute difference between 0.21 dB and 0.20 dB), we can say, pragmatically:

$$\boxed{\textit{return loss} = (9.66 \pm 0.21)\ \text{dB}}$$

This result agrees with the result obtained for this example in ANA_tips No 4.

Example 2

The measured $|S_{11}|$ of a nominal near-matched load was found to be 0.0062 ± 0.0054.

1. $|S_{11}|_{\min} = 0.0062 - 0.0054 = 0.0008$, and $|S_{11}|_{\max} = 0.0062 + 0.0054 = 0.0116$.
2. The equivalent logarithmic (*i.e.* return loss) values for $|S_{11}|$, $|S_{11}|_{min}$ and $|S_{11}|_{max}$ are found using equation (1): return loss = 44 dB; minimum return loss = 39 dB; maximum return loss = 62 dB.
3. The upper and lower uncertainty intervals for the return loss are: 62 – 44 = 18 dB, and 44 - 39 = 5 dB
4. Here we are unable to neglect the very large amount of asymmetry in the uncertainty interval (*i.e.* 13 dB, being the absolute difference between 18 dB and 5 dB), so we need to say:

$$\boxed{return\ loss = 44\left\{\pm\begin{matrix}18\\5\end{matrix}\right\}dB}$$

However, if we use the formula given in ANA_tips No 4 (equation 5) to find the uncertainty in return loss, we obtain the following result:

$$\boxed{\textit{return loss} = (44 \pm 8)\ \text{dB}}$$

Comments

The above result clearly disagrees with the result obtained using the method presented in this ANA_tips Note. The reason is that the simple formulae presented in ANA_tips No 4 cannot take account of the asymmetric uncertainties which result when the non-linearity of equations (1) and (2) become significant (*i.e.* when the uncertainty of measurement is relatively large compared to the value being measured). It is therefore advisable to use the method presented in this ANA_tips Note when the uncertainty of measurement is relatively large.

Also, strictly speaking (as mentioned in ANA_tips No 4), uncertainty statements should be converted from expanded to standard uncertainties *before* propagating uncertainties, and then converted back again afterwards. Again, this can cause problems when uncertainties are relatively large (*i.e.* where the non-linearity in the equations becomes significant). To acquire a deeper understanding of what happens under these circumstances, it may be more appropriate to examine what happens to the *distributions* of the measured quantities when they are transformed from logarithmic to linear formats, and vice-versa, rather than the uncertainty intervals themselves. Such considerations will be discussed in another ANAMET publication, to follow shortly!