Measuring the capacitance of coaxial open-circuits

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• AIM to obtain traceable measurements of capacitance coefficients for coaxial open-circuit VNA cal standards

• Standard model of coaxial open-circuits
• Least squares fitting of model to VRC data
• Propagation of uncertainty from phase of VRC to capacitance coefficients using Monte Carlo
• Typical results for a GPC-7 open-circuit
Model of coaxial open-circuit (1)

\[ \Gamma = \Gamma_0 \exp(-2\gamma l) \]
Model of coaxial open-circuit (2)

For a flush open-circuit:

\[
\Gamma_0 = \frac{Z - Z_0}{Z + Z_0} \quad Z = \frac{1}{j\omega C}
\]

\[
\Gamma_0 = \frac{\frac{1}{j\omega C} - Z_0}{\frac{1}{j\omega C} + Z_0} = \frac{1 - j\omega C Z_0}{1 + j\omega C Z_0}
\]

\[
\Gamma_0 = \frac{\sqrt{1 + (\omega C Z_0)^2} \exp\left(j \tan^{-1}(-\omega C Z_0)\right)}{\sqrt{1 + (\omega C Z_0)^2} \exp\left(j \tan^{-1}(\omega C Z_0)\right)} = \exp\left(-j 2 \tan^{-1}(\omega C Z_0)\right)
\]

Magnitude

\[|\Gamma_0| = 1\]

Phase

\[\phi_0 = -2 \tan^{-1}(\omega C Z_0)\]
Model of coaxial open-circuit (3)

\[ \phi_0 = -2 \tan^{-1}(\omega CZ_0) \]

Approximate frequency dependence of capacitance by a cubic polynomial

\[ C \equiv C(f) \approx C_0 + C_1 f + C_2 f^2 + C_3 f^3 \]

Express phase in degrees

\[ \phi_0(f) = -\frac{360}{\pi} \tan^{-1}\left(2\pi Z_0 \left(C_0 f + C_1 f^2 + C_2 f^3 + C_3 f^4\right)\right) \]
Fitting the model to VRC data (1)

- For a flush open-circuit, model can be written

\[
\tan\left(\frac{\phi_0}{C}\right) = b_0 f + b_1 f^2 + b_2 f^3 + b_3 f^4 = P(f)
\]

\[b_i = 2\pi Z_0 C_i\]

- Measure phase of VRC at \(m\) frequencies \(\{(z_i, y_i) : i = 1, \ldots, m\}\)

- Obtain best fit cubic polynomial to transformed data \(\{(z_i, q_i) : i = 1, \ldots, m\}\)

where

\[q_i = \tan\left(\frac{y_i}{C}\right)\]

(using weighted least squares)
Fitting the model to VRC data (2)

- Choose weights for transformed data $u(q_i)$ so that

$$
\frac{y_i - \phi(z_i)}{u(y_i)} = \frac{q_i - P(z_i)}{u(q_i)}
$$

- Applying L’Hopital’s rule this leads to

$$
u(q_i) = \left| \frac{1}{C} \right| \sec^2 \left( \frac{y_i}{C} \right) u(y_i)$$
1 Specify the measurement model that relates the input quantities (phase of VRC at \( m \) frequencies) to the output quantities (capacitance coefficients - \( C_0, C_1, C_2, C_3 \)).

2 Assign a joint distribution to the input quantities of the measurement model. The input quantities are the VRC phase values measured at \( m \) frequencies. Assume phase values at different frequencies are uncorrelated.
3 Generate a large random sample of size \(n\) from the joint distribution of the input quantities (the ‘input sample’)

\[
\begin{bmatrix}
\phi_1(f_1) & \cdots & \phi_1(f_m) \\
\vdots & \ddots & \vdots \\
\phi_n(f_1) & \cdots & \phi_n(f_m)
\end{bmatrix}
\]

4 Apply the measurement model to each point of the input sample to obtain a large random sample from the distribution of the output quantities (the ‘output sample’)

\[
\begin{bmatrix}
C_0(1) & \cdots & C_3(1) \\
\vdots & \ddots & \vdots \\
C_0(n) & \cdots & C_3(n)
\end{bmatrix}
\]
5 Extract the required uncertainty information from the output sample (e.g. by sorting and trimming to give a coverage interval)
95% prediction interval for phase

- From Monte Carlo Sample of Capacitance coefficients, a sample of phase values at frequency $F$ can be obtained

$$
\begin{bmatrix}
\phi_1(F) \\
\vdots \\
\phi_n(F)
\end{bmatrix}
$$

- From this a 95% prediction interval for the phase at frequency $F$ can be derived
Results – GPC-7 (1)
### Results – GPC-7 (2)

<table>
<thead>
<tr>
<th></th>
<th>NPL capacitance coefficients derived from PIMMS</th>
<th>Manufacturer’s generic capacitance coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0 /10^{15} \text{ F}$</td>
<td>91.2 ± 1.7</td>
<td>87.2</td>
</tr>
<tr>
<td>$C_1 /10^{25} \text{ F Hz}^{-1}$</td>
<td>6.9 ± 5.6</td>
<td>17.0</td>
</tr>
<tr>
<td>$C_2 /10^{35} \text{ F Hz}^{-2}$</td>
<td>-5.4 ± 5.7</td>
<td>-15.1</td>
</tr>
<tr>
<td>$C_3 /10^{45} \text{ F Hz}^{-3}$</td>
<td>6.1 ± 1.8</td>
<td>8.9</td>
</tr>
</tbody>
</table>
Comparison of phase predicted by manufacturer’s capacitance coefficients with phase predicted by NPL capacitance coefficients (dashed curves define NPL 95% prediction interval)
• Maximum difference between the phase predicted by the manufacturer’s and NPL’s capacitance coefficients is about 0.7° at 16 GHz

• This is significant –
  – It corresponds to a length error of about 50 µm
  – It corresponds to a magnitude error of about

\[ u(\Gamma) = \sin(u(\phi)) = 0.012 \]
Effect of uncertainty in knowledge of open-circuit
Conclusion

- Traceable values have been obtained for open-circuit capacitance coefficients
- These can differ significantly from the manufacturer’s generic values
- Use of traceable rather than generic capacitance coefficients could result in reduced measurement uncertainties for DUTs