Bilaniuk and Wong

a) 112 point equation

$$c = 1.40238742 \times 10^{3} + 5.03821344\,T - 5.80539349 \times 10^{-2}\,T^{2} + 3.32000870 \times 10^{-4}\,T^{3} - 1.44537900 \times 10^{-6}\,T^{4} + 2.99402365 \times 10^{-9}\,T^{5}$$

b) 36 point equation

$$c = 1.40238677 \times 10^{3} + 5.03798765\,T - 5.80980033 \times 10^{-2}\,T^{2} + 3.34296650 \times 10^{-4}\,T^{3} - 1.47936902 \times 10^{-6}\,T^{4} + 3.14893508 \times 10^{-9}\,T^{5}$$

c) 148 point equation

$$c = 1.40238744 \times 10^{3} + 5.03836171\,T - 5.81172916 \times 10^{-2}\,T^{2} + 3.34638117 \times 10^{-4}\,T^{3} - 1.48259672 \times 10^{-6}\,T^{4} + 3.16585020 \times 10^{-9}\,T^{5}$$

Bilaniuk and Wong (1993,1996) converted Del Grosso and Mader's 1972 data to the 1990 International Temperature Scale and then produced three sets of coefficients depending on the number of temperature points which were converted and taken into account in their data fitting routines.

Range of validity: 0-100 ^{O}C at atmospheric pressure

Marczak

$$c = 1.402385 \times 10^{3} + 5.038813\,T - 5.799136 \times 10^{-2}\,T^{2} + 3.287156 \times 10^{-4}\,T^{3} - 1.398845 \times 10^{-6}\,T^{4} + 2.787860 \times 10^{-9}\,T^{5}$$

Marczak (1997) combined three sets of experimental measurements, Del Grosso and Mader (1972), Kroebel and Mahrt (1976) and Fujii and Masui (1993) and produced a fifth order polynomial based on the 1990 International Temperature Scale.

Range of validity: 0-95^{O}C at atmospheric pressure

Which Equation?

Differences between the Bilaniuk and Wong 148 point equation and Marczak's equation are of the order of 0.02 ms^{-1} or better at most temperatures, and either equation is suitable for the most accurate work. The Marczak equation has the advantage that all coefficients are expressed to 6 decimal places rather than the 8 decimal places of Bilaniuk and Wong.

Lubbers and Graaff's simplified equations

A simple equation for use in the temperature interval 15-35^{O}C

a) $c = 1404.3 + 4.7T - 0.04\,T^2$

Range of validity: 15-35^{O}C at atmospheric pressure

claimed accuracy - maximum error 0.18 ms^{-1}

b) $c = 1405.03 + 4.624T - 3.83 \times 10^{-2}\,T^2$

Range of validity: 10-40^{O}C at atmospheric pressure

Lubbers and Graaff (1998) produced these simple equations with a restricted temperature range for medical ultrasound applications, including tissue mimicking materials and test objects. Within the quoted temperature ranges they claim that the maximum error is approximately 0.18 ms^{-1} in comparisons with experimental data and more detailed equations such as Bilaniuk and Wong (1993,1996).

Belogol'skii, Sekoyan et al: speed of sound as a function of temperature and pressure

$$c(T,P) = c(T,0) + M_1(T)(P- 0.101325) + M_2(T)(P- 0.101325)^2 + M_3(T)(p - 0.101325)^3$$

$$c(T,0) = a_{00} + a_{10}T + a_{20}T^2 + a_{30}T^3 + a_{40}T^4 + a_{50}T^5$$
$$M_1(T) = a_{01} + a_{11}T + a_{21}T^2 + a_{31}T^3$$
$$M_2(T) = a_{02} + a_{12}T + a_{22}T^2 + a_{32}T^3$$
$$M_3(T) = a_{03} + a_{13}T + a_{23}T^2 + a_{33}T^3$$

T = temperature in degrees Celsius
P = pressure in megapascals

Range of validity: 0-40^{O}C, 0.1 - 60 MPa
This version uses the 1990 International Temperature Scale

Belogol'skii, Sekoyan et al (1999) made their own measurements of sound speed as a function of pressure and temperature and also used the equation of Bilaniuk and Wong (1996) for sound speed at atmospheric pressure.

Table of Coefficients

Coefficient	Numerical value
a_{00}	1402.38744
a_{10}	5.03836171
a_{20}	$-5.81172916 \times 10^{-2}$
a_{30}	$3.34638117 \times 10^{-4}$
a_{40}	$-1.48259672 \times 10^{-6}$
a_{50}	$3.16585020 \times 10^{-9}$
a_{01}	1.49043589
a_{11}	$1.077850609 \times 10^{-2}$
a_{21}	$-2.232794656 \times 10^{-4}$
a_{31}	$2.718246452 \times 10^{-6}$
a_{02}	$4.31532833 \times 10^{-3}$
a_{12}	$-2.938590293 \times 10^{-4}$
a_{22}	$6.822485943 \times 10^{-6}$
a_{32}	$-6.674551162 \times 10^{-8}$
a_{03}	$-1.852993525 \times 10^{-5}$
a_{13}	$1.481844713 \times 10^{-6}$
a_{23}	$-3.940994021 \times 10^{-8}$
a_{33}	$3.939902307 \times 10^{-10}$

Equations of state of water and steam

It is possible to calculate sound speeds from the thermodynamic equations of state for water and steam. For more information, please refer to the International Association for the Properties of Water and Steam (1995) and Saul and Wagner (1989). However, some acousticians may agree with Marczak (1997) that , "…the speed of sound in water can be calculated using the equation of state proposed by the International Association for the Properties of Steam (IAPS); the procedure, however, is labor consuming and leads to results of insufficient accuracy."

For some experimental data on the equation of state of water to 200^{O}C and 3.5 GPa, see Wiryana, Slutsky and Brown (1998).

Lakes: pure water or sea water?

Chen and Millero (1977) point out that lake water is by no means pure water, especially when precise pressure, volume and temperature properties are considered. They argue that the properties of lake water can be determined from the equation of state for sea water provided that the total mass fraction of dissolved salts in sea water and lake water are equated.

It is also important to recognise that water may vary in density owing to variations in its isotopic composition. Marczak (1997) quoting Kell (1977) argues that an increase of 1.5 p.p.m in density (caused by the presence of deuterium oxide) results in an increase of 1 p.p.m in the speed of sound. Variations in density due to variations in isotopic composition of water can reach 20 p.p.m - leading to sound speed variations of up to 13 p.p.m.

Dispersion

Most of the experimental results for sound speed in pure water which have been reported in the literature have been acquired at MHz frequencies only. All the empirical equations which are listed in this technical guide are based on this high-frequency data. There is little information on sound speed at much lower frequencies. For further discussion on dispersion and the Kramers-Kronig relationship between phase velocity and attenuation, please refer to O'Donnell, Jaynes and Miller (1981).

Speed of sound in sea-water

In you require information on the speed of sound in sea water, we have a web-page devoted to this topic.

Any comments or suggestions about further speed of sound equations?

Please contact:

Justin Ablitt
Tel: +44 (0)20 8943 6695
Fax: +44 (0)20 8943 6217
E-mail: justin.ablitt@npl.co.uk

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