Converting between logarithmic and linear formats for reflection and transmission coefficients

Nick M. Ridler
National Physical Laboratory

This 'tip' presents some simple equations for converting between the common logarithmic and linear formats which are used to express reflection and transmission coefficients (*i.e.* *S*-parameters). We begin by defining, for an *n*-port device, the relationship between logarithmic and linear units, as follows:

$$\alpha = -20\log_{10}\left|S_{ij}\right| \tag{1}$$

where S_{ij} are scattering parameters, with $i = 1, \ldots, n$, and $j = 1, \ldots, n$, and α is the equivalent logarithmic quantity (*e.g.* return loss, attenuation, etc), measured in dB.

The linear *S*-parameters for reflection coefficient measurements are given when $i = j$. The equivalent logarithmic quantity for the magnitude of any reflection coefficient is return loss. The linear *S*-parameters for transmission coefficient measurements are given when $i \neq j$. The equivalent logarithmic quantity for the magnitude of any transmission coefficient depends on the type of device being measured. For example, this might be attenuation for a two-port device.

Equation (1) can be re-arranged to given the magnitude of the *S*-parameter in terms of the equivalent logarithmic quantity.

$$\left|S_{ij}\right| = 10^{-\alpha/20} \tag{2}$$

Now, from the law of propagation of uncertainty,

$$u^2(\alpha) \approx \left(\frac{d\alpha}{d\left|S_{ij}\right|}\right)^2 u^2(\left|S_{ij}\right|) \tag{3}$$

where $u(\alpha)$ and $u(|S_{ij}|)$ are the uncertainties in the logarithmic and linear quantities, respectively. So, from equation (1):

$$\frac{d\alpha}{d\left|S_{ij}\right|} = -\frac{8.686}{\left|S_{ij}\right|} \tag{4}$$

therefore:

$$u(\alpha) \approx 8.686 \times \frac{u(\left|S_{ij}\right|)}{\left|S_{ij}\right|} \tag{5}$$

and, using equation (2):

$$u(\left|S_{ij}\right|) \approx \frac{1}{8.686} \times 10^{-\alpha/20} \times u(\alpha) \tag{6}$$

Equations (1), (2), (5) and (6) can be used to convert between logarithmic and linear representations of reflection and transmission coefficients. Two examples are given overleaf.

Example 1

The measured $|S_{11}|$ of a nominal 2.0 VSWR mismatch termination was found to be 0.3288 ± 0.0078. Equation (1) is used to find the equivalent return loss:

$$return\ loss = -20\log_{10}(0.3288) = 9.66\ dB$$

and equation (5) to find the uncertainty in return loss:

$$u(return\ loss) \approx 8.686 \times \frac{0.0078}{0.3288} = 0.21\ dB$$

Therefore, we have:

return loss = (9.66 ± 0.21) dB

Example 2

The measured attenuation of a nominal 20 dB pad was found to be (19.848 ± 0.026) dB. Equation (2) is used to find the equivalent $|S_{21}|$:

$$|S_{21}| = 10^{-19.848/20} = 0.10177$$

and equation (6) to find the uncertainty in $|S_{21}|$:

$$u(|S_{21}|) \approx \frac{1}{8.686} \times 10^{-19.848/20} \times 0.026 = 0.00030$$

Therefore, we have:

$\|S_{21}\|$ = 0.10177 ± 0.00030

Clearly, for a reciprocal device, $|S_{12}|$ can be used in place of $|S_{21}|$.

Comments

The above equations can be very useful for providing a quick and easy method of converting measurement results from a linear to a logarithmic format, and vice versa. However, there are circumstances when their application becomes inappropriate. For example, if the size of $u(|S_{ij}|)$ is significant with respect to the size to $|S_{ij}|$ (*e.g.* of the order of 50% or more), then the equivalent uncertainty interval in the logarithmic format actually becomes asymmetric. This situation can occur when measuring the reflection coefficient of a well-matched load at low frequencies ($|S_{11}| \rightarrow 0$).

Also, strictly speaking, uncertainty statements should be converted from expanded uncertainties (*e.g.* at a 95% level of confidence) to standard uncertainties *before* applying the law of propagation of uncertainty, and then converted back again afterwards. Not doing this may cause problems when the uncertainties being treated are relatively large.

Under the above circumstances, it may be more appropriate to use alternative techniques. These will be discussed in another ANA_tips Note, to follow shortly!